The Bombay University

MATRICULATION EXAMINATION PAPERS

IN

MATHEMATICS

(From 1878 to 1904),

WITH FULL SOLUTIONS AND AN APPENDIX,

AND

The M. S. F. Papers

IN

MATHEMATICS,

WITH SOLUTIONS,

BY

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PREFACE.

This book contains full solutions of the last twenty years' Mathematical Papers of the Bombay Matriculation Examination. To these are added School Final Examination Papers in Mathematics up to date, with solutions of all the questions in Arithmetic and some harder ones in Algebra and Geometry.

The answers to questions of S. F. E. Algebra, of which solutions are not given, have been furnished by Mr. Pranjiwan V. Vassaivala, of the New High School, Bombay.

In some cases where several examples are similar in type, different methods of solution have been given and duplicate solutions of some examples have also been furnished. Some typical examples from various University Examination Papers with full solutions have been added at the end of the book in the form of an appendix, which, it is hoped, will be useful to students.

Notwithstanding great care bestowed in bringing out the work, errors may have crept in, and I shall feel highly obliged to those who will bring them to the notice of the Publishers. My warmest thanks are due to Mr. Erachshah F. B. Karani for much cordial assistance in the revision of proof-sheets.

Bombay, 20th August 1898.

LIST OF ABBREVIATIONS USED IN THE BOOK.

Ang. for Angle.
Cir. „ Circle.
Perp. „ Perpendicular.
‖m. „ Parallelogram
Prll. „ Parallel.
Rect. AB, BC for Rectangles contained by AB, BC.
Rt. for Right.
Sqr. for Square.
Str. „ Straight.
∵ „ Because.
∴ „ Therefore.
△ or *Tr.* for Triangle.
= for Equal to.
> „ Greater than.
< „ Less than.

University of Bombay.

MATRICULATION EXAMINATION.

1878-79.

WEDNESDAY, 20TH NOVEMBER.

T. COOKE, M.A., M.I., LL.D.
J. T. HATHORNTHWAITE, M.A.
GOVIND VITHAL KARKARAY, B.A.

[*The black figures to the right indicate full marks.*]

Arithmetic.

1. Seven men find a lump of gold weighing 13lbs. 7½ oz. Troy. What will be each man's share, supposing gold to be worth £3 17s. 10½d. per oz.? **5**

2. Simplify— **5**

$$1\frac{1}{11}-\frac{1-\frac{7}{22}}{2-\frac{1}{8}}+\frac{1\frac{2}{5}}{3\frac{1}{2}}-\frac{5\frac{3}{8}}{6\frac{1}{4}}\times\left\{\frac{1}{5}-\frac{\frac{1}{2}-\frac{1}{8}}{4\frac{3}{4}-3\frac{2}{9}}\right\}$$

3. Find the value of— **7**

$\cdot387$ of £8 16s. 3d. + $6\frac{1}{2}$ of $\frac{16}{65}$ of 7s. $8\frac{1}{2}$d. + $\frac{7}{11}$ of 1d.

4. What is the length of the edge of a cubical cistern which contains as much as a rectangular one whose edges are 154 ft. 11 ins., 70 ft. 7ins., and 53 ft. 1 in. **7**

5. In 1861, 3 towns had populations of 17,650, 19,600 and 18,760 respectively. In 1871 the population of the first had decreased 18%. that of second had increased 21% while the population of the third had increased by 4,691; find the change per cent. in the population of the third town.

6. A bankrupt has goods worth Rs. 9,750 ; and had they realized their full value, his creditors would have received 13 annas in the rupee, but $\frac{2}{5}$ths were sold at 17·5% and the remainder at 23·75% below their value. What sum did the goods fetch and what dividend was paid? 9

7. What sum will amount to £1,591 13s. 2·16d. in 3 years at compound interest; the interest for the first, second and third years being 3, 2, and 1% respectively. 9

8. Find the true discount on £2,750 due 2 years hence at $4\frac{1}{4}$ per cent. 6

9. If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys, working together for 8 days, earn £22, what will be the earnings of 8 men and 6 women working together for 10 days? 9

10. A person having a certain sum of money to invest finds that an investment in a railway stock bearing 5% interest at $117\frac{1}{2}$ will yield him £29 more annually than an investment in the three per cents. at $92\frac{1}{4}$. How much money has he to invest? 9

Algebra.

1. Remove the brackets from the expression— 5

$$\{m-n-\overline{3x-2y}\}-\{2x+5y-\overline{n+m}\}$$

and enclose the last three terms of the expression— $a-b+c-2d-1$ in a bracket with a negative sign.

2. Find the quotient which arises from dividing the third power of $10a^2$ by the square root of one million times a^{12}. 4

3. Extract the square root of— 6

$$x^4-6x^3y+13x^2y^2-12xy^3+4y^4.$$

4. Find the G. C. M. of— 7

$$21x^3-26x^2+8x \text{ and } 6x^2-x-2.$$

5. Find the L. C. M. of— 7

$x^2-1,\ x^2+2x-3$ and x^3-7x^2+6x.

6. Reduce to their simplest forms the following expressions— 9

(i) $a+\left(\frac{b-a}{1+ba}\right)\times\frac{a}{b}\div\left(1-a\frac{b-a}{1+ba}\right)$

(ii) $\dfrac{3\frac{1}{4}-\frac{1}{8}(x-2)}{1\frac{1}{12}+(x-\frac{3}{4})}$

7. Solve equations— 9

(i) $(x-a)(x-b)=ab-x^2$

(ii) $\sqrt{x+4}+\sqrt{2x+9}=\sqrt{3x+25}$

8. Solve the equation—

$$\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+1\frac{22}{35}$$

$$xy=\frac{3}{3}\frac{2}{4}(y-x)$$

9. From a certain sum of money I took away one-third part and put in its stead Rs. 50; from the sum thus increased I took away one-fourth part and put in its stead Rs. 70. I then found I had Rs. 120: what was the original sum? 10

10. A number consists of two digits whose sum is 8; another number is obtained by reversing the digits. If the product of these two is 1,855, find the number. 10

Euclid.

1. Define *figure, circle, right angle, scalene triangle, oblong, rhombus, gnomon, segment of circle, sector of circle.* 5

What do you mean by an *axiom* and what by a *postulate*? Distinguish them carefully.

2. If at a point in a straight line two other straight lines on opposite sides of it make the adjacent angles together equal to two right angles, these two straight lines shall be in one and the same straight line. 5

3. The opposite sides and angles of a parallelogram are equal to one another. Shew that the sum of the perpendiculars from an interior point upon the four sides is constant. 6

4. Bisect a triangle by a straight line drawn from a given point in the base. 11

5. If a straight line be divided into two equal parts and also into two unequal parts the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line. 5

6. Describe a square that shall be equal to a given rectilineal figure. 5

7. AB is a diameter of a circle. C any point in its circumference, AC, BC produced meet the tangents at B and A in D and E and the tangent at C meets the same tangents in F and G: show that FG is half of BD and AE taken together. 11

8. Similar segments of circles on equal straight lines are equal to one another. 11

In the base BC of an isosceles triangle ABC, any point D is taken: show that the circle described about the triangles ABD and ACD are equal.

9. If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them shall be equal to the rectangle contained by the segments of the other. [Prove that case only in which one straight line passing through the centre, cuts the other, which does not pass through the centre, not at right angles.] 5

10. Describe a circle about a given triangle. 11

Having given three points not in the same straight line, describe a circle such that all the tangents to it from the given points shall be equal.

SOLUTIONS.

Arithmetic.

1. 13 lbs. $7\frac{1}{2}$oz. $= \frac{327}{2}$oz.

$£3$ $17s.$ $10\frac{1}{2}d. = £\frac{623}{160}$, the cost of 1 oz.

$$1 \text{ oz} : \frac{327}{2} \text{ oz} :: £\frac{623}{160} = £\frac{327 \times 623}{2 \times 160}$$

$$7 \text{ men} : 1 \text{ man} :: £\frac{327 \times 623}{2 \times 160} = £\frac{327 \times 89}{2 \times 160}$$

$£90$ $18s. 11\frac{1}{4}d.$ the share of one man. *Ans.*

2. $$\frac{12}{11} - \frac{15}{22} \times \frac{3}{5} + \frac{7}{5} \times \frac{2}{7} - \frac{45}{8} \times \frac{4}{25}\left(\frac{1}{5} - \frac{1}{6} \div \frac{55}{36}\right)$$

$$= \frac{12}{11} - \frac{9}{22} + \frac{2}{5} - \frac{9}{10}\left(\frac{1}{5} - \frac{6}{55}\right)$$

$$= \frac{12}{11} - \frac{9}{22} + \frac{2}{5} - \frac{9}{110} = \frac{110}{110} = 1. \quad \textit{Ans.}$$

3. $$\frac{387}{1{,}000} \times £\frac{141}{16} + \frac{13}{2} \times \frac{16}{65} \times \frac{185}{24}s. + \frac{7}{11}d.$$

$$= £\frac{54{,}567}{16{,}000} + \frac{37}{3}s. + \frac{7}{11}d.$$

$$= £3\ 8s.\ 2\frac{101}{200}d. + 12s.\ 4d. + \frac{7}{11}d. = £4\ 0s.\ 7\frac{311}{2{,}200}d. \quad \textit{Ans.}$$

4. The contents of the rect. cistern.

$$= \left(\frac{1859}{12} \times \frac{847}{12} \times \frac{637}{12}\right) \text{ cub. ft.}$$

∴ the length of the edge of the cubic cistern

$$= \sqrt[3]{\frac{1{,}859}{12} \times \frac{847}{12} \times \frac{637}{12}}$$

$$= \sqrt[3]{\frac{11 \times 13 \times 13}{12} \times \frac{11 \times 11 \times 7}{12} \times \frac{7 \times 7 \times 13}{12}}$$

$$= \sqrt[3]{\frac{(11)^3 \times (7)^3 \times (13)^3}{(12)^3}} = \frac{11 \times 7 \times 13}{12} = \frac{1{,}001}{12} \text{ ft.}$$

$=$ 83ft. 5in. *Ans.*

5. 100 : 17,650 :: 18=3,177 decrease in the population of the first town.

100 : 19,600::21=4,166 increase in the population of the second town.

∴ increase in the total population of the three towns =4,166+4,692−3,177=5,630.

The original population =17650+19600+18760=56010

∴ 56,010 : 100::5,630=$\frac{56,300}{5,601}$=$10\frac{290}{5,601}$ increase %. *Ans.*

6. Rs. 9,750×$\frac{2}{5}$=Rs. 3,900 worth goods sold at 17·5 % below their value, *i. e.*, at 82·5 % of their price.

∴ 9,750−3,900=Rs. 5,850 worth goods sold at 23·57 % below their value, *i. e.*, at 76·25 per cent. of their price.

Rs. 100 : Rs. 3,900:: Rs. 82½= Rs. $\frac{6,435}{2}$

Rs. 100 : Rs. 5,850 :: Rs. 76¼= Rs. $\frac{35,685}{8}$

Rs. $\frac{6,435}{2}$+Rs. $\frac{35,685}{8}$=Rs. $\frac{61,425}{8}$= Rs. 7,678⅛ the goods fetched. *Ans.*

Had he realised the full value of the goods, *i.e.*, Rs. 9,750 he would have paid 13 annas in the rupee.

∴ Re. $\frac{13}{16}$: Rs. 9,750 :: Re. 1=Rs. 12,000 amount of debt but the creditors were paid only Rs. $\frac{61,425}{8}$

∴ Rs. 12,000 : Re. 1::Rs. $\frac{61,425}{8}$=Re. $\frac{247}{3,840}$

=10 annas $2\frac{1}{30}$ pies. *Ans.*

7. £100 : £1 ::£103=£1·03 amount of £1 for the 1st year.

£100 : £1·03 ::£102=£1·0506 „ „ £1 „ 2nd year.

£100: £1·0506::£101=£1·061106„ „ £1 „ 3rd year.

£1591 13*s.* 2·16*d.*=£1591·659.

∴£1·061106. amount : £1,591·659. amount :: £1 sum =£1,500 sum. *Ans.*

8. $4\frac{1}{4}\times 2=$ £$8\frac{1}{2}$ interest on £100 for 2 years.

£$108\frac{1}{2}$: £2,750 :: £$8\frac{1}{2}$ discount. = £215 8*s*. $9\frac{15}{217}$. *Ans.*

9. 4 men = 7 women = 14 boys ∴ 2 men = 7 boys.

6 men + 10 women + 14 boys = (21 + 20 + 14) or 55 boys.

8 men + 6 women = (28 + 12) or 40 boys.

$\left.\begin{array}{l}\text{Boys } 55 : 40 \\ \text{Days } 8 : 10\end{array}\right\}$ :: £22 = £20. *Ans.*

10. Let £$92\frac{1}{4}$ be invested; then

£$117\frac{1}{2}$: £$92\frac{1}{4}$ :: £5 = £$\frac{369}{94}$ interest on the first investment.

£$92\frac{1}{4}$: £$92\frac{1}{4}$ :: £3 = £3 interest on the second investment.

∴ £$\frac{369}{94}$ − £3 = £$\frac{87}{94}$ difference in the two incomes.

∴ £$\frac{87}{94}$: £29 :: £$92\frac{1}{4}$ = £2,890 10*s*. *Ans.*

Algebra.

1. $(m-n-3x+2y)-(2x+5y-n-m)$

$= m-n-3x+2y-2x-5y+n+m.$

$= 2m-5x-3y.$ *Ans.*

$a-b-(2d-c+1).$ *Ans.*

2. $\dfrac{(10a^2)^3}{\sqrt{1000000a^{12}}}=\dfrac{1000a^6}{1000a^6}=1.$ *Ans.*

3.
$$\begin{array}{r|l}
 & x^4-6x^3y+13x^2y^2-12xy^3+4y^4 \;\big|\; x^2-3xy+2y^2 \\
 & x^4 \\
2x^2-3xy & -6x^3y+13x^2y^2 \\
 & -6x^3y+9x^2y^2 \\
2x^2-6xy+2y^2 & 4x^2y^2-12xy^3+4y^4 \\
 & 4x^2y^2-12xy^3+4y^4
\end{array}$$

$x^2-3xy+2y^2.$ *Ans.*

4. $21x^3-26x^2+8x$

$=x(21x^2-26x+8)=x(21x^2-14x-12x+8)$

$=x\{7x(3x-2)-4(3x-2)\}=x(3x-2)(7x-4)$

$6x^2-x-2=6x^2-4x+3x-2=(2x+1)(3x-2)$

Hence the G. C. M. $=3x-2.$ *Ans.*

5. $$x^2-1=(x+1)(x-1)$$
$$x^2+2x-3=(x+3)(x-1)$$
$$x^3-7x^2+6x=x(x^2-7x+6)=x(x^2-6x-x+6)$$
$$=x(x-6)(x-1)$$
$\therefore$ the L. C. M. of the three expressions $=$

$x(x-1)(x+1)(x+3)(x-6)$. *Ans.*

6. (i) $$a+\frac{b-a}{1+ba}\times\frac{a}{b}\div\left(1-\frac{ab-a^2}{1+ba}\right)$$
$$=a+\frac{b-a}{1+ba}\times\frac{a}{b}\div\frac{1+a^2}{1+ba}=a+\frac{b-a}{1+ba}\times\frac{a}{b}\times\frac{1+ba}{1+a^2}$$
$$=a+\frac{a(b-a)}{b(1+a^2)}=\frac{ab+a^3b+ab-a^2}{b(1+a^2)}$$
$$=\frac{a^3b+2ab-a^2}{b(1+a^2)}=\frac{a(a^2b+2b-a)}{b(1+a^2)}$$ *Ans.*

(ii) $$\frac{\frac{13}{4}-\frac{x}{3}+\frac{2}{3}}{\frac{13}{12}+x-\frac{3}{2}}=\frac{\frac{47}{12}-\frac{x}{3}}{x-\frac{5}{12}}=\frac{47-4x}{12}\times\frac{12}{12x-5}$$
$$=\frac{47-4x}{12x-5}$$ *Ans.*

7. (i) $(x-a)(x-b)=ab-x^2$

$\therefore\ x^2-ax-bx+ab=ab-x^2\ \therefore 2x^2-ax-bx=0$

$\therefore\ x(2x-a-b)=0\ \therefore\ x=0$ or

$2x-a-b=0\ \therefore 2x=a+b\ \therefore\ x=\frac{a+b}{2}$. *Ans.*

(ii) $\sqrt{x+4}+\sqrt{2x+9}=\sqrt{3x+25}$

Square both sides.

$\therefore\ x+4+2x+9+2\sqrt{(x+4)(2x+9)}=3x+25$

$\therefore\ 2\sqrt{(x+4)(2x+9)}=12\ \therefore\ \sqrt{(x+4)(2x+9)}=6$

Squaring both sides, we have $(x+4)(2x+9)=36$

$\therefore\ 2x^2+17x+36=36\ \therefore 2x^2+17x=0$

$\therefore\ x\ (2x+17)=0$

$\therefore\ x=0$ or $2x+17=0\ \therefore\ x=-\frac{17}{2}$. *Ans.*

8. $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{57}{35}$.............. ......(i)

$xy=\frac{35}{34}(y-x)$ .... (ii)

$\therefore\ \frac{4}{x}-\frac{5}{y}=\frac{1}{y}+\frac{1}{x}+\frac{57}{35}$ or $\frac{3}{x}-\frac{6}{y}=\frac{57}{35}$

$\therefore \frac{1}{x}-\frac{2}{y}=\frac{19}{35}$.................. .... (i)

Now divide (ii) by xy

$\therefore\ 1=\frac{35}{34x}-\frac{35}{34y}$..................... . ... (ii)

Multiply the 1st by $\frac{35}{34}$ and subtract it from the 2nd.

$\therefore \frac{35}{34x}-\frac{35}{34y}=1$.......................... (ii)

$\frac{35}{34x}-\frac{70}{34y}=\frac{19}{34}$.......................... (i)

$\therefore\ \frac{35}{34y}=\frac{15}{34}\quad \therefore \frac{7}{y}=3\quad \therefore\ y=\frac{7}{3}$. *Ans.*

Substituting the value of y in (i), we get

$\frac{1}{x}-2\times\frac{3}{7}=\frac{19}{35}\therefore \frac{1}{x}=\frac{19}{35}+\frac{6}{7}=\frac{49}{35}=\frac{7}{5}\therefore 7x=5\ \therefore x=\frac{5}{7}$. *Ans.*

9. Let x rupees be the original sum.

Rs. $\frac{x}{3}$ is taken away ;

$\therefore$ Rs. $\frac{2x}{3}$ remains, to which is added Rs. 50.

$\therefore$ Rs. $\frac{2x}{3}$ + 50 is the sum thus increased. From this $\frac{1}{4}$ is taken away $\therefore$ Rs. $\frac{3}{4}(\frac{2x}{3}+50)$ remains, which, with Rs. 70, gives Rs. $\frac{3}{4}(\frac{2x}{3}+50)+70$. And this is equal to Rs. 120

$\therefore\ \frac{3}{4}(\frac{2x}{3}+50)+70=120$

$\therefore \frac{x}{2}+\frac{75}{2}=50\ \therefore\ \frac{x}{2}=50-\frac{75}{2}$

$x=100-75=25.$

$\therefore$ Rs. 25 is the original sum. *Ans.*

10. Let x be the digit in the tens' place and y the digit in the units' place.

$\therefore$ the number formed $= 10x+y$ and the number formed by reversing the digits $= 10y+x$

The sum of the digits $= 8 \;\therefore\; x+y = 8$ (i)

and $(10x+y)(10y+x) = 1855$ (ii)

From (i) we have $x = 8-y$.

Substitute the value of x in (ii). Hence we get

$\{10(8-y)+y\}\{10y+(8-y)\}=1855$ (ii)

$\therefore (80-9y)(9y+8) = 1855$

$\therefore 720y-81y^2+640-72y = 1855$

$\therefore 81y^2-648y+1215 = 0 \therefore y^2-8y+15 = 0$

$\therefore y^2-5y-3y+15 = 0 \therefore (y-3)(y-5) = 0$

$\therefore y = 3$ *or* 5. Hence $x = 5$ or 3

$\therefore$ the number $= 53$ or 35. *Ans.*

Euclid.

1. An *axiom* is a theorem, the truth of which is admitted without proof. A *postulate* is a problem, the possibility of which is admitted to be self-evident, and to require no proof. "An *axiom* is a truth of so self-evident a character that it cannot be made clearer by any attempted demonstration of it. In the *postulates* we demand as a basis of reasoning that the geometrical operations they refer to, shall be considered to be perfectly done. This is to prevent our demonstrations being impugned on account of imperfect drawing. They also demand that no proof shall be required of the operations stated being effected."

2. Euclid I., 14.

(Fig. 1) Let x and y be any two points from which there fall xQ, xG, xR, xH and yF, yL, yZ, yM prep. to the four sides of pm. $ABCD$.

$\because$ xQ and xR are prep. to the parallel str. lines CB and D $\therefore$ xQ is in the same str. line with xR, *i.e.*, QxR str. line. Similarly, GxH, FyZ, LyM are str. lines. Be-

cause the angles QRZ and $RZF = 2$ rt. angles, $\therefore$ QR is prll. to FZ (I. 28) and RZ is parallel to QF. $\therefore$ $QFZR$ is a pm. $\therefore$ $RQ = FZ$. Similarly,
$GH = LM$; $\therefore$ $GH + QR = LM + FZ$, *i.e.*, $xG + xH + xQ + xR = yL + yF + yM + yZ$. Similarly, the sum of the perpendiculars drawn from any other point will be the same $\therefore$ it is constant.

3. Euclid I., 34.

4. (Fig. 2). Let ABC be the given triangle and D the given point point in AB. Bisect BC in E (I. 10). Join DE. Through A draw AF parallel to DE (I 31).
Join AE, DF. Then because DE is prll.
to AF, $\therefore$ tr. ADE = tr. DEF (I. 37)
to each of these equals add tr. BDE.
$\therefore$ tr. ABE = tr. DBF.
but tr. $ABE = \frac{1}{2}$ tr. ABC. (I. 38)
$\therefore$ tr. $DBF = \frac{1}{2}$ tr. ABC.

5. Euclid II., 5.

6. Euclid II., 14.

7. (Fig. 3). $FC = FB$. (III., 17 cor.) $\therefore$ ang. FBC = ang. FCB (I. 5), but ang FCB. =
ang. GCE. (I. 15), and ang. FBC = ang. GEC (I. 29) because AE and BD are parallel, being at right angles to the diameter.
$\therefore$ ang. ECG = ang. CEG $\therefore$ $GE = GC$ (I. 6) and $GC = GA$ (III. 17 cor.) $\therefore$ $GC = \frac{1}{2}$ AE. Similarly $CF = \frac{1}{2} DB$ $\therefore$ $FG = \frac{1}{2}(BD + AE)$.

8. Euclid III. 24.

(Fig. 4). $AB = AC$ (*hyp.*). $\therefore$ ang. ABD = ang. ACD (I. 5) $\because$ the segments ABD and ACD are similar (III. def. 15) $\therefore$ they are equal (III., 24)
Hence the circle $ABDE$ = circle $ACDF$.

10. Euclid IV., 5.

(Fig. 5). Let the given points A, B, C be equidistant from one another. Join AB, BC, CA. Then ABC is an equila-

teral triangle. Describe a circle about ABC (IV. 5) and draw tangents at A, B, C, *viz.*, DE, EF, FD respectively. Then the resulting figure shall be equilateral, *i. e.*, all the tangents shall be equal.

Each of the angs. EAB, EBA=ang. ACB (III. 32) =an angle of an equilateral tr.$=\frac{1}{3}$ of two right angles.

$\therefore$ ang. $E=\frac{1}{3}$ of two right angles (I, 32) $\therefore$ also the angs. at D and F are each $\frac{1}{3}$ of two right. angs. $\therefore$ tr. DEF is equiangular.

$\therefore$ it is equilateral (I. 6, cor), *i. e.*, the tangents DE, EF, FD from the given points A, B, C are equal.

1879-80

WEDNESDAY, 19TH NOVEMBER.

THEODORE COOKE, M.A., M.I., LL.D.

GOVIND VITHAL KURKARAY, B.A.

[*The figures to the right indicate full marks.*]

Arithmetic and Algebra.

1. Add the following numbers: Eighty four thousand three hundred and one; nine hundred and thirty-three thousand; forty seven million six thousand three hundred; and subtract from the result two million eighty-one thousand and eighty. 4

2. Explain the terms—*Measure*, *Common Measure* and *Greatest Common Measure* and prove that every common measure of dividend and divisor is a measure of the remainder. 6

3. Find the value of—

$\cdot 45$ of £1 3*s*. 9*d*. $+\cdot 2\dot{5}\dot{7}$ of £11 5*s*. 6*d*. $+\cdot 3125$ of £5 6

4. Find the value of—

$$\frac{\frac{7}{10}-\frac{2}{5}}{\frac{9}{12}+\frac{1}{3}}\div\frac{\frac{1}{3}}{9\frac{1}{2}};\text{ and also of }\frac{1}{2}+\frac{2}{9}-\frac{4}{15}+\frac{5}{18}$$ 6

5. If by selling wine at Rs. 6 per gallon I lose 25 per cent., at what rate must I sell it to gain 25 per cent. ? 8

6. A person borrows £130 on the 5th of March, and pays back £ 132 10*s*. 6*d*. on the 18th October : find the rate of interest charged. 7

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7. Reduce to the simplest form— 4

$$a^2+2d^2-(2e^2-b^2)-\{(d^2-e^2-c^2)+(d^2-e^2)\}$$

8. Find the square root of— 6

$$\frac{x^2}{y^2}+\frac{y^2}{x^2}-\left(\frac{x}{y}+\frac{y}{x}\right)+\frac{9}{4}.$$

9. Find the G. C. M. of— 6

$$2x^2-xy-6y^2 \text{ and } 3x^2-8xy+4y^2.$$

10. Add together— 6

$$\frac{x(x+3)}{(x+1)(x+2)} \text{ and } \frac{2}{3x(x+2)}$$

And find the value of the result when $x=\frac{1}{2}$.

11. Find the value of x and y from the equations— 7

$$ax+by=c^2 ; \frac{a}{b+y}-\frac{c}{a+x}=0$$

12. *A* and *B* invest equal sums in speculation ; *A* gains Rs. 1,000 and B loses so much that his money is now two-thirds of *A*'s money. If each gave the other one-third of his present sum, *B*'s loss would be diminished by one-half. What did each adventure ? 9

WEDNESDAY, 19th NOVEMBER.

(2 P.M. to 5. P.M.)

Euclid.

1. The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another. 7

2. *ABCD* is a parallelogram. A straight line *EF*, drawn parallel to the diagonal *AC*, meets *AD*, *DC* or these produced in *E* and *F* respectively : show that the triangle *ABE* is equal to the triangle *BCF*. 10

3. A, B, C, are three points in a straight line such that AB is equal to BC. Show that the sum of the perpendiculars from A and C on any straight line which does not pass between A and C is double the perpendiculars from B on the same straight line. 10

4. Describe a square that shall be equal to a given rectilineal figure. 7

5. In any triangle the square on the two sides are together double of the squares on half the base and on the line joining the middle point of the base with the opposite angle. 15

6. Draw a straight line from a given point either without or in the circumference which shall touch a given circle. 7

7. If from any point without a circle there be drawn two straight lines, one of which cuts the circle and the other meets it, and if the rectangle contained by the whole line which cuts the circle and the part of it without the circle be equal to the square on the line which meets the circle, the line which meets the circle shall touch it. 8

8. Inscribe a circle in a given triangle. 6

9. Give without proof the construction for inscribing an equilateral and equiangular pentagon in a given circle. 5

SOLUTIONS.

Arithmetic and Algebra.

1. 84,301+933,000+47,006,300=48,023,601. *Ans.*
48,023,601 − 2,081,080=45,942,521. *Ans.*

2. The *Measure* of a number is the number which divides the given number exactly.

The *Common Measure* of two or more given numbers is a which divides each of them without a remainder.

The *Greatest Common Measure* of two or more given numbers is the greatest number which divides each of them without a remainder.

Every common measure of dividend and divisor is a measure of the remainder: for, let 20 be the divisor and 32 be the dividend, then 4 is the common measure of 20 and 32. $\therefore$ 32 − 20 = (8 times 4) − (5 times 4) = (8 − 5) times 4 = 3 times 4; thus 4 measures (32 − 20).

Or thus:—Let a be contained m times in x and n times in y: then $ma=x$ and $na=y$, $\therefore$ $x-y = ma - na = a(m-n)$ *i. e.*, a is contained $m-n$ times in $x-y$, or a measures $x-y$ by the units in $m-n$.

3. $\cdot 45 = \frac{9}{20}$; £1 3*s*. 9*d*. = £$\frac{19}{16}$; $\cdot 2\dot{5}\dot{7} = \frac{17}{66}$ and

£11 5*s*. 6*d*. = £$\frac{451}{40}$; ·3125 of £5 = £1 11*s*. 3*d*.

$\therefore$ £$\frac{9}{20}\times\frac{19}{16}$ = £$\frac{171}{320}$ = 10*s*. $8\frac{1}{4}$*d*., $\frac{17}{66}\times\frac{451}{40}$ = £ $\frac{7,667}{2,640}$ £2 18*s*. 1*d*.

$\therefore$ 10*s*. $8\frac{1}{4}$*d*. + £2 18*s*. 1*d*. + £1 11*s*. 3*d* = £5 0*s*. $0\frac{1}{4}$*d*. *Ans.*

4. $\dfrac{\frac{1}{30}}{\frac{57}{60}} \div \dfrac{\frac{1}{3}}{\frac{19}{2}} = \frac{1}{30}\times\frac{60}{57}\times\frac{19}{2}\times\frac{3}{1} = 1.$ *Ans.*

$\frac{1}{2}+\frac{2}{9}-\frac{4}{15}+\frac{5}{18} = \frac{45+20-24+25}{90} = \frac{66}{90} = \frac{11}{15}.$ *Ans.*

5. Rs. 75 S.P. : Rs. 6 S.P. :: Rs. 100 C.P. = Rs. 8 C.P.
Rs. 100 C.P. : Rs. 8 C.P. :: Rs. 125 S.P. = Rs. 10 S.P. *Ans.*

6. Interest for (26 + 30 + 31 + 30 + 31 + 31 + 30 + 18) or 227 days.
£132 10*s*. 6*d*. — £130 = £2 10*s*. 6*d*. = £$2\frac{21}{40}$ = £$\frac{101}{40}$

$$\left.\begin{array}{lcl}\text{£}130 & : & \text{£}100 \\ \text{Days } 227 & : & \text{Days } 365\end{array}\right\} :: \text{£}\frac{101}{40} = 3\frac{1453}{11804}.\ \textit{Ans.}$$

7. The expression

$$= a^2+2d^2-2c^2+b^2-\{d^2-e^2-c^2+d^2-e^2\}$$
$$= a^2+2d^2-2e^2+b^2-d^2+e^2+c^2-d^2+e^2$$
$$= a^2+b^2+c^2. \quad \textit{Ans.}$$

8. The expression

$$= \left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+2\right)-\left(\frac{x}{y}+\frac{y}{x}\right)+\frac{1}{4}$$
$$= \left\{\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2+2\left(\frac{x}{y}\right)\left(\frac{y}{x}\right)\right\}-\left(\frac{x}{y}+\frac{y}{x}\right)+\frac{1}{4}$$
$$= \left(\frac{x}{y}+\frac{y}{x}\right)^2-\left(\frac{x}{y}+\frac{y}{x}\right)+\frac{1}{4}$$
$$= \left\{\left(\frac{x}{y}+\frac{y}{x}\right)-\frac{1}{2}\right\}^2=\left(\frac{x}{y}-\frac{1}{2}+\frac{y}{x}\right)^2$$

$\therefore$ the square root $=\frac{x}{y}-\frac{1}{2}+\frac{y}{x}$. *Ans.*

9. $2x^2-xy-6y^2=2x^2-4xy+3xy-6y^2$
$=(x-2y)(2x+3y)$
$3x^2-8xy+4y^2=3x^2-6xy-2xy+4y^2$
$=(3x-2y)(x-2y)$

Hence the G.C.M. $=x-2y$. *Ans.*

10. $$\frac{x(x+3)}{(x+1)(x+2)}+\frac{2}{3x(x+2)}=\frac{3x^2(x+3)+2(x+1)}{3x^2(x+1)(x+2)}$$
$$=\frac{3x^3+9x^2+2x+2}{3x^3+9x^2+6x}$$

Now $x=\frac{1}{2}$, $\therefore$ the expression

$$=\frac{\frac{3}{8}+\frac{9}{4}+1+2}{\frac{3}{8}+\frac{9}{4}+3}=1. \quad \textit{Ans.}$$

11. $ax+by=c^2$ .... (i)

$a(a+x)-c(b+y)=0$ .... (ii)

i.e. $a^2+ax-bc-cy=0 \therefore ax-cy=bc-a^2$...(ii)

Substracting (ii) from (i), we get

$$by+cy=c^2+a^2-bc$$

$\therefore \quad y(b+c)=c^2+a^2-bc \therefore y=\frac{c^2+a^2-bc}{b+c}$. *Ans.*

Substituting the value of y in (i), we get

$$ax+b\,\frac{(a^2+c^2-bc)}{b+c}=c^2\ \therefore ax=c^2-\frac{b(a^2+c^2-bc)}{b+c}$$

$$=\frac{bc^2+c^3-ba^2-bc^2+b^2c}{b+c}=\frac{c^3-a^2b+b^2c}{b+c}$$

$$\therefore\ x=\frac{c^3-a^2b+b^2c}{a(b+c)},\quad \textit{Ans.}$$

12. Let Rs. x be the investment of each, and Rs. y the loss to B.

Then A has Rs. $x+1{,}000$ and B has Rs. $x-y$.

$\therefore$ by the question $x-y=\frac{2}{3}(x+1{,}000)$.........(i)

B gives $\frac{1}{3}$ of $(x-y)$ $\therefore \frac{2}{3}(x-y)$ is left with him, and he obtains $\frac{1}{3}(x+1{,}000)$ from A $\therefore$ B now possesses $\frac{2}{3}(x-y)+\frac{1}{3}(x+1000)$, and this sum, by the question, is equal to $x-\frac{y}{2}$

$\therefore\ \frac{2}{3}(x-y)+\frac{1}{3}(x+1000)=x-\frac{y}{2}$......(ii)

$\therefore\ 4x-4y+2x+2{,}000=6x-3y\ \therefore\ y=$ Rs. 2,000

$\therefore\ x=$ Rs. 8,000, $\therefore$ Rs. 8,000 investment of each. *Ans.*

Euclid.

1. Euclid I., 43.

2. (Fig 5*a*.) The parallelograms $ACFG$ and $ACHE$ are equal $\because$ they are on the same base AC and between the same parallels (I. 35), Again, the parallelogram $ACFG=2$ tr. BCF (I. 41), and the parallelogram $ACHE=$ twice tr. AEB (I. 41) $\therefore$ tr. $AEB=$tr. BCF (ax. 7).

3. (Fig 6.) Let XZ, ZY be the projections of AB, BC on any straight line PQ. Through A draw AHK parallel to PQ meeting BZ, CY in H and K respectively. Now AX, BZ, CY are prll. $\because$ they are perpendicular to PQ (I. 28).

$\therefore$ the figures XH and HY are parallelograms.

$\therefore\ AX=HZ=KY$ (I. 34)

$\therefore\ AX+KY=2HZ.$

But through B, the middle point of AC, a side of the tr. ACK, BH has been drawn parallel to the side CK $\therefore$ BH bisects AK and is half of CK.

$\therefore$ $AX+KY+CK=2\,(HZ+BH)$ i.e. $AX+CY=2BZ$.

4. Euclid II. 14.

5. (Fig. 7.) Let ABC be a tr. and AD the line drawn from the vertex A to the bisection point D of the base BC. From A draw AE perpendicular to BC (I. 12).

Then, in the obtuse-angled tr. ABD, the square on AB exceeds the squares on AD, DB by twice the rect. BD, DE (II. 12) and in the acute-angled tr., ADC, the square on AC is less than the squares on AD, DC by twice the rect. CD. DE. (II. 13); but the rect. BD, DE=rect. CD, DE

$\therefore$ the squares on AB, AC are double of the squares on AD, DB ($\because$ $DB=DC$).

6. Euclid III. 17.

7. Euclid III. 37.

8. Euclid IV. 4.

9. Euclid IV. 11.

1880-81.

Arithmetic and Algebra.

S. Cooke, M.A., F.G.S., Assoc. M. Inst. C. E.
The Rev. D. Mackichan, M.A., B.D.

1. Simplify the following expressions:—

$$2+\frac{1}{5+\frac{1}{1\frac{1}{2}}}\,; \qquad \frac{4{\cdot}5\times 2{\cdot}\dot{3}}{5{\cdot}341\dot{3}}\times 2\,\frac{253}{875},$$

and add together the results.

2. Three boys agree to start together and run, until all come together again, round a circular court 15 yards in circumference. One runs at the rate of six, the second seven, and the third eight miles an hour. In how many seconds will the race end? 10

3. If 3 soldiers or 10 coolies can dig 155 cubic feet of earth in 5 days, how many coolies must be employed to assist 7 soldiers in removing 600 cubic feet of earth so as to get it done in 4 days. 12

4. In what time will Rs. 2,250 amount to Rs. 2,565 at 7 per cent. per annum. 10

5. A merchant sells a lakh of Rupees out of the 4 per cents. at 16 discount, and invests the proceeds while exchange is at 2*s.* 1*d.* in the 3 per cent. consols at 96. What income does he derive therefrom? 12

6. Find the continued product of the following quantities:— 6

$$a+b+c,\ -a+b+c,\ a-b+c,\ a+b-c;$$
$$a+b\sqrt{-1},\quad a-b\sqrt{-1}.$$

7. From the sum of the squares of $\frac{1}{a-b}$ and $\frac{1}{a+b}$, substract the square of $\frac{2b}{a^2-b^2}$. 6

8. Find the G. C. M. of the quantities forming the numerator and denominator of the fraction $\frac{x^4-15x^2+28x-12}{2x^3-15x+14}$ and reduce the fraction to its lowest terms. 8

9. Find the value of x, y, and z in the following set of simultaneous equations:— 12

$$x+y+z=6$$
$$3x-y+2z=7$$
$$4x+3y-z=7$$

10. Determine the *time* between ten and twelve o'clock, at which the hour and minute hands of a common clock are exactly together. 10

11. One student said to another: 'If you give me half your money I shall have a hundred rupees.' The other replied: 'I shall have a hundred rupees if you give me a third of your money.' How much had each? 8

Euclid.

1. State the four cases in which Euclid proves that two triangles are equal in every respect. 6

Two isosceles triangles stand on opposite sides of the same base: show that the straight line joining their vertices bisects their common base at right angles.

2. Define parallelogram, square and rhombus. 8

Prove that the opposite sides and angles of a parallelogram are equal to one another and that the diagonal bisects it.

Show that the diagonals of a rhombus bisect each other at right angles.

3. Find a point in a straight line such that straight lines drawn to it from two given points on the same side of the given straight line may make equal angles with the given straight line. Show that the sum of these two lines is less than that of any other pair of lines drawn by joining the two given points to any other point in the given straight line. 10

4. If a straight line be divided into two parts the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts. 10

In a right angled triangle, of which the right angle is the vertex, a perpendicular is let fall from the right angle on the base: prove that the square on this perpendicular is equal to the rectangle contained by the segments into which it divides the base.

5. From a given point without a circle draw a straight line which will touch the circle. 9

Show that two tangents can be drawn from the given point to the circle and that they are equal to one another.

6. If two circles touch one another externally the straight line joining their centres passes through the point of contact. 10

Two equal circles touch each other externally and through the point of contact chords are drawn, one to each circle, at right angles to each other: prove that the straight line joining the other extremities of these chords is equal and parallel to the straight line joining the centres of the circles.

7. If from any point without a circle two straight lines be drawn one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle and the part of it without the circle is equal to the square on the line which touches it. 11

Hence show that if two circles intersect each other their common tangent is bisected by the line joining the points of intersection produced.

8. Describe the circle about a given triangle. 11
Prove that, if the centre of this circle coincides with the centre of the inscribed circle, the triangle is equilateral.

SOLUTIONS.

1 $$2+\frac{1}{5+\frac{1}{1\frac{1}{2}}}=2+\frac{1}{5+\frac{1}{\frac{3}{2}}}=2+\frac{1}{5+\frac{2}{3}}$$

$$=2+\frac{3}{17}=2\tfrac{3}{17}.\quad \textit{Ans.}$$

$$\frac{4\frac{5}{10}\times 2\frac{3}{9}}{5\frac{3072}{9000}}\times\frac{2{,}003}{875}=\frac{4\frac{1}{2}\times\frac{7}{3}}{\frac{2{,}003}{375}}\times\frac{2{,}003}{875}$$

$$=\frac{9}{2}\times\frac{7}{3}\times\frac{375}{2{,}003}\times\frac{2{,}003}{875}=\frac{9}{2}=4\tfrac{1}{2}.\quad \textit{Ans.}$$

$$\therefore\ \frac{37}{17}+\frac{9}{2}=\frac{74+153}{34}=\frac{227}{34}=6\frac{23}{34}.\quad \textit{Ans.}$$

2. yds. $6 \times 1,760 : 15 :: 60 \times 60$ secs. $= \frac{225}{44}$ number of seconds in which the first boy walks round the circular court.

yds. $7 \times 1,760 : 15 :: 60 \times 60$ secs. $= \frac{675}{154}$, number of seconds in which the second boy walks round the circular court.

yds. $8 \times 1,760 : 15 :: 60 \times 67$ secs. $= \frac{675}{176}$ number of seconds in which the third boy walks round the circular court.

When the boys meet together again at the starting point, each will have walked round the circular court an exact number of times; therefore to find the number of seconds in which the race will end, find the least number which contains each of the fractions $\frac{225}{44}, \frac{675}{154}, \frac{675}{176}$ an exact number of times, *i.e.*, find the L.C.M. of the fractions, *i.e.*, the L.C.M. of the numerators divided by the G.C.M. of the denominators.

$\therefore$ the L. C. M. of 225, 675, 675 = 675.
The G. C. M. of 44, 154, 176 = 22.

$\therefore$ the required L. C. M. $= \frac{675}{22} = 30\frac{15}{22}$.

$\therefore$ The race will end in $30\frac{15}{22}$ seconds. *Ans.*

3. To dig 155 c. ft., 10 coolies are required for 5 days.

$\therefore$ to dig 1 c. ft., $\frac{10}{155}$ coolie is required for 5 days,

to dig 1 c. ft. $\frac{10 \times 5}{155}$ coolie is required for 1 day.

to dig 600 c. ft. $\frac{5 \times 10 \times 600}{155}$ coolies are required for 1 day.

to dig 600 c. ft., in 4 days $\frac{10\times600\times5}{155\times4}$ coolies are required.

$= \frac{1,500}{31}$ coolies are required.

Or thus—

$$\left.\begin{array}{l}\text{(Direct) } 155 : 600\\ \text{(Inverse) } 4 : \quad 5\end{array}\right\} :: 10 \text{ coolies} = \frac{1,500}{31}$$

But, 7 soldiers, *i.e.*, (3 : 10 :: 7) $\frac{70}{3}$ coolies are already employed, $\therefore \frac{1,500}{31} - \frac{70}{3} = \frac{2,330}{93} = 25\frac{5}{93}$, *i.e.*, 26 coolies are required. *Ans.*

4. Rs. 2.565 is the amount.
Rs. 2,250 is the principal.
Rs. 315 is the interest on Rs. 2,250.
Rs. 2250 : 100 :: 315 = Rs. 14 interest obtained on Rs 100 for the required time.
But in one year Rs. 100 produces Rs. 7,
$\therefore$ Rs. 7 int. : Rs. 14 :: 1 year = 2 years. *Ans.*

5. The price of Rs. 100 stock is at 16 discount, *i.e.* Rs. 84 cash.
Rs. 100 stock : 100,000 :: Rs. 84 cash = Rs. 84,000 cash

Re. 1 = 25d. $= £\frac{25}{240} = £\frac{5}{48}$

$\therefore$ Rs. 100 : Rs. 84000 :: £ $\frac{5}{48}$ = £8,750.

Now £8,750 is invested in the three per cents. at 96.

$\therefore$ £ 96 : £8,750 :: £3 $= \frac{4,375}{16}$ = £273 8s. 9*d.* *Ans.*

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6 $(a+b+c)(-a+b+c)$
$= \{(b+c)+a\}\{(b+c)-a\}$
$= (b+c)^2-a^2=b^2+2bc+c^2-a^2$
$(a-b+c)(a+b-c)$

$= \{a-(b-c)\}\{a+(b-c)\}$
$= a^2-(b-c)^2 = a^2-b^2+2bc-c^2$
$\therefore\ (b^2+2bc+c^2-a^2)(a^2-b^2+2bc-c^2)$
$= \{2bc-(a^2-b^2-c^2)\}\{2bc+(a^2-b^2-c^2)\}$
$= 4b^2c^2-(a^2-b^2-c^2)^2$
$= 4b^2c^2-a^4-b^4-c^4+2a^2b^2+2a^2c^2-b^2c^2$
$= 2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4.$ *Ans.*

$(a+b\sqrt{-1})(a-b\sqrt{-1})$
$= a^2-b^2(-1) = a^2+b^2.$ *Ans.*

7. The sum of the squares of $\dfrac{1}{a-b}$ and $\dfrac{1}{a+b}$

$$= \left(\frac{1}{a-b}\right)^2 + \left(\frac{1}{a+b}\right)^2 = \frac{1}{(a-b)^2} + \frac{1}{(a+b)^2}$$
$$= \frac{(a+b)^2+(a-b)^2}{(a+b)^2(a-b)^2} = \frac{2(a^2+b^2)}{(a^2-b^2)^2}$$

Now the square of $\dfrac{2b}{a^2-b^2} = \dfrac{4b^2}{(a^2-b^2)^2}$

$$\therefore\ \frac{2(a^2+b^2)}{(a^2-b^2)^2} - \frac{4b^2}{(a^2-b^2)^2} = \frac{2(a^2+b^2)-4b^2}{(a^2-b^2)^2}$$
$$= \frac{2(a^2-b^2)}{(a^2-b^2)^2} = \frac{2}{a^2-b^2}.$$ *Ans.*

8. $x^4-15x^2+28x-12$
$= x^4-2x^3+2x^3-4x^2-11x+22x+6x-12$
$= x^3(x-2)+2x^2(x-2)-11x(x-2)+6(x-2)$
$= (x-2)(x^3+2x^2-11x+6)$
and $x^3+2x^2-11x+6$
$= x^3-2x^2+4x^2-8x-3x+6$
$= x^2(x-2)+4x(x-2)-3(x-2)$
$= (x-2)(x^2+4x-3)$
$\therefore$ the numerator $= (x-2)(x-2)(x^2+4x-3)$
The denominator $2x^3-15x+14$
$= 2x^3-4x^2+4x^2-8x-7x+14$
$= 2x^2(x-2)+4x(x-2)-7(x-2)$
$= (x-2)(2x^2+4x-7)$. $\therefore$ the G. C. M. $= x-2$.

The fraction

$$=\frac{(x-2)(x-2)(x^2+4x-3)}{(x-2)(2x^2+4x-7)}$$

$$=\frac{(x-2)(x^2+4x-3)}{2x^2+4x-7}. \quad \textit{Ans.}$$

9. Adding the 1st and the 2nd equations together, we get

$$\begin{array}{l} x+y+z=6\ldots\ldots\ldots\ldots(\text{i}) \\ \underline{3x-y+2z=7}\ldots\ldots\ldots\ldots(\text{ii}) \\ \quad 4x+3z+13\ldots\ldots\ldots\ldots(\text{iv}) \end{array}$$

Multiply (i) by 3, and subtract it from (iii) thus:—

$$\begin{array}{l} 4x+3y-z=7\ldots\ldots\ldots\ldots(\text{iii}) \\ \underline{3x+3y+3z=18}\ldots\ldots\ldots(\text{i}) \\ \quad x-4z=-11 \ \ldots\ldots\ldots(\text{v}) \end{array}$$

Now multiply (v) by 4 and subtract it from (iv), thus:—

$$\begin{array}{l} 4x+3z \ = \ 13 \\ \underline{4x-16z \ = \ -44} \\ \qquad 19z \ = \ 57 \ \therefore \ z=3 \end{array}$$

Substitute the value of z in (iv)

$\therefore\ 4x+3\times 3 = 13 \ \therefore\ 4x = 4 \ \therefore\ x = 1.$

Substitute the value of x and z in (i).

$\therefore\ 1+y+3 = 6 \ \therefore y = 6-4 = 2$

$\therefore\ x = 1,\ y = 2,\ z = 3.$ *Ans.*

10. Let x be the number of minute divisions passed over by the minute-hand after 10 o'clock. The minute hand in 60 min. moves 60 divisions and the hour hand only 5; therefore the minute-hand moves 12 times as fast as the hour-hand. Now, when the minute-hand moves x divisions, the number of minute divisions passed over by the hour-hand $=\frac{x}{12}$. Now at 10 o'clock, the minute-hand is 10 divisions in advance of the hour-hand; $\therefore$ in order that the two hands may be exactly together, the minute-hand must pass over 50 more divisions.

Hence $x=\frac{x}{12}+50 \therefore 12x-x=600 \therefore 11x=600 \therefore x=54\frac{6}{11}$.

Thus the hands are coincident at $54\frac{5}{11}$ minutes past 10 o'clock. *Ans.*

Again $x = \frac{x}{12} + 55$ $\therefore$ $x = 60$, *i. e.*, the hands will be again coincident at 60 mins. past 11 o'clock, *i. e.*, at 12 o'clock. *Ans.*

11. Let x rupees be the sum with the first student and y rupees with the second.

Then $x + \frac{y}{2} = 100$ and $y + \frac{x}{3} = 100$

$$\therefore\ 2x + y = 200 \ldots\ldots\ldots \text{(i)}$$
$$x + 3y = 300 \ldots\ldots\ldots \text{(ii)}$$

Multiply the 2nd by 2 and subtract it from the 1st.

$$\therefore\ 2x + y = 200$$
$$2x + 6y = 600, \ \therefore\ y = 80 \text{ ; whence } x = 60.$$
$$-5y = -400$$

$\therefore$ the first student had Rs. 60 and the second had Rs. 80. *Ans.*

Euclid.

1. In Props. 4, 8 and 26 (two cases) of the First Book, Euclid proves that two triangles are identically equal.

(Fig. 8). In trs. *ACD*, *BCD*, $\because$ *AC*=*CB* (hyp.), *CD* is common, and *AD*=*BD* (hyp.) $\therefore$ ang *ACD*=ang *BCD* (I.) 8). Again in trs, *ACE*, *BCE*, $\because$ *AC*=*CB*, *CE* is common and ang. *ACE* = ang. *BCE* (proved) $\therefore$ *AE*=*EB* (I. 4) and ang. *AEC*=ang. *CEB* (I. 4); but these are adjacent angles $\therefore$ they are rt. angs. (def. 7) $\therefore$ *AB* is bisected in *E* at right angles.

2. Euc. I. 34.

(Fig. 9.) In trs. *ABD*, *BCD*, $\because$ *AB*=*BC* and *BD* is common, and *AD*=*DC* $\therefore$ ang *ABD*=ang *DBC* (I. 8). Again, in trs. *ABE*, *BEC*, $\because$ *AB*=*BC*, *BE* is common. And ang. *ABE*=ang. *EBC* (proved) $\therefore$ *AE*=*EC* (I. 4), and ang. *AEB*=ang. *BEC* (I. 4), but they are adjacent angles, $\therefore$ they are right angles (def. 7). In the same way it can be proved that *BE*=*ED*; $\therefore$ *BD*, *AC* bisect each other at right angles.

3. (Fig. 10) Let A and B be the given points and MN the given str. line. Draw AC perpendicular to MN, and produce it to D, making $DC = AC$ (I. 3). Join DB, cutting MN at P. Then P shall be the required point.

(i) Join AP. In the trs. ACP and DCP $\because AC = DC$ (constr.), CP is common and ang. $ACP =$ ang. DCP, being right angles, $\therefore$ trs. ACP, DCP, are equal in all respects (I, 4) $\therefore$ ang. $APC =$ ang. DPC. but ang. $DPC =$ ang BPN (I. 15); $\therefore$ ang $APC =$ ang. BPN.

(ii) Take any other point Q in MN. Join AQ, BQ, DQ. The sides DQ, QB are together greater than DB (I. 20). But $AP = PD$ (I. 4). Similarly $AQ = QD$, $\therefore AP + BP = BD$; and $AQ + BQ = BQ + QD$, $\therefore AQ + BQ$ is greater than $AP + BP$, *i. e.*, AP, PB are less than any other two lines which can be drawn from A and B, to any other point Q in MN.

4. Euclid II. 4.

(Fig. 11) Let ABC be a triangle having the right angle at A and let AD be drawn perpendicular to BC.

Now, $BC^2 = BD^2 + DC^2 + 2$ rect. $BD.DC$ (II. 4). But $BC^2 = BA^2 + AC^2$ (I. 47) $= AD^2 + BD^2 + AD^2 + DC^2$ (I. 47), $\therefore 2AD^2 + BD^2 + DC^2 = BD^2 + DC^2 + 2$ rect. $BD.DC$ (ax. I). $\therefore 2AD^2 = 2$ rect. $BD.DC$ $\therefore AD^2 =$ rect. $BD.DC$. Q.E.D.

5. Euclid III. 17.

(Fig. 12). By III. 17, we prove that AB is a tangent to the circle. Similary it may be shewn that AC is a tangent. Now, show that $AB = AC$. Find E the centre of the circle.

Join EB, EC. In trs. AEB, AEC, angs. ABE, ACE are rt. angs. (III. 18), $\therefore AE^2 = AB^2 + BE^2 = AC^2 + CE^2$ (I. 47). But $BE^2 = CE^2$, $\because BE = CE$; $\therefore AB^2 = AC^2$. $\therefore AB = AC$.

6. Euclid III. 12.

(Fig. 13). BC passes through the point of contact A (III. 12). $\because DAE =$ one rt. ang. (hyp.) $\therefore$ angs. DAB, EAC together $=$ one rt. ang. but ang. $BAD =$ ang. BDA (I. 5), and ang. $CAE =$ ang. CEA (I. 5). $\therefore$ angs. BAD, BDA, CAE,

CAE. together $=2$ rt. angs., $\therefore$ angs. *DBA, ECA* together $=2$ rt. angs. (I. 32) $\therefore$ *DB* and *EC* are prll. (I. 28), and they areequal. $\therefore$ *DE* and *BC* are also equal and prll. (I. 33).

7. Euclid III. 36.

Fig. 14. Now $CE^2=$ rect. *EA, EB* (III. 36) and $DE^2=$ rect. *E.A.EB*, (III. 36). $\therefore CE^2=DE^2$ $\therefore CE=DE$. *Q. E. D.*

8. Euclid IV. (Fig. 15). Let *K* be the centre of the concentric circles. Now, as in IV. 5. we may prove that $KA=KB=KC$ $\therefore$ *KA, KB, KC* are the bisectors of angs. *BAC, ABC, BCA.*

Now, $KA=KB$ $\therefore$ ang. $KAB=$ ang. *KBA*. But ang. *BAC* $=$ twice ang. *KAB* and ang. $ABC=$ twice ang. *KBA*. $\therefore$ ang. $ABC=$ ang. *BAC* (ax. 6.)

Similarly, we may prove ang. $BAC=$ ang. $ACB=$ ang. *ABC*. $\therefore$ triangle *ABC* is equilateral (I.,6, cor.)

1881-82.

WEDNESDAY, 23RD NOVEMBER.

Govind Vithal Kurkaray, B. A.

Khan Bahadur Bamanji Sorabji, L. C. E.

Arithmetic and Algebra.

1. If the income-tax be 7*d*. in the pound in the 1st half of the year and $3\frac{1}{2}d$. in the second, what is the net income of a gentleman whose gross annual receipts are £1,542 10*s*. 6*d*. 7

2. A passenger train going 41 miles an hour and 431 feet long overtakes a goods train on a parallel line of rails; the goods train is going 28 miles an hour and is 713 feet long. How long does the passenger train take in passing the other? 9

3. Find the cost of painting the outside of a cubical box whose edge is 3·5 feet at 1·3s. per square yard. 9

4. A person invests Rs. 48,000 in the 4 per cents at 80 and at the end of each year invests the dividend which becomes due in the same stock. Supposing the funds to remain at 80 for 3 years; find his dividend at the end of the third year. 8

5. Define *Discount*. If the Discount on Rs. 2,261 5a. 4p. due at the end of a year and a half be Rs. 128, what is the rate of interest? 7

6. Find the square root of $\frac{\cdot 00125}{\cdot 18}$; and the cube root of 423564·751. 7

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7. Divide $a^2x^8+(2ac-b^2)x^4+c^2$ by ax^4+c-bx^2. 7

8. Resolve $4a^2b^2-(a^2+b^2-c^2)^2$ into four factors. 7

9. Find the square root of $\frac{x^2}{y^2}+\frac{y^2}{4x^2}-\frac{x}{y}+\frac{y}{2x}-\frac{3}{4}$. 6

10. Find the G. C. M. and L. C. M. of— 8

a^4-x^4 and $a^3-a^2x-ax^2+x^3$

11. A train carrying three classes of passengers at 6as. 4as. and 3as. has eight times as many third class passengers as there are of the second class and seven times as many second class passengers as there are of the first class: the whole sum received is Rs. 290 6as. How many first class tickets were issued? 9

12. Solve the equations :—

(i) $\frac{2x-9}{27}+\frac{x}{18}-\frac{x-3}{4}=8\frac{1}{3}-x$

(ii) $1\cdot 2x-\frac{18x-\cdot 05}{\cdot 5}=\cdot 4x+8\cdot 9$

(iii) $\left.\begin{aligned}\frac{x+6}{y}&=\frac{3}{4}\\ \frac{x}{y-2}&=\frac{1}{2}\end{aligned}\right\}$

Euclid.

1. Define scalene triangle, quadrilateral, semicircle, rectilineal figure, rectangle, superficies. 3

2. Prove that parallelograms upon the same base and between the same parallels are equal to one another. 8

3. Of all trangles having the same vertical angle, and whose bases pass through a given point, the least is that whose base is bisected in the given point. 9

4. Divide a given straight line into two parts so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

5. If a striaght line be divided into any two parts the rectangles contained by the whole and each of the parts are together equal to the square on the whole line. 6

6. In equal circles, equal straight lines cut off equal arcs; the greater equal to the greater and less to the less. 10

7. If AD and CE be drawn perpendicular to the sides BC, AB. of the triangle ABC, and DE be joined prove that the angles ADE and ACE are equal to each other. 10

8. Inscribe an equilateral and equiangular quindecagon in a given circle. 10

9. In a given circle inscribe a trainleg whose angles shall be as $\frac{1}{6} : \frac{1}{3} : \frac{1}{2}$. 10

SOLUTIONS.

Arithmetic and Algebra.

1. The average income-tax for one year on £ 1

$$= \frac{7d + 3\frac{1}{2}d}{2} = (7 + \tfrac{7}{2}) \times \tfrac{1}{2} = 5\tfrac{1}{4}d.$$

1£ $= 240d$.} $\therefore$ $240 - 5\frac{1}{4} = 234\frac{3}{4}d$. is the net income from £1

: £1,542. 10s. 6d :: £$\frac{234\frac{3}{4}}{240}$ = Net inc. £1,508 15s. $7\frac{119}{160}d$. *Ans.*

2. The passenger train gains $41-28=13$miles per hour upon the goods train.

Now we are to find the time in which the hindmost surface of the passenger train will meet the foremost surface of the goods train. As the distance between these two surfaces is the sum of the length of the two trains, *i.e.*, 431 feet $+713$ feet $=1,144$ feet, 1,144 feet must be gained by the passenger train.

13 miles$=13\times1,760\times3=68,640$ feet.

68,640 ft. : 1,144 ft. :: 1hr. $=\frac{1,144}{6,8640}$

$=\frac{1}{60}$hr. $=1$ minute. *Ans.*

3. The edge $=3\frac{5}{10}=\frac{7}{2}$ feet, $\therefore$ the area of one side only $=\frac{7}{2}\times\frac{7}{2}=\frac{49}{4}$ sq. feet; and as there are 6 sides in all,

the total area $=\frac{49}{4}\times6=\frac{147}{2}$ sq. feet.

sq. ft. 9 : sq. ft. $\frac{147}{2}$ :: $\frac{4}{3}=\frac{147}{2}\times\frac{4}{3}\times\frac{1}{9}$

$=\frac{98}{9}=10s.\ 10\frac{2}{3}d.$ *Ans.*

4. Rs. 80 : Rs. 48,000 :: Rs. 4 = Rs. 2,400, dividend for the 1st year

. Total sum invested in the same stock during the 2nd year = Rs. 48,000 + Rs. 2,400 = Rs. 50,400

$\therefore$ Rs. 80 : Rs. 50,400 :: Rs. 4=Rs. 2,520 dividend for the 2nd year.

Total sum invested in the same stock during the 3rd year =Rs. 50,400 Rs. + 2,520=Rs. 52,920

$\therefore$ Rs. 80 : Rs. 52,920 :: Rs. 4=Rs. 2,646 dividend for the 3rd year. *Ans.*

5. The discount of a sum of money is the interest on the Present Worth of that sum, calculated from the present time to the time when the sum would be properly payable.

Rs.	a.	p.	
2,261	5	4	Debt.
128	0	0	Discount
2,133	5	4	Present Value.

Rs. 2,133 5a. 4p.: Rs. 100 :: Rs. 128 = Rs. 6 Int.
$\frac{3}{2}$ yrs. : 1 :: Rs. 6 = Rs. 4 rate of interest. *Ans.*

6. $$\sqrt{\frac{\cdot 00125}{\cdot 18}} = \sqrt{\frac{\cdot 00125 \times 2}{\cdot 18 \times 2}} = \sqrt{\frac{\cdot 0025}{\cdot 36}} = \frac{\cdot 05}{\cdot 6}$$
$$= \cdot 08\dot{3}. \quad \textit{Ans.}$$

$$\sqrt[3]{423564 \cdot 751}$$

$$\begin{array}{rr|l}
3 \times 70^2 = & 14700 & 42\dot{3}56\dot{4}\cdot 751 \;\underline{|\; 75\cdot 1} \\
3 \times 70 \times 5 = & 1050 & 343 \\
5^2 = & 25 & \overline{80564} \\
 & & 78875 \\
 & \overline{15775} & \overline{1689751} \\
3 \times 750^2 = & 1687500 & 1689751 \\
3 \times 750 \times 1 = & 2250 & \overline{} \\
1^2 = & \underline{1} & \\
 & 1689751 & \quad 75\cdot 1. \ \textit{Ans.}
\end{array}$$

Or thus—

$$\begin{array}{r|r|l}
215 & 14700 & 423,564,751 \;\underline{|\; 75\cdot 1} \\
\underline{10} & \underline{1075} & \underline{343} \\
2251 & 15775 & 80564 \\
 & \underline{25} & \underline{78875} \\
 & 1687500 & 1689751 \\
 & \underline{2251} & \underline{1689751} \\
 & 1689751 & 75\cdot 1. \ \textit{Ans.}
\end{array}$$

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7. $$\begin{aligned}
& a^2x^8 + (2ac - b^2)x^4 + c^2 \\
= \ & a^2x^8 + 2acx^4 - b^2x^4 + c^2 \\
= \ & (a^2x^8 + 2acx^4 + c^2) - b^2x^4 \\
= \ & (ax^4 + c)^2 - (bx^2)^2 \\
= \ & (ax^4 + c + bx^2)(ax^4 + c - bx^2) \\
\therefore \ & \frac{(ax^4 + c + bx^2)(ax^4 + c - bx^2)}{ax^4 + c - bx^2} \\
= \ & ax^4 + c + bx^2. \quad \textit{Ans.}
\end{aligned}$$

8. $$\begin{aligned}
& 4a^2b^2 - (a^2 + b^2 - c^2)^2 \\
= \ & \{2ab + (a^2 + b^2 - c^2)\}\{2ab - (a^2 + b^2 - c^2)\} \\
= \ & (2ab + a^2 + b^2 - c^2)(2ab - a^2 - b^2 + c^2)
\end{aligned}$$

Now, $2ab+a^2+b^2-c^2=(a^2+2ab+b^2)-c^2$
$= \quad (a+b)^2-c^2=(a+b-c)(a+b+c)$
and $2ab-a^2-b^2+c^2=c^2-(a^2-2ab+b^2)$
$= \quad c^2-(a-b)^2=(c+a-b)(c-a+b)$
$\therefore$ the four factors are
$(a+b+c)(b+c-a)(a+c-b)(a+b-c)$. *Ans.*

9. $\frac{x^2}{y^2}+\frac{y^2}{4x^2}-\frac{x}{y}+\frac{y}{2x}-\frac{3}{4}$

$$=\left(\frac{x^2}{y^2}+\frac{y^2}{4x^2}-1\right)-\left(\frac{x}{y}-\frac{y}{2x}\right)+\frac{1}{4}$$

$$=\left(\frac{x}{y}\right)^2+\left(\frac{y}{2x}\right)^2-2\left(\frac{x}{y}\right)\left(\frac{y}{2x}\right)-\left(\frac{x}{y}-\frac{y}{2x}\right)+\frac{1}{4}$$

$$=\left(\frac{x}{y}-\frac{y}{2x}\right)^2-\left(\frac{x}{y}-\frac{y}{2x}\right)+\frac{1}{4}$$

$=\left(\frac{x}{y}-\frac{y}{2x}-\frac{1}{2}\right)^2$ $\therefore$ the square root $=\frac{x}{y}-\frac{y}{2x}-\frac{1}{2}$. *Ans.*

10. a^4-x^4
$=(a^2-x^2)(a^2+x^2)=(a+x)(a-x)(a^2+x^2)$
$a^3-a^2x-ax^2+x^3=a^2(a-x)-x^2(a-x)=(a^2-x^2)(a-x)$
$=(a+x)(a-x)^2$
$\therefore$ the G. C. M. and L. C. M. of—
$(a+x)(a-x)(a^2+x^2)$ and $(a+x)(a-x)^2$
$=(a+x)(a-x)$ and $(a-x)^2(a+x)(a^2+x^2)$ respectively.
Ans.

11. Let x be the number of first class passengers; then $7x$ is the number of 2nd class passengers and $8\times 7x$, *i. e.*, $56x$, the number of 3rd class passengers.

$\therefore x\times 6+7x\times 4+56x\times 3=290\times 16+6$
$\therefore 6x+28x+168x=4{,}646$ $\therefore 202x=4{,}646$ $\therefore x=23$.
$\therefore$ 23 first class tickets were issued. *Ans.*

12. (i) $\frac{2x-9}{27}+\frac{x}{18}-\frac{x-3}{4}=\frac{25}{3}-x$

$8x-36+6x-27x+81=900-108x$
$\therefore 8x+6x-27x+108x=900+36-81$
$\therefore 95x=855$ $\therefore x=9$ *Ans.*

(ii) $1{\cdot}2x-\dfrac{{\cdot}18x-{\cdot}05}{{\cdot}5}={\cdot}4x+8{\cdot}9$

Multiply both sides by ·5.

$\therefore\ {\cdot}6x-{\cdot}18x+{\cdot}05={\cdot}2x+4{\cdot}45$

$\therefore\ {\cdot}6x-{\cdot}18x-{\cdot}2x=4{\cdot}45-{\cdot}05$

$\therefore\ {\cdot}22x=4{\cdot}4\quad \therefore x=20.$ *Ans.*

(iii) $\dfrac{x+6}{y}=\dfrac{3}{4}$ .... (i)

$\dfrac{x}{y-2}=\dfrac{1}{2}$ .... (ii)

$4x+24=3y\ \therefore\ 4x-3y=-24$ ......(i)

$2x=y-2\ \therefore\ 2x-y=-2$ ..... (ii)

Multiply the 2nd by 2, and subtract it from the 1st.

$$\begin{aligned}\therefore\ 4x-3y&=-24\\ 4x-2y&=-\ \ 4\\ \hline -\ y&=-20\end{aligned}$$

$\therefore\ y=20$; whence $x=9$. *Ans.*

Euclid.

1. *Vide* Euclid. (Definitions)
2. Euclid I. 35.
3. (Fig. 16.) Let trs. ABC, AEF, have the same vert. ang. and let the bases BC, EF pass through the same point D, EF is bisected in D. Draw EH prll. to AF (I. 31). Now in trs. EHD, DCF, $\because$ ang. EDH = ang. CDF (I. 15) and ang. HED = ang. DFC (I. 29) and $ED=DF$ (hyp.) $\therefore$ trs. are equal in all respects (I. 26). Add $AEDC$ to each of the equals. $\therefore$ tr. AFE = fig. $AEHC$, *i. e.*, tr. AFE is less than tr. ABC.
4. Euclid II., 11.
5. Euclid II., 2.
6. Euclid III., 28.
7. (Fig. 17.) It follows from III. 21 that the locus of e vertices of triangles drawn on the same base with equal ·cal angles is an arc of a circle,

6. (i) If $\left(a+\frac{1}{a}\right)^2 = 3$, prove that $a^3+\frac{1}{a^3}=0$.

(ii) If $x+y=2z$ show that $\frac{x}{x-z}+\frac{y}{y-z}=2$. 8

7. Resolve into factors the following expression :— $6x^2+5x-6$, $3x^2-10x-8$, $9x^4-82x^2y^2+9y^4$. 8

8. Find the G. C. M. of

$x^4-3x+20$ and $5x^4-3x^2+64$. 8

9. Extract the cube root of—

$$x^3+\frac{8}{x^3}-12x^2-\frac{48}{x^2}+54x+\frac{108}{x}-112$$

10. Solve the equations— 10

(i) $\frac{a}{x+a}+\frac{b}{x+b}=\frac{a-c}{x+a-c}+\frac{b+c}{x+b+c}$

(ii) $\frac{1}{2}(7x+1)-\frac{1}{3}(17-2x)=\frac{1}{4}(5x+1)$

11. A father's age is four times that of his eldest son and five times that of his younger son; when the elder son has lived to three times his present age the father's age will exceed twice that of his younger son by 3 years. Find their present ages. 10

12. A cottage costs Rs. 1,500 to build. At what rent must it be let to pay 5% clear, after allowing 10% of receipts of repairs? 8

Euclid.

1. Prove that the straight lines which join the extremities of two equal and parallel straight lines towards the same part are also themselves equal and parallel. 6

Why is the restriction contained in the words "towards the same parts," necessary.

2. From the extremities of a straight line AB perpendiculars AC and BD are drawn on opposite sides of it, such that AC and BD are together equal to AB. Shew t the straight line CD always makes the same angle AB. 10

3. The complements of a parallelogram which are about the diameter of any parallelogram are equal to one another. 7

If the complements be squares determine their relation to the whole parallelogram.

4. All the exterior angles of any rectilineal figure made by producing the sides successively in the same direction are equal to four right angles. 5

5. Describe a square that shall be equal to a given rectilineal figure. 8

6. Prove that the square on any straight lines drawn from the vertex of an isosceles triangle to the base is less than the square on a side of a triangle by the rectangle contained by the segments of the base. 9

7. In a circle the angle in a semi-circle is a right angle; but the angle in a segment greater than a semi-circle is less than a right angle and the angle in a segment less than a semi-circle is greater than a right angle. 8

8. In a circle the extremities of two radii at right angles to each other are joined. Prove that the angle in a segment so formed is equal to one right angle and a half. 9

9. About a given circle describe a triangle equiangular to a given triangle. 7

10. The square inscribed in a circle is equal to half the square described about the same circle. 6

SOLUTIONS.

1. £·596875. = ·596875 × 20*s* = 11·9375*s*.
=11*s*. ·9375 × 12*d* = 11*s*. 11·25*d*. *Ans.*

11 poles 4 yds. $4\frac{1}{2}$ inches $= \frac{4,653}{2}$ inches.

1 mile = 1 × 1,760 × 3 × 12 inches = 63,360 inches

$\therefore \frac{4,653}{2} \times \frac{1}{63360} = \frac{4653}{126720} = \cdot 03671875$. *Ans.*

2. The train goes 48 miles in one hour.

$\therefore$ it runs $\frac{48 \times 1760}{60}$ yards, or 1,408 yards in one minute.

The telegraph posts are 58 yards apart, $\therefore$ going 1408 yards in one minute, he will pass $\frac{1408}{58}$ posts

$= 24\frac{8}{29}$, *i. e.*, 24 posts. *Ans.*

3. 5 boys can do the work of 3 men,

$\therefore$ 40 boys can do the work of 24 men,

$\therefore$ 40 boys + 15 men can do as much as 39 men ;

and 20 boys and 20 men can do the work of 32 men.

39 men can do the work in 8 weeks,

$\therefore$ 1 man can do the work in 8×39 weeks,

$\therefore$ 32 men can do the work in $\frac{8 \times 39}{32} = \frac{39}{4}$ weeks

$= 9\frac{3}{4}$ weeks.

i. e., 20 boys and 20 men can do the work in $9\frac{3}{4}$ weeks. *Ans.*

The wages of 3 boys = wages of 2 men ;

$\therefore$ the wages of 1 boy = wages of $\frac{2}{3}$ man,

$\therefore$ wages of 40 boys = wages of $\frac{40 \times 2}{3}$

$= \frac{80}{3}$ men and wages of 20 boys = wages of $\frac{40}{3}$ men.

$\therefore$ wages of 40 boys and 15 men = wages of $\left(\frac{80}{3} + 15\right) = \frac{125}{3}$ men ;

and wages of 20 boys and 20 men = wages of $\left(\frac{40}{3} + 20\right)$ or $\frac{100}{3}$ men.

Now the wages of $\frac{125}{3}$ men working for 8 weeks = £350

$\therefore$ the wages of $\frac{100}{3}$ men working for $9\frac{3}{4}$ weeks will be

$$\left(\frac{125}{3} \times 8 : \frac{100}{3} \times 9\tfrac{3}{4} :: £350\right)$$

$$= \frac{39 \times 35}{4} = £341\ 5s. \quad \textit{Ans.}$$

4. $£54311\ 5s\ 11\frac{1}{4}d = £5431\ \frac{1}{4}\frac{1}{4} = £\frac{347635}{64}$

1 year : 6 yrs. :: $£4\frac{1}{4} = £\frac{51}{2}$

$£100 + £\frac{51}{2} = £\frac{251}{2}$ amount of £100

$£\frac{251}{2}$ amt. : $\frac{347635}{64}£$ amt. :: sum. £100=

£4,328 2s. 6d. *Ans.*

(5) 77 yds. 1 ft. 9 in. = 2,793 inches.

7 yds. 2 ft. 4 in. = 280 inches.

$\therefore$ sum of the areas of both squares $=(2793)^2+(280)^2$ sq. in.

$=7800849$ sq. in $+$ 78400 sq. in $=$ 7879249 sq. in.

$\therefore$ the side of the third sqr. $= \sqrt{7{,}879{,}249}$ inches.

```
     |7879249 | 2807
     |4       |______
  48 |387
     |384
5607 |  39249
     |  39249
```

2,807 inches=77 yds. 2 ft. 11 inches *Ans.*

6. (i) $\left(a+\frac{1}{a}\right)^2 = a^2+2+\frac{1}{a^2}=3.$

$\therefore\ a^2+2-3+\frac{1}{a^2}=0$, *i.e.*, $a^2-1+\frac{1}{a^2}=0.$

Now $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)\left(a^2-1+\frac{1}{a^2}\right)$

$=\left(a+\frac{1}{a}\right)\times 0=0$, which was to be proved.

(ii) $\frac{x}{x-z}+\frac{y}{y-z}=\frac{xy-xz+xy-yz}{xy-yz-xz+z^2}$

$\frac{2xy-z(x+y)}{xy+z^2-z(x+y)}$; but $x+y=2z$

$\therefore\ =\frac{2xy-z\times 2z}{xy+z^2-z\times 2z}=\frac{2xy-2z^2}{xy-z^2}$

$=\frac{2(xy-z^2)}{xy-z^2}=2$, which was to be proved.

7. $6x^2+5x-6=6x^2+9x-4x-6$

$=3x(2x+3)-2(2x+3)=(3x-2)(2x+3)$. *Ans.*

$3x^2-10x-8=3x^2-12x+2x-8$

$=\quad 3x(x-4)+2(x-4)=(3x+2)(x-4)$. *Ans.*

$9x^4-82x^2y^2+9y^4=9x^4-81x^2y^2-x^2y^2+9y^4$

$=\quad 9x^2(x^2-9y^2)-y^2(x^2-9y^2)=(9x^2-y^2)(x^2-9y^2)$

$=\quad (3x+y)(3x-y)(x+3y)(x-3y)$. *Ans.*

8. $x^4+3x+20$

$=\quad x^4+3x^3-3x^3+4x^2-9x^2+5x^2-12x+15x+20$

$=\quad x^4+3x^3+4x^2-3x^3-9x^2-12x+5x^2+15x+20$

$=\quad x^2(x^2+3x+4)-3x(x^2+3x+4)+5(x^2+3x+4)$

$=\quad (x^2-3x+5)(x^2+3x+4)$

$5x^4-3x^3+64$

$=5x^4+12x^3-15x^3+20x^2-36x^2+16x^2-48x+48x+64$

$=5x^4-15x^3+20x^2+12x^3-36x^2+48x+16x^2-48x+64$

$=5x^2(x^2-3x+4)+12x(x^2-3x+4)+16(x^2-3x+4)$

$=(5x^2+12x+16)(x^2-3x+4)$

Hence, the G. C. M. $=x^2-3x+4$. *Ans.*

Or thus: $x^4-3x+20)\ 5x^4-3x^3+64\ (5$

$$\begin{array}{r}
5x^4-15x+100 \\ \hline
-3x^3+15x-36
\end{array}$$

$-3(x^3-5x+12)\quad x^4-3x+20\ (x$

$$\begin{array}{r}
x^4-5x^2+12x \\ \hline
5x^2-15x+20
\end{array}$$

$5(x^2-3x+4)\,x^3-5x+12\ (x+3$

$$\begin{array}{r}
x^3+4x-3x^2 \\ \hline
3x^2-9x+12 \\
3x^2-9x+12 \\ \hline
\end{array}$$

$\therefore$ the G. C. M. $=x^2-3x+4$. *Ans.*

9.

$$
\begin{array}{r|l}
 & x^3-12x^2+54x-112+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3}\left(x-4+\frac{2}{x}\right. \\
3\times x\times -4 = \quad -12x & x^3 \\
(-4)^2 = \quad +16 & \\
3x^2-12x+16 & -12x^2+54x-112 \\
 & -12x^2+48x-64 \\
3(x-4)^2 = 3x^2-24x+48 & \\
3(x-4)\times\frac{2}{x} = \quad +6-\frac{24}{x} & 6x-48+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3} \\
\left(\frac{2}{x}\right)^2 = \quad +\frac{4}{x^2} & 6x-48+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3} \\
3x^2-24x+54-\frac{24}{x}+\frac{4}{x^2} &
\end{array}
$$

$\therefore\ x-4+\frac{2}{x}$. *Ans.*

10. (i) $\frac{a}{x+a}-\frac{a-c}{x+a-c}=\frac{b+c}{x+b+c}-\frac{b}{x+b}\ \therefore\ \frac{ax+a^2-ac-ax-a^2+cx+ac}{(x+a)(x+a-c)}$

$=\frac{bx+b^2+cx+bc-bx-b^2-cb}{(x+b+c)(x+b)};\ \therefore\ \frac{cx}{(x+a)(x+a-c)}=\frac{cx}{(x+b+c)(x+b)}$

$\therefore\ (x+a)(x+a-c)=(x+b)(x+b+c)\ \therefore\ x^2+2ax-cx+a^2-ac=x^2+2bx+cx+b^2+bc$

$\therefore\ 2ax-2bx-2cx=b^2+bc-a^2+ac\ \therefore\ 2x(a-b-c)=b^2-a^2+bc+ac$

$=(b+a)(b-a)+c\ (b+a)=(b+a)(b-a+c)=-(b+a)(a-b-c)$

$$\therefore\ x=\frac{-(b+a)(a-b-c)}{2(a-b-c)}=\frac{-(b+a)}{2}\quad \textit{Ans.}$$

(ii) $\frac{1}{5}(7x+1)-\frac{1}{3}(17-2x)=\frac{1}{4}(5x+1)$

$84x+12-340+40x=75x+15$

$84x+40x-75x=-12+340-15$

$\therefore\ 49x=343\quad \therefore\ x=7.$ *Ans.*

11. Let x be the age of the eldest son; then $4x$ is the age of the father, and $\frac{4x}{5}$ age of the younger son. When the elder son has lived to three times his present age, *i. e.*, when he is $3x$ years old, *i. e.*, after $3x-x$ or $2x$ years, the father's age will be $4x+2x$ or $6x$ years, and the younger son's age will be $\frac{4x}{5}+2x$ or $\frac{14x}{5}$ years.

Now, by the question, $6x=\frac{28x}{5}+3$

$\therefore\ 30x=28x+15 \quad \therefore\ 2x=15,\ x=7\frac{1}{2}$

$\therefore\ 4x=30,\ \frac{4x}{5}=6$

$\therefore$ the father's age $=30$ years,
the eldest son's age $=7\frac{1}{2}$ years, and
the younger son's age $=6$ years. } *Ans.*

12. Let x Rupees be the rent of the cottage.

$\therefore$ 10% of x Rs. for repairs $=\frac{10x}{100}$, *i. e.*, Rs. $\frac{x}{10}$,

$\therefore$ Rs. $\left(x-\frac{x}{10}\right)$, *i. e.*, Rs. $\frac{9x}{10}$ he gets

Rs. 1,500 : Rs. 100 :: $\frac{9x}{10}=\frac{3x}{50}$, which is equal to Rs. 5 by the question.

$\therefore\ \frac{3x}{50}=5 \quad \therefore x=\frac{250}{3}=83\frac{1}{3}$

Hence, Rs. $83\frac{1}{3}$ is the rent. *Ans.*

Euclid.

1. Euclid I, 33.

The restriction contained in the words "towards the same parts" is necessary, for, if not, we may join the extremities *A*, *C* and *B*, *D* by *AC* and *BD*, or the extremities *A*, *D* and *B*, *C* (towards opposite parts) by *AD* and *BC* as in figure 18.

2. Euclid I. 43.

If the complements be sqrs. the plm. would be divided into four equal squares. $\therefore$ each figure is one-fourth of the whole.

3. (Fig. 19.) From AB cut off $AL = AC$. Join CL and produce it to meet BD in D.

$\because$ $AL = AC$ and angle LAC is a rt. angle, $\therefore$ $ALC =$ half a rt. angle = angle BLD (I. 15); and $\because$ LBD is a right angle, $\therefore$ LDB is half a right angle = ang. BLD; $\therefore$ $BL = BD$ (I. 6) and $AL = AC$. $\therefore$ $AC + BD = AB$ and the inclination of any other line FG will be the same as that of CD, *i.e.*, 45° in order that $AB = AC + BD$.

4. Euclid 2nd Corollary of I. 32.

5. Euclid II. 14.

6. (Fig. 20). Let ABC be an isosceles tr. and from the vertex of A, let AD be drawn to any point D in BC. Draw AE perp. to BC (I. 12). Then in trs. ABE, AEC, ang. ABE $=$ang. ACE (I. 5), ang. $AEB =$ ang. AEC (ax. 11), $AB = AC$ (hyp.) $\therefore$ trs. are equal (I. 26) $\therefore$ $BE = EC$.

Now rect. $BD.DC + ED^2 = EC^2$ (II. 5); to each of these equals add AE^2

$\therefore$ rect. $BD.DC + AE^2 + ED^2 = AE^2 + EC^2$.

$\therefore$ rect. $BD.DC + AD^2 = AC^2$ (I. 47).

7. Euclid III. 31.

(Fig. 21.) Take any point E in the segment AEB and join AE, EB, EC. Then ang. ADB is a rt. ang. (hyp.) $\therefore$ ang. ABD, BAD together = one rt. ang. (I. 32); but $BD = DA$ $\therefore$ ang. $BAD =$ ang. DBA (I. 5) $\therefore$ ABD is half a rt. ang. but ang. $ABD =$ ang. AEC (III. 21) $\therefore$ ang. AEC is half a rt. ang. and ang. BEC in a semicircle is a rt. ang. (III. 31) $\therefore$ ang. $BEA = 1\frac{1}{2}$ rt. ang.

8. Euclid IV. 3.

9. (Fig. 22.) Find *K*, the centre of the circle, and draw diameters *AC, BD.*

Then, as in IV. 7, it can be shewn that the figures *KE KG, KF, KH* are plms., and they are double of trs. *AKB, KDC, AKD, KBC*, respectively (1. 34) ∴ sq. *EFGH* is double of sq. *ABCD.*

1883-84.

Arithmetic and Algebra.

MONDAY, 26TH NOVEMBER.

GOVIND VITHAL KURKURAY, B. A.

KHAN BAHADUR BOMANJEE SORABJEE, L.C.E., A.M.I.C.E.

1. (*a*) Express in figures:—Sixteen billion seventy-five million forty thousand and two. **16**

(*b*) Simplify the expression :—

$$\left(\frac{1\frac{5}{17}-\frac{5}{7}}{1\frac{2}{5}+\frac{2}{15}}\right) \div \left(\frac{\frac{8}{11}-\frac{3}{13}}{1\frac{1}{3}+\frac{8}{25}}\right)$$

(*c*) Find the value of—

$$3\cdot75 \times 5s.\ 6d. + 5\cdot05 \times £3\ 1s.\ 8d. + 5\cdot0\dot{7} \times 7s.\ 6d. + 3\cdot\dot{1}\dot{3}5 \times £2\ 1s.\ 3d.$$

2. At the examination of a school one-tenth of the children were presented in the 6th standard, one-ninth in the 5th standard, one-eighth in the 4th, one-sixth in 3rd, one-fifth in the 2nd ; and the remainder 107 in the 1st standard. How many were presented altogether, and how many in each of the other standards? **9**

3. In a bicycle race of 2 miles over a circular course of 1 furlong, the winner in his last round overtook the second at a point in his 15th round: their paces were as 159 to 149. At what distance was this point from the winning post? **10**

4. Find the expenses of an excursion, which includes 5,782 miles of railway at $\frac{3}{4}d.$ per mile, 517 miles of carriage at $10\frac{1}{2}d.$ per mile, and 57 days of hotel keep at 14*s.* 3*d.* per day, allowing 5 guineas for extras. 9

5. Divide 1·04 by ·000078125; and prove your result by vulgar fraction. 10

Find the square root of 8658·3025 and the cube root of 573·571.

---

6. Shew that—

$$\frac{x}{a} + \left(\frac{z-x}{b}\right) \text{ and } \frac{z^2}{a^2+b^2} + \frac{a^2+b^2}{a^2b^2}\left(x - \frac{za^2}{a^2+b^2}\right)^2 \qquad 10$$

are identical expressions, such that one can be deduced from the other.

7. Divide $x^3 + 8y^3 - 27z^3 + 18xyz$ by $x-3z+2y$ and resolve into factors of the first degree $a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$. 10

8. A criminal having escaped from prison travelled 10 hours before his escape was known. He was pursued so as to be gained upon 3 miles an hour. After his pursuers had travelled 3 hours they met an express going at the same rate as themselves, who met the criminal 2 hours 24 minutes before. In what time after the commencement of the pursuit will they overtake him? 12

9. Divide the number 127 into four such parts, that the first increased by 18, the second diminished by 5, the third multiplied by 6 and the fourth divided by $2\frac{1}{2}$, shall all be equal. 8

10. Solve the equation:— 12

$$\frac{17-3x}{5} - \frac{4x+2}{3} = 5 - 6x + \frac{7x+14}{3};$$

and find the value of—

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}, \text{ when } x = \frac{4ab}{a+b}.$$

Euclid.

1. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram shall be double of the triangle. 7

2. AB is hypotenuse of a right-angled triangle ABC: find a point D in AB such that DB may be equal to the perpendicular from D on AC. 12

3. In obtuse-angled triangles if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and obtuse angle. 10

4. In a triangle whose vertical angle is a right angle, a straight line is drawn from the vertex perpendicular to the base: shew that the square on this perpendicular is equal to the rectangle contained by the segments of the base. 10

5. The angles in the same segment of a circle are equal to one another. 8

6. Two tangents are drawn to a circle at the opposite extremties of a diameter and cut off from a third tangent a portion AB: if C be the centre of the circle, shew that ACB is a right angle. 11

7. Describe a circle about a given triangle. 7

8. Inscribe an equilateral and equiangular pentagon in a given circle. 10

SOLUTIONS.

1. (*a*) 16,000,075,040,002. *Ans.*

(*b*) $\dfrac{\frac{22}{17}-\frac{5}{7}}{\frac{7}{5}+\frac{2}{19}}=\dfrac{69}{119}\times\dfrac{95}{143}$

$\dfrac{\frac{9}{11}-\frac{3}{13}}{\frac{4}{5}+\frac{9}{35}}=\dfrac{95}{143}\times\dfrac{69}{119}$

$\therefore\ \dfrac{\frac{69}{119}\times\frac{95}{143}}{\frac{95}{143}\times\frac{69}{119}}=\dfrac{69}{119}\times\dfrac{95}{143}\times\dfrac{143}{95}\times\dfrac{119}{69}=1.$ *Ans.*

(c) $3{\cdot}75$ of $5\frac{1}{2}s. = 3\frac{3}{4}\times 5\frac{1}{2}s. = \dfrac{15}{4}\times\dfrac{11}{2}=\dfrac{165}{8}s. =$ £1 0*s.* $7\frac{1}{2}d.$

$5{\cdot}05$ of £3 1*s.* 8*d.* $= 5\dfrac{5}{100}\times$ £$\dfrac{37}{12}=$ £15 11*s.* 5*d.*

$5{\cdot}\dot{0}\dot{7}$ of 7*s.* 6*d.* $= 5\dfrac{7}{99}\times 7\frac{1}{2}s. =$ £1 18*s.* $0\frac{4}{11}d.$

$3{\cdot}1\dot{3}\dot{5}$ of £2 1*s.* 3*d.* $= 3\dfrac{134}{990}\times$ £$\dfrac{33}{16}=$ £6 9*s.* 4*d.*

$\therefore$ the sum of the different results = £24 19*s.* $4\frac{1}{22}d.$ *Ans.*

2. Let 1 be the total number.

$\therefore\ 1-\left(\dfrac{1}{10}+\dfrac{1}{9}+\dfrac{1}{8}+\dfrac{1}{6}+\dfrac{1}{5}\right)$, *i.e.*, $1-\dfrac{253}{360}=\dfrac{107}{360}$

= the remaining number which is equal to 107 children.

$\therefore\ \dfrac{107}{360} : 1 :: 107$ children = 360 children, the total number. *Ans.*

$\therefore\ \dfrac{1}{10}\times 360$, *i.e.*, 36 children were in the 6th standard.

$\dfrac{1}{9}\times 360$, *i.e.*, 40 „ „ „ 5th „

$\dfrac{1}{8}\times 360$, *i.e.*, 45 „ „ „ 4th „

$\dfrac{1}{6}\times 360$, *i.e.*, 60 „ „ „ 3rd „

$\dfrac{1}{5}\times 360$, *i.e.*, 72 „ „ „ 2nd „

3. The race of 2 miles = race of 16 furlongs.

The circular course is of 1 furlong ∴ the race is of 16 rounds.

The winner has run over 1 round more than the other by the question. Also, while the winner takes 159 rounds, the loser takes only 149 rounds ∴ the winner gains 10 rounds over the loser.

10 : 1 :: 159 = 15·9 rounds taken.

∴ the winner is (16 − 15·9) or ·1 furlong behind the winning post.

$\cdot 1 = \frac{1}{10}$ furlong; $\frac{1}{10} \times 660 = 66$ feet the distance of the point from the winning post.

4. 1 mile : 5,782 miles :: $\frac{3}{4}d.$ = £18 1s. $4\frac{1}{2}d.$
1 mile : 517 miles :: $10\frac{1}{2}d.$ = £22 12s. $4\frac{1}{2}d.$
1 day : 57 days :: 14s. 3d. = £40 12s. 3 d.
Extra expenses = £5 5s. 0 d.
£86 11s. 0 d.
£86 11s. *Ans.*

5. $\frac{1\cdot040000000}{\cdot000078125} = 13312.$ *Ans.*

$1\frac{4}{100} \div \frac{78125}{1000000000} = \frac{26}{25} \times \frac{1000000000}{78125} = 13132.$ *Ans.*

```
      |8658 3025|93·05
      |81
  183 | 558
      | 549
18605 |  93025
      |  93025
```

93·05. *Ans.*

```
                 |753·571|9·1
3 × 90² = 24300  |729
3 × 90 × 1 = 270 | 24571
        1² =   1 |
           24571 | 24571
```

9·1. *Ans.*

6. $\dfrac{z^2}{a^2+b^2}+\dfrac{a^2+b^2}{a^2b^2}\left(x-\dfrac{za^2}{a^2+b^2}\right)^2$

Now $\left(x-\dfrac{za^2}{a^2+b^2}\right)^2=x^2-\dfrac{2xza^2}{a^2+b^2}+\dfrac{z^2a^4}{(a^2+b^2)^2}$

$\therefore$ the expression

$$=\frac{z^2}{a^2+b^2}+\frac{x^2(a^2+b^2)}{a^2b^2}-\frac{2xz}{b^2}+\frac{z^2a^2}{b^2(a^2+b^2)}$$

$$=\frac{z^2}{a^2+b^2}+\frac{z^2a^2}{b^2(a^2+b^2)}+\frac{x^2(a^2+b^2)}{a^2b^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2b^2(a^2+b^2)+z^2a^2(a^2+b^2)}{b^2(a^2+b^2)^2}+\frac{x^2(a^2+b^2)}{a^2b^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2b^2+z^2a^2}{b^2(a^2+b^2)}+\frac{x^2(a^2+b^2)}{a^2b^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2}{b^2}+\frac{x^2(a^2+b^2)}{a^2b^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2}{b^2}+\frac{x^2a^2}{a^2b^2}+\frac{x^2b^2}{a^2b^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2}{b^2}+\frac{x^2}{b^2}+\frac{x^2}{a^2}-\frac{2xz}{b^2}$$

$$=\frac{z^2}{b^2}-\frac{2xz}{b^2}+\frac{x^2}{b^2}+\frac{x^2}{a^2}$$

$$=\frac{z^2-2xz+x^2}{b^2}+\frac{x^2}{a^2}$$

$=\left(\dfrac{x}{a}\right)^2+\left(\dfrac{z-x}{b}\right)^2$, which was to be shown.

☞ N.B.—The question is misprinted in the University Calendar; it should be stated, thus :—

Shew that $\left(\dfrac{x}{a}\right)^2+\left(\dfrac{z-x}{b}\right)^2$ and $\dfrac{z^2}{(a^2+b)^2}$, &c., are identical expressions.

7. $x^3+8y^3-27z^3+18xyz$

$= x^3+(2y)^3-(3z)^3+18xyz.$

Now, $a^3+b^3=(a+b)^3-3ab(a+b)$.

$\therefore x^3+(2y)^3=(x+2y)^3-6xy(x+2y)$.

$\therefore$ the expression

$= (x+2y)^3-6xy(x+2y)-(3z)^3+18xyz.$

$= \{(x+2y)^3-(3z)^3\}-6xy(x+2y)+18xyz.$
$= (x+2y-3z)\{(x+2y)^2+3z(x+2y)+9z^2\}-6xy(x+2y-3z).$
$= (x+2y-3z)\{(x+2y)^2+3z(x+2y)+9z^2-6xy\}$
$\therefore = (x+2y-3z)(x^2+4xy+4y^2+3xz+6yz+9z^2-6xy)$
$= (x+2y-3z)(x^2-2xy+4y^2+3xz+6yz+9z^2)$
$\therefore$ the expression divided by $x+2y-3z$
$=x^2-2xy+4y^2+3xz+6yz+9z^2$. *Ans.*

The expression
$= a^2b+a^2c+b^2c+ab^2+ac^2+bc^2+abc+abc$
$= a^2b+ab^2+a^2c+abc+abc+b^2c+ac^2+bc^2$
$= ab(a+b)+ac(a+b)+bc(a+b)+c^2(a+b)$
$= (a+b)(ab+ac+bc+c^2)$
Now, $ab+ac+bc+c^2=a(b+c)+c(b+c)$
$=(b+c)(a+c)$
$\therefore$ the expression $=(a+b)(b+c)(c+a)$

8. Let x miles per hour be the rate at which the criminal walks, and y hours be the time when the thief is overtaken after the commencement of the pursuit.

The criminal is pursued so as to be gained upon 3 miles per hour; $\therefore$ the pursuers and the express (who goes at the same rate as the pursuers) go over $(x+3)$ miles in one hour.

When the pursuers had travelled 3 hours, they met the express who met the criminal 2 hours 24 minutes before $\therefore$ the criminal travelled 10 hours before his escape was known and 3 hours $-$ $2\frac{2}{5}$ hours or $\frac{3}{5}$ hour when he met the express, *i. e.*, in all he travelled $10\frac{3}{5}$ hours when he met the express. The pursuers and the express travelled 3 hours $+2\frac{2}{5}$ hours, *i. e.*, $5\frac{2}{5}$ hours together. Hence

$$10\tfrac{3}{5}\times x=5\tfrac{2}{5}(x+3)\therefore 53x = 27x+81\therefore 26x=81$$

$$\therefore x=\frac{81}{26} \text{ and } x+3=\frac{81}{26}+3=\frac{159}{26}$$

$$\text{Again } \frac{81}{26}(y+10)=\frac{159}{26}y$$

$$81y+810=159y\ldots 78y=810\ \therefore y=\frac{810}{78}=10\tfrac{5}{13}\text{hrs.} \quad \textit{Ans.}$$

9. Let x be the first part, then $x+18$=second part diminished by 5 $\therefore$ the second part$=x+23$. Again, $x+18$=third part multiplied by 6 $\therefore$ the third part$=\dfrac{x+18}{6}$ and $x+18$ =fourth part divided by $2\frac{1}{2}$ $\therefore$ the fourth part$=\dfrac{5(x+18)}{2}$.

$$\text{Hence, } x+x+23+\frac{x+18}{6}+\tfrac{5}{2}(x+18)=127$$

$$\therefore 6x+6x+138+x+18+15x+270=762$$

$$\therefore 28x=336 \therefore x=12$$

$$\left.\begin{array}{rl} \therefore x \text{ the first part} &= 12 \\ x+23 \text{ the second part} = 12+23 &= 35 \\ (x+18) \text{ the third part} = \dfrac{12+18}{6} &= 5 \\ \tfrac{5}{2}(x+18) \text{ the fourth part} = \tfrac{5}{2}(12+18) &= 75 \end{array}\right\} \textit{Ans.}$$

10. Clearing off fractions we get

$$51-9x-20x-10=75-90x+35x+70$$

$$\therefore -9x+20x+90x-35x=75+70-51+10$$

$$\therefore 26x=104 \therefore x=4. \quad \textit{Ans.}$$

The expression

$$=\frac{(x+2a)(x-2b)+(x+2b)(x-2a)}{(x-2a)(x-2b)}$$

$$=\frac{x^2+2ax-2bx-4ab+x^2+2bx-2ax-4ab}{x^2-2ax-2bx+4ab}$$

$$=\frac{2x^2-8ab}{x^2-2x(a+b)+4ab}. \quad \text{But } x=\frac{4ab}{a+b}, \text{ i.e.,}$$

$$x(a+b)=4ab \therefore \text{the expression}$$

$$=\frac{2x^2-8ab}{x^2-2\times 4ab+4ab}$$

$$=\frac{2(x^2-4ab)}{x^2-4ab}=2. \quad \textit{Ans.}$$

Euclid.

1. Euclid I. 41.

2. (Fig. 23). Bisect ang. ABC by BE, meeting AC in E (I. 9). From E draw ED at right angles to AC, meeting AB in D. Then D is the required point.

$\because$ DE and BC are perp. to AC, $\therefore$ they are parallel (I., 82, cor.) $\therefore$ ang. DEB = ang. EBC (I. 29); but ang. EBC = ang. EBD (constr.) $\therefore$ ang. $DEB = EBD$ $\therefore$ $DE = DB$ (I. 6).

3. Euclid II. 12.

4. *Vide* Question 4 of 1880.

5. Euclid III. 21.

6. (Fig. 24.) AP, AD are tangents to cir. DPQ : $\therefore AD = AP$ (III., 17 Cor.)

In trs. APC, ADC, $AD=AP$, $PC=CD$, ang. APC= ang. ADC, each being a rt. ang. (III. 18), $\therefore$ ang. PAC=ang. CAD (I. 4). Similarly ang. DBC=ang. CBQ $\therefore$ ang. CAD, DBC= $\frac{1}{2}$ angs. DAP, DBQ. But angs. DAP, DBQ = 2 ang. $\because$ in the quadrilateral, angs. P, Q = 2 rt. angs. (III. 18). $\therefore$ angs. CAD, DBC = $\frac{1}{2}$ of 2 rt. ang. = one rt. ang. $\therefore$ ang. ACB is a rt. ang. (I. 32).

7. Euclid IV. 5.

8. Euclid IV. 11.

1884-85.

Arithmetic and Algebra.

WEDNESDAY, 15TH NOVEMBER.

GOVIND V. KURKARAY, B.A.

REV. R. SCOTT, M.A.

1. Reduce to a vulgar fraction $\cdot\dot{4}2857\dot{1}$. 8

Divide 301·6 by ·416.

Find the value of $\cdot475 \times £1 + \cdot\dot{4}\dot{2} \times £2\ 17s.\ 9d.$

2. A merchant buys 1,260 maunds of corn, $\frac{1}{5}$ of which sells at a gain of 5 %, $\frac{1}{3}$ at a gain of 8 %, and the re- 9

mainder at a gain of 12 %. If he had sold the whole at a gain of 10 %, he would have obtained £22 13*s.* more. What was the cost price per maund ?

3. A room 10ft. 6in. high, 22 ft. long, and 14 ft. broad is painted up to $\frac{1}{3}$ of its height and the remaining $\frac{2}{3}$ papered. The painting is charged at $7\frac{1}{2}d.$ per sq. yd., the paper costs 5*s.* 2*d.* per sq. yd. and the work of papering is charged at 2*d.* per sq. yd. How much will the whole cost amount to ? 9

4. A person sells out £3,850 4 per cent. stock at 104 and invests the proceeds in another stock at 143. If the dividend on this be $5\frac{3}{4}$ %, what will be the change in his income ? 9

5. What must be the rate of interest in order that the discount on £387 7*s.* $7\frac{1}{2}d.$ payable at the end of 3 years may be £41 10*s.* $1\frac{1}{8}d.$

---

6. Divide $(4x^3-3a^2x)^2+(4y^3-3a^2y)^2-a^6$ by $x^2+y^2-a^2$. 9

7. If $(a+b)(b+c)(c+d)(d+a)=(a+b+c+d)$ $(bcd+cda+dab+abc)$, then prove that $ac=bd$. 9

8. Show that $(a-b)(a-c)+2(b-c)(b-a)+2(c-b)$ $(c-a)$ is the sum of three squares. And resolve $10x^2-23x-5$ into the simplest factors. 9

9. Find G. C. M. of $7x^4-10ax^3+3a^2x^2-4a^3x+4a^4$ and $8x^4-13ax^3+5a^2x^2-3a^3x+3a^4$. 9

10. A certain number consisting of two digits, becomes 110 when the number obtained by reversing the digits is added to it; also the first number exceeds unity by 5 times the excess of the second number over unity. What is the number ? 9

11. A person walks from *A* to *B*, a distance of $7\frac{1}{2}$ miles, in 2 hours $17\frac{1}{2}$ minutes and returns in 2 hours and 20 minutes, his rate of walking uphill, downhill and on a level road being 3, $3\frac{1}{2}$, $3\frac{1}{4}$ miles per hour respectively. Find the length of the level road between *A* and *B*. 12

Euclid

1. Prove that a parallelogram of which one angle is 8
a right angle and the two adjacent sides are equal is a square. Show also that its diagonals bisect each other at right angles.

2. If the square described on one of the sides of a 12
triangle be equal to the sum of the squares described on the other two sides, prove that the angle contained by these two sides is a right angle.

Show that the straight line drawn from the right angle to the middle of the hypotenuse is equal to half the hypotenuse.

3. Divide a straight line into two parts so that the 6
rectangle contained by the whole and one of the parts may be equal to the square on the other part.

4. Define tangent.

Show that the shortest straight line which can be 9
drawn from the circumference of a circle from any point within or without is perpendicular to the tangent at the point where it meets the circumference.

5. Prove that the opposite angles of a quadrilateral 7
inscribed in a circle are together equal to two right angles.

Can a rhombus be inscribed in a circle?

6. Prove that the angle in a semi-circle is a right 12
angle.

Show that the circles described on any two sides of a triangle as diameters intersect in the third side.

7. Describe an isosceles triangle having the angles 10
at the base double the vertical angle.

8. Investigate a method of measuring the angles of 11
an equiangular polygon of a given number of sides.

Show that in a regular polygon of twelve sides each angle is five times the angle subtended by one of the sides at the centre of the circumscribed circle.

SOLUTIONS.

Arithmetic and Algebra.

1. $\frac{\cdot 428571}{999999}=\frac{3}{7}$. *Ans.* (See Appendix.)

$\frac{301\cdot6}{\cdot416}=\frac{301600}{416}=725$. *Ans.*

$\cdot475$ of £1 $=\frac{475}{1000}$ of £1 $=$ £$\frac{19}{40}$.

$\cdot\dot{4}\dot{2}=\frac{42}{99}=\frac{14}{33}$; $\frac{14}{33}$ of £2 17*s.* 9*d.* $=\frac{14}{33}\times$ £ $\frac{231}{80}=$ £ $\frac{49}{40}$.

£ $\frac{19}{40}+$ £ $\frac{49}{40}-$ £ $\frac{68}{40}=$ £1 14*s.* *Ans.*

2. Let the C. P. of one maund be £1. Then the profit derived by selling $\frac{1}{5}$ of the corn, *i. e.*, 1,260 $\times\frac{1}{5}$, or 252 mds., at a gain of 5% would be £$\frac{252\times5}{100}$, or £$\frac{63}{5}$.

The profit derived by selling $\frac{1}{3}$, *i. e.*, 1,260 $\times\frac{1}{3}$, or 420 mds., at a gain of 8 % would be £$\frac{420\times8}{100}$ or £$\frac{168}{5}$.

The profit derived by selling the remainder, *i. e.*, 1,260—(252+420), or 588 mds., at a gain of 12 % would be £ $\frac{588\times12}{100}$, or £$\frac{1764}{25}$.

$\therefore$ the total profit = £$\frac{63}{5}$ + £$\frac{168}{5}$ + £$\frac{1,764}{25}$ = £$\frac{2,919}{25}$.

By selling the whole at a gain of 10 % the gain would be £$\frac{1,260\times10}{100}$, or £126.

$\therefore$ The difference of the profits would be £126 $-$ £$\frac{2,919}{25}$, or £$\frac{231}{25}$ when the C. P. of 1 md. is £1. But the diff. between the profits is £22 13*s.* or £$\frac{345}{20}$; $\therefore$ the C. P. per md. $=\frac{453}{20}\times$ £$\frac{25}{231}=$ £2 9*s.* $0\frac{24}{77}$*d.* *Ans.*

Or thus:—

Gain on $\frac{1}{5}$ = cost of $\frac{5}{100} \times \frac{1}{5}$ or $\frac{1}{100}$

Gain on $\frac{1}{3}$ = cost of $\frac{8}{100} \times \frac{1}{3}$ or $\frac{2}{75}$

Gain on remainder, *i. e.*, on $1-(\frac{1}{5}+\frac{1}{3})$ or $\frac{7}{15}$ = cost of $\frac{12}{100} \times \frac{7}{15} = \frac{7}{125}$

∴ gain on the whole = cost of $\frac{1}{100} + \frac{2}{75} + \frac{7}{125}$ or $\frac{139}{1,500}$ of the whole.

If the whole were sold at a gain of 10 %, the gain would be equal to the cost of $\frac{10}{100}$ or $\frac{1}{10}$ of the whole.

∴ the difference of the profits in the two cases = cost of $\frac{1}{10} - \frac{139}{1500}$ or $\frac{11}{1500}$ of the whole.

But, by the question, the cost of $\frac{11}{1500}$ is £22 13*s.* or £$\frac{453}{20}$.

∴ the cost of the whole = £$\frac{453}{20} \times \frac{1,500}{11}$ = £$\frac{453 \times 75}{11}$.

∴ 1,260 mds. : 1 md. : : $\frac{453 \times 75}{11} = \frac{2,265}{924}$ = £2 9*s.* $0\frac{24}{77}$*d.* *Ans.*

3. If l = length, b = breadth, and h = height of the walls then the area of the four walls would be represented by $2h(b+l)$.

∴ the area of the walls $= 2 \times \frac{21}{2}(22+14) = 756$ sqr. ft.

$\frac{1}{3} \times 756 = 252$ sq. ft., area to be painted.

∴ $756-252$ or 504 sq. ft., area to be papered.

9 sq. ft. : 252 sq. ft. : : $7\frac{1}{2}d. = 17s.$ $6d.$, cost of painting one-third of the height.

The cost of papering 1 sq. yd. = $5s.$ $2d. + 2d. = 5s.$ $4d = 5\frac{1}{3}s.$

∴ 9 sq. ft. : 504 sq. ft. : : $5\frac{1}{3}s.$ = £14 18*s.* 8*d.*, cost of papering two-thirds of the height.

the total cost = £14 18*s.* 8*d.* + 17*s.* 6*d.* = £15 16*s.* 2*d.* *Ans.*

4. £100 : £3,850 : : £104 = £4,004, sum realized.

£100 : £3,850 : : £4 = £154, income.

£143 : £4,004 : : £5$\frac{3}{4}$ = £161, new income.

∴ £161 − £154 = £7, change in the income. *Ans.*

5. £387 7*s.* 7$\frac{1}{5}$*d.* Debt.

41 10*s.* 1$\frac{1}{5}$*d.* Discount.

£345 17*s.* 6*d.* Present value.

£345 17*s.* 6*d.* = £$\frac{2,767}{8}$; £41 10*s.* 1$\frac{1}{5}$*d.* = £$\frac{8,301}{200}$. £$\frac{2767}{8}$ P. W. : £100 : : $\frac{8,301}{200}$ Int. = £12 Int.

3 years : 1 : : £12 = 4. 4% rate of interest. *Ans.*

6. $(4x^3-3a^2x)^2+(4y^3-3a^2y)^2-a^6 = 16x^6-24a^2x^4+9a^4x^2+16y^6-24a^2y^4+9a^4y^2-a^6$

$=16(x^6+y^6)-24a^2(x^4+y^4)+9a^4(x^2+y^2)-a^6,$

$$(x^2+y^2)-a^2\,\big)\ 16(x^6+y^6)-24a^2(x^4+y^4)+9a^4(x^2+y^2)-a^6\ \big(16(x^4+y^4-x^2y^2)-8a^2(x^2+y^2)+a^4.$$

$$\underline{16(x^6+y^6)-16a^2(x^4+y^4-x^2y^2)}$$

$$-8a^2(x^4+y^4+2x^2y^2)+9a^4(x^2+y^2)$$

$$\underline{-8a^2(x^4+y^4+2x^2y^2)+8a^4(x^2+y^2)}$$

$$a^4(x^2+y^2)-a^6$$

$$\underline{a^4(x^2+y^2)-a^6}$$

$16(x^4+y^4-x^2y^2)-8a^2(x^2+y^2)+a^4.$ *Ans.*

7. $(a+b+c+d)(bcd+cda+dab+abc)$

$= \{(a+b)+(c+d)\}\{cd(b+a)+ab(d+c)\}$

$= cd(a+b)^2+cd(a+b)(c+d)+ab(a+b)(c+d)+ab(c+d)^2$

$\therefore$ the given expression becomes—

$cd(a+b)^2+ab(c+d)^2$

$=(a+b)(b+c)(c+d)(d+a)-ab(a+b)(c+d)-cd(a+b)(c+d)$

$=(a+b)(c+d)\{(b+c)(d+a)-ab-cd\}$

$=(a+b)(c+d)(bd+ab+cd+ac-ab-cd)$

$=(a+b)(c+d)(bd+ac)=bd(a+b)(c+d)+ac(a+b)(c+d)$

$\therefore cd(a+b)^2-ac(a+b)(c+d)=bd(a+b)(c+d)-ab(c+d)^2$

$\therefore (a+b)\{(a+b)cd-ac(c+d)\}=(c+d)\{bd(a+b)-ab(c+d)\}$

$\therefore (a+b)(acd+bcd-ac^2-acd)=(c+d)(abd+b^2d-abc-abd)$

$\therefore (a+b)(bcd-ac^2) =(c+d)(b^2d-abc)$

$\therefore c(a+b)(bd-ac) =b(c+d)(bd-ac)$

$\therefore c(a+b)=b(c+d) \quad \therefore ac+bc=bc+bd$

$\therefore ac=bd$, which was to be proved.

8. In the expression

$2(a-b)(a-c)+2(b-c)(b-a)+2(c-b)(c-a)$,

$(b-a)=-(a-b)$, $(c-b)=-(b-c)$ and $c-a=-(a-c)$

$\therefore =2(a-b)(a-c)-2(b-c)(a-b)+2(b-c)(a-c)$

$=(a-b)(a-c)+(a-b)(a-c)-(b-c)(a-b)-(b-c)(a-c)+(b-c)(a-c)+(b-c)(a-c)$

$=\{(a-b)(a-c)-(b-c)(a-b)\}+\{(a-b)(a-c)+(b-c)(a-c\}+\{(b-c)(a-c)-(b-c)(a-b)\}$

$=(a-b)\{(a-c)-(b-c)\}+(a-c)\{(a-b)+(b-c)\}+(b-c)\{(a-c)-(a-b)\}$

$=(a-b)(a-b)+(a-c)(a-c)+(b-c)(b-c)$

$=(a-b)^2+(a-c)^2+(b-c)^2$, *i. e.*, the given expression is a sum of three perfect squares.

$10x^2-23x-5$

$= 10x^2-25x+2x-5$

$= 5x(2x-5)+(2x-5)$

$=(5x+1)(2x-5)$. *Ans.*

9. $7x^4-10ax^3+3a^2x^2-4a^3x+4a^4$

$= 7x^4-7ax^3-3ax^3+3a^2x^2-4a^3x+4a^4$

$= 7x^3(x-a)-3ax^2(x-a)-4a^3(x-a)$

$= (x-a)(7x^3-3ax^2-4a^3)$

and $7x^3-3ax^2-4a^3$

$= 7x^3-7ax^2+4ax^2-4a^3$

$= 7x^2(x-a)+4a(x^2-a^2)$

$= (x-a)\{7x^2+4a(x+a)\}$

$= (x-a)(7x^2+4ax+4a^2)$

$\therefore$ the expression $=(x-a)(x-a)(7x^2+4ax+4a^2)$

Again, $8x^4-13ax^3+5a^2x^2-3a^3x+3a^4$

$= 8x^4-8ax^3-5ax^3+5a^2x^2-3a^3x+3a^4$

$= 8x^3(x-a)-5ax^2(x-a)-3a^3(x-a)$

$= (x-a)(8x^3-5ax^2-3a^3)$

and $8x^3-5ax^2-3a^3$

$= 8x^3-8ax^2+3ax^2-3a^3=8x^2(x-a)+3a(x^2-a^2)$

$= (x-a)\{8x^2+3a(x+a)\}= (x-a)(8x^2+3ax+3a^2)$

$\therefore$ the expression$=(x-a)(x-a)(8x^2+3ax+3a^2)$

$\therefore$ the H. C. F. $=(x-a)(x-a)$, *i. e.*, $(x-a)^2$. *Ans.*

10. Let x be the digit in the tens' place and y the digit in the units' place; then $10x+y$ is the number, and the number obtained by reversing the digits $=10y+x$.

$\therefore$ by the 1st condition $10x+y+10y+x=110$

$\therefore$ $11x+11y=110$ $\therefore$ $x+y=10$ ....(i)

The excess of the second number over unity$=10y+x-1$; five times the excess of the second number over unity $=5(10y+x-1)$.

Now, by the 2nd condition $10x+y > 1$ by $5(10y+x-1)$

$\therefore$ $10x+y=1+5(10y+x-1)$

$\therefore$ $10x+y-50y-5x=-4$

$\therefore$ $5x-49y=-4$ ....(ii)

Multiplying the first equation by 5 and subtracting it from the second we get—

$$\begin{array}{r} 5x-49y=-\ 4 \\ \underline{5x+5y=\ \ 50} \\ -54y=-54 \end{array} \therefore y=1;\ \text{hence } x=9$$

$\therefore$ the number $=91$. *Ans.*

11. Let x miles be the distance uphill and y miles the distance downhill while walking from A to B; then as the whole distance from A to B is equal to $7\frac{1}{2}$ miles, the length of the level road will be $7\frac{1}{2}-x-y$ miles.

$\therefore$ the total time taken by the person to walk the whole distance $=\frac{x}{3}+\frac{y}{3\frac{1}{2}}+\frac{7\frac{1}{2}-x-y}{3\frac{1}{4}}$, which, by the question, is equal to 2 hrs. $17\frac{1}{2}$ min.

$$\therefore \frac{x}{3}+\frac{2y}{7}+\frac{30-4x-4y}{13}=\frac{55}{24}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Now, when the person walks from B to A, the distance uphill $=y$ miles and the distance downhill $=x$ miles, therefore the length of the level road travelled will be $7\frac{1}{2}-x-y$ miles $\therefore$ the total time taken to traverse the whole distance from B to $A=\frac{x}{3\frac{1}{2}}+\frac{y}{3}+\frac{7\frac{1}{2}-x-y}{3\frac{1}{4}}$, which, by the question, is equal to 2 hrs. 20 mins.

$$\therefore \frac{2x}{7}+\frac{y}{3}+\frac{30-4x-4y}{13}=\frac{7}{3}\ldots\ldots\ \ldots\ldots\ldots\ldots\text{(ii)}$$

If we add the second equation to the first, we get—

$$\frac{x}{7}-\frac{y}{7}=-\frac{1}{8}.\ i.\ e.,\ 8x-8y=-7\ldots\ldots\ldots\ldots\text{(iii)}$$

And the second equation, when simplified, is equal to $78x+91y+630-84x-84y=637$,

$$i.\ e.,\ -6x+7y=7\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Multiplying the third by 3 and the second by 4 and adding the two together, we get—

$$\begin{array}{r} 24x-24y=-21 \\ \underline{-24x+28y=\ \ 28} \\ 4y=\ \ \ 7 \end{array} \therefore y=\frac{7}{4}=1\frac{3}{4}\ \therefore\ x=\frac{7}{8}$$

Hence the length of the level road between A and B $=7\frac{1}{2}-\frac{1}{8}-1\frac{3}{4}=4\frac{1}{8}$ miles. *Ans.*

Euclid.

1. Euc. I. 46.

(Fig. 25.) ∵ in trs. ABE, DEC, ang. $ABE=$ang. EDC (I. 29), ang. $AEB=$ang. DEC (I. 15), and $AB=DC$, ∴ the trs. are equal in all respects (I. 26). ∴ $AE=EC$ and $DE=EB$, *i. e.*, the diagonals are bisected at E.

Again, ∵ in trs. ADE, ABE, $AB=AD$, and AE is common, $DE=BE$ (proved) ∴ the trs. are equal in all respects (I. 8) ∴ ang. $DEA=$ang. AEB; but they are adjacent angles ∴ they are rt. angs. (def. 7) ∴ AC, BD bisect each other at right angles.

2. Euc. I. 48.

(Fig. 27.) Let ABC be a triangle, right angled at B, and let D be the middle point of AC, the hypotenuse. Join BD. From D draw DE, DF perp. to AB, BC respectively (I. 12). Angs. EBF, BFD are equal to two rt. angs. ∴ EB is prll. to DF (I. 28); similarly BF is prll. to ED, ∴ $EBFD$ is a plm. ∵ AB is prll. to DF and AC meets them ∴ ang. FDC = ang. DAE (I. 29). Again, ED is prll. to BC and AC meets them, ∴ ang. $ADE=$ang. DCF (I. 29). Now in trs. ADE, DCF ang. $DAE=$ang. CDF and ang. $ADE=$ang. DCF, and $AD=DC$ (hyp.), ∴ $AE=DF$ (I. 26); but $DF=EB$ (I. 34), ∴ $AE=EB$ (ax. 1). Hence in trs. AED, BED, $AE=ED$, ED is common, and ang. $AED=$ang. BED (ax. 11), ∴ $AD=BD$ (I. 4), *i.e.*, $BD=\frac{1}{2}AC$.

3. Euclid II. 11.

4. From III., 7, 8, it can be shewn that the shortest line is a part of the diameter, or, the diameter produced, ∴ the tangent at the extremity of the diameter is a line at right angles to it, *i. e.*, the shortest line is perpendicular to the tangent (III. 16).

5. Euclid III. 22.

A rhombus cannot be inscribed in a cir. ∵ the opposite angs. must be together equal to two rt. angs. (III. 22), which is not the case.

6. Euclid III. 31.

If we draw a perp. from the vertex to the base, both the circles on the sides of the tr. as diameters must pass through the foot of the perp. (III. 31).

7. Euclid IV. 10.

8. Suppose that the regular polygon has n number of sides, ∴ by the 1st corollary to I. 32, n angles $+4$ rt. angs. $=2n$ rt. angs. ∴ n angles$=(2n-4)$ rt. angs. ∴ each angle of the figure $=\frac{2n-4}{n}$ right angles.

Hence, one angle of a regular polygon of 12 sides $=\frac{2\times12-4}{12}$, *i. e.*, $1\frac{2}{3}$ right angles.

Again, all the angs. at centre are equal to 4 rt. angs. (I. 15, cor. 2), and as all angs. can be shewn to be equal to one another, one angle of the regular polygon of 12 sides$=\frac{4}{12}$ *i. e.*, $\frac{1}{3}$ of a rt. ang., *i. e*, $\frac{1}{5}$ of the polygon.

1885-86.

WEDNESDAY, 2ND DECEMBER.

Arithmetic and Algebra.

KAWASJI JAMSHEDJI SANJANA, M.A.
KRISHNAJI BALVANT WAGLE, M.A.

1. Reduce $\frac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}$ of 2 guineas $+\frac{7}{12}\times\frac{9\div\dot{1}}{14\times3}$ of 4 crowns $-\frac{\cdot8\dot{3}\text{ of }1\frac{5}{7}-\frac{1}{7}}{1\frac{5}{7}}$ of £1 to the decimal of 5 guineas. 9

and prove that $\frac{6+5}{11+7}$ is greater than $\frac{6}{11}$ and less than $\frac{5}{7}$.

2. A man contracts to perform a piece of work in 30 days and immediately employs 15 men on it; at the end of 24 days the work is only half done. How many boys should be given to assist them that the contract may be fulfilled, each boy working two-fifths as much as each man? 7

3. A person buys 80 tons of coal and after selling them again at 1*s*. 6*d*. per sack finds that he has gained £4; had he sold them for 1*s*. 4*d*. per sack, he would have lost £6. Find the weight of each sack and the cost price per ton. 11

4. A field of 7 acres is sown with wheat, barley and maize, the areas of the crops being respectively as $2\frac{1}{2} : 3\frac{1}{2} : 4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field? 10

5. If the three per cents. are at $92\frac{3}{4}$ and the four per cents. at $123\frac{1}{4}$, in which should one invest; and how much is one investing when the difference in income is a shilling? 9

---

6. Find the continued product of—

x^2-2y^2, $x^2-2xy+2y^2$, x^2+2y^2, and $x^2+2xy+2y^2$. 7

7. Explain the terms *factor*, *multiple*, and *common measure*. 7

Resolve into factors, and thence find the L. C. M. of—

$a^2+6ab+5b^2$, $a^3-a^2b-ab^2+b^3$, and $a^2+5ab-6b^2$.

8. Simplify the following fractional expressions:— 10

$$\left(x+\frac{16x-27}{x^2-16}\right) \div \left(x-1+\frac{13}{x+4}\right); \text{ and}$$

$$\frac{\sqrt{a^4-4a^3+12a^2-16a+16}}{\sqrt[3]{a^9+24a^6+192a^3+512}}.$$

9. Solve the equations—

(i) $\frac{2x}{3}-\frac{1-\frac{1}{2}x}{4x}=\frac{2x+7}{12}-\frac{1-x}{2}$

(ii) $\begin{cases}\frac{p}{x}+\frac{q}{y}=0.\\ px+qy=r.\end{cases}$

10. I bought a horse and a carriage for £90 ; I sold 3
the horse at a gain of 12 per cent. and the carriage at a loss of 4 per cent. and gained on the whole 6 per cent. Find the prime cost of the carriage.

11. A man walks one-third of the distance from A 10
to B, at the rate of a miles per hour and the remainder at the rate of $2b$ miles per hour, and travelling back from B to A at the rate of $3c$ miles per hour takes the same time. Prove that $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$.

Euclid.

1. If a straight line fall on two parallel straight lines 16
it makes the alternate angles equal to one another and exterior angle equal to the interior and opposite angle on the same side and also the two interior angles on the same side together equal to two right angles.

Prove that straight lines bisecting two adjacent angles of a parallelogram intersect at right angles.

2. The opposite sides and angles of a parallelogram 15
are equal to one another.

Given the three middle points of the sides of a triangle, construct it.

3. If a straight line be divided into any two parts, the 8
square on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

4. What are similar segments of circles? If two circles touch each other any straight lines drawn through the point of contact will cut off similar segments. 10

5. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles. 13

A six-sided figure, $ABCDEF$, is inscribed in a circle, prove that the sum of the angles A, C, E is equal to the sum of the angles B, D, F.

6. Inscribe a circle in a given triangle. 13

Describe a circle touching three given straight lines, two of which are parallel.

SOLUTIONS.

Arithmetic and Algebra.

1. $$\frac{\frac{5}{2}-\frac{5}{6}}{\frac{5}{2}+\frac{5}{6}} = \frac{5}{3} \times \frac{3}{10} = \frac{1}{2}$$

$\frac{1}{2}$ of 2 guineas $= \frac{1}{2}$ of $42s. = 21s.$

$$\frac{7}{12} \times \frac{9\times 10}{14\times 3} \times 20s. = 25s.$$

$$\frac{83-8}{90},\ i.\ e.,\ \frac{\frac{75}{90}\times\frac{12}{7}-\frac{1}{7}}{\frac{10}{7}} = \frac{9}{7}\times\frac{7}{10} = \frac{9}{10}$$

$$\frac{9}{10}\times 20s. = 18s. \qquad \therefore 21s. + 25s. - 18s. = 28s.$$

$$5 \text{ half-guineas} = \frac{5}{2}\times 21s. = \frac{105}{2}\,s.$$

$$\therefore 28 \times \frac{2}{105} = \frac{8}{15} = \cdot 5\dot{3}. \quad \textit{Ans.}$$

$$\frac{6+5}{11+7} = \frac{11}{18}$$

$$\frac{11}{18} > \frac{6}{11} \quad \because \frac{121}{18\times 11} > \frac{108}{18\times 11}$$

$$\frac{11}{18} < \frac{5}{7} \because \frac{77}{18\times 7} < \frac{90}{18\times 7}.$$

2. Half the work is done by 15 men in 24 days ∴ in one day half the work can be done by 24×15 men. Now, according to the contract, the remaining half of the work must be finished in $30-24$, or 6, days; therefore, we are to find out the number of men who will finish half the work in 6 days.

24×15 men can do half in a day ∴ $\frac{24 \times 15}{6}$ men can do half in 6 days, *i. e.*, 60 men in all are required.

∴ $60-15$, or 45, additional number of men required. By the question each boy works $\frac{2}{5}$ths as much as each man.

∴ $45 \times \frac{5}{2} = \frac{225}{2} = 112\frac{1}{2}$, *i. e.*, 113 boys are to be engaged. *Ans.*

3. In the first case the man has got £4 more
,, second ,, ,, ,, £6 less.

∴ he gets £10 less in the second case than in the first. But in the second case he gets 2*d.* less (1*s.* 6*d.*—1*s.* 4*d.*) per sack

∴ a difference of 2*d.* per sack made a difference of £10 on the whole.

$2d. : 10 \times 240d. :: 1 \text{ sack} = 1{,}200$ sacks.

By the question 1,200 sacks weigh 80 tons ∴ the weight of one sack $= \frac{80}{1{,}200} = \frac{1}{15}$ of a ton $= \frac{1}{15} \times 20 = \frac{4}{3} = 1\frac{1}{3}$ cwts. *Ans.*

1 sack . 1,200 sacks $:: 1\frac{1}{2}s. = 1{,}800s. = £90$, selling price of 1,200 sacks.

Now, the selling price is £90 and the gain is £4 ∴ the cost price of 1,200 sacks, *i. e.*, 80 tons $= £90 - £4 = £86$.
80 tons : 1 ton $:: £86 = £1$ 1*s.* 6*d.*, cost price of one ton. *Ans.*

4. Altogether there are $2\frac{1}{2} + 3\frac{1}{2} + 4\frac{1}{2} = 10\frac{1}{2}$ parts.
$10\frac{1}{2}$ acres : 7 :: $2\frac{1}{2}$ acres of wheat $= \frac{5}{3}$ acres of wheat
$10\frac{1}{2}$ acres : 7 :: $3\frac{1}{2}$ acres of barley $= \frac{7}{3}$,, ,, barley
$10\frac{1}{2}$ acres : 7 :: $4\frac{1}{2}$ acres of maize $= 3$,, ,, maize

The value of 1 acre of wheat $= £7$, and the values of wheat, barley and maize are proportional to $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$,

$£2\frac{1}{2} : £7 :: £3\frac{1}{2} = \frac{£49}{5}$, price per acre of barley.

$£2\frac{1}{2} : £7 :: £4\frac{1}{2} = \frac{63}{5}$ „ „ „ maize.

$1 : \frac{5}{3} :: £7 = £\frac{35}{3}$ price of $\frac{5}{3}$ acres of wheat

$1 : \frac{7}{3} :: £\frac{49}{5} = £\frac{343}{15}$ „ „ $\frac{7}{3}$ „ barley

$1 : 3 :: £\frac{63}{5} = £\frac{189}{5}$ „ „ 3 „ maize

$\therefore$ the worth of all the crops in the field

$= £\frac{35}{3} + £\frac{343}{15} + £\frac{189}{5} = £72\frac{1}{3}$. *Ans.*

(5) Let $£92\frac{3}{4}$ be invested in both cases.

$\therefore £92\frac{3}{4} : £92\frac{3}{4} :: £3 = £3$ income in the first case.

$£123\frac{1}{4} : £92\frac{3}{4} :: £4 = \frac{1484}{493} = £3\frac{5}{493}$, income in the second case.

As the income in the second case is greater than that in the first, one should invest in the second.

$£3\frac{5}{493} - £3 = \frac{5}{493}$ difference in income when $£92\frac{3}{4}$ is invested

$£\frac{5}{493} : £\frac{1}{20} :: £92\frac{3}{4} = £457\frac{103}{200} = £457\ 5s.\ 1\frac{4}{5}d.$ *Ans.*

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6. $(x^2-2y^2)(x^2+2y^2)=x^4-4y^4$ and
$(x^2-2xy+2y^2)(x^2+2xy+2y^2)$
$=\{(x^2+2y^2)-2xy\}\{(x^2+2y^2)+2xy\}$
$=(x^2+2y^2)^2-4x^2y^2=x^4+4x^2y^2+4y^4-4x^2y^2$
$=x^4+4y^4$
$\therefore (x^4-4y^4)(x^4+4y^4)=x^8-16y^8$. *Ans.*

7. When a quantity consists of the product of two or more quantities each of the latter is called a *factor* (or maker) of the product.

A *multiple* of a quantity is any quantity which contains it without remainder.

A *common measure* of two or more quantities is one which divides each of them without remainder.

$$a^2+6ab+5b^2=a^2+5ab+ab+5b^2$$
$$=a(a+5b)+b(a+5b)=(a+b)(a+5b)$$
$$a^3-a^2b-ab^2+b^3=a^2(a-b)-b^2(a-b)$$
$$=(a^2-b^2)(a-b)=(a-b)^2(a+b)$$
$$a^2+5ab-6b^2=a^2+6ab-ab-6b^2$$
$$=a(a+6b)-b(a+6b)=(a-b)(a+6b)$$

$\therefore$ the L.C.M. of $(a+b)(a+5b)$, $(a-b)^2(a+b)$, $(a-b)(a-6b)$
$=(a-b)^2(a+b)(a+5b)(a+6b)$. *Ans.*

8. $$x+\frac{16x-27}{x^2-16}=\frac{x^3-27}{x^2-16}=\frac{(x-3)(x^2+3x+9)}{(x-4)(x+4)}$$

$$x-1+\frac{13}{x+4}=\frac{x^2+3x+9}{x+4}$$

$$\therefore\quad \frac{(x-3)(x^2+3x+9)}{(x-4)(x+4)}\times\frac{x+4}{x^2+3x+9}=\frac{x-3}{x-4}.\quad \textit{Ans.}$$

$$\sqrt{a^4-4a^3+12a^2-16a+16}$$

$$a^4-4a^3+12a^2-16a+16=a^4-4a^3+8a^2+4a^2-16a+16$$
$$=a^4-2a^2(2a-4)+(2a-4)^2=\{a^2-(2a-4)\}^2$$

$\therefore$ the square root $=a^2-2a+4$.

$$\sqrt[3]{a^9+24a^6+192a^3+512}$$

$$a^9+24a^6+192a^3+512=(a^3)^3+3.(a^3)^2 8+3a^3(8)^2+(8)^3$$
$=(a^3+8)^3$ $\therefore$ the cube root $=a^3+8$

$\therefore$ the whole expression $=\dfrac{a^2-2a+4}{a^3+8}$

$$=\frac{a^2-2a+4}{(a+2)(a^2-2a+4)}=\frac{1}{a+2}.\quad \textit{Ans.}$$

9. (i) $\frac{2x}{3}-\frac{2-x}{8x}=\frac{2x+7}{12}-\frac{1-x}{2}$

$16x^2-6+3x=4x^2+14x-12x+12x^2$

$3x-2x=6 \quad \therefore \quad x=6.$ *Ans.*

(ii) $\frac{p}{x}+\frac{q}{y}=0$; $px+qy=r$

$py+qx=0$ .... (i)

$px+qy=r$ .... (ii)

Multiply (ii) by q and (i) by p, and substract one from the other.

$pqx+p^2y=0$

$$\frac{pqx+q^2y=qr}{y(p^2-q^2)=-qr} \therefore y=\frac{-qr}{-(q^2-p^2)}=\frac{qr}{q^2-p^2}. \quad \textit{Ans.}$$

From the first equation $qx=-py$

$$\therefore x=\frac{-p}{q}\times y=\frac{-p}{q}\times\frac{-qr}{p^2-q^2}=\frac{pr}{p^2-q^2}. \quad \textit{Ans.}$$

10. Let £ x be the prime cost of the carriage, then £ $(90-x)$ is the prime cost of the horse.

£100 : £$(90-x)$ :: £12 $=$ £$\frac{3(90-x)}{100}$ gain on the horse.

£100 : £x :: £4 $=$ £$\frac{x}{25}$ loss on carriage.

£100 : £90 :: £6 $=$ £$\frac{27}{5}$ gain on the whole

$\therefore \frac{3(90-x)}{25}-\frac{x}{25}=\frac{27}{5}$

$\therefore 3(90-x)-x=135 \quad \therefore x=33\frac{3}{4}$

$\therefore$ £$33\frac{3}{4}$ is the prime cost of the carriage. *Ans.*

11. Let x miles be the distance from A to B. He rides $\frac{x}{3}$ miles at the rate of a miles per hour. $\therefore$ he takes $\frac{x}{3a}$ hours.

Again he rides $\frac{2x}{3}$ miles at the rate of $2b$ miles per hour

$\therefore$ he takes $\frac{x}{3b}$ hours.

Travelling from B to A, *i.e.*, x miles, at the rate of $3c$ miles per hour, he takes $\frac{x}{3c}$ hours.

$\therefore$ $\frac{x}{3a}+\frac{x}{3b}=\frac{x}{3c}$ $\therefore \frac{1}{a}+\frac{1}{b}=\frac{1}{c}$, which was to be proved.

Euclid.

1. (*a*) Euclid I. 29.

(*b*) (Fig. 27). Let $ABCD$ be the parallelogram, and let angs. A, D be bisected by AE, DE meeting in E. $\because$ AB is prll. to DC and AD meets them, $\therefore$ angs. BAD, ADC together = 2 rt. angs. (I. 29); but as they are bisected, angs. EAD, EDA together = one rt. ang. $\therefore$ ang. AED = one rt. ang. (I. 32).

2. Euclid I., 34.

(Fig. 28). Let D, E, F be the given points. It is required to make a triangle whose sides shall be bisected at those points.

Join ED, DF, EF. Through D draw ADC prll. to EF through E draw AEB prll. to DF, and through F draw BFC prll. to ED (I. 31). Then ABC shall be the required tr. $\because$ EF is prll. to AC and DF prll. to AB (constr.) $\therefore$ $AEFD$ and $DEFC$ are plms. $\therefore$ $AD=EF$, and $EF=DC$ (I. 34) $\therefore$ $AD=DC$. Similarly it can be proved that $AE = BE$ and $BF=FC$ $\therefore$ ABC is the required triangle.

3. Euclid II., 7.

4. (Fig. 29). Let AB, AC be the diameters. Draw ADE cutting the circles in D and E. Join DB, CE. Then angs. ADB, AEC in semicircles are rt. angs. (III. 31).

$\therefore$ ang. ADB=ang. AEC, (ax. 11), and they are alternate, $\therefore$ BD is prll. to CE.(I. 27), and angs. DAB, EAC being vertically opposite are equal (I. 15).

$\therefore$ the angs. DBA, ACE are equal (I. 32), *i. e.*, the segments DBA, ECA are similar (def. 15, Bk. III).

N.B.—The proof is similar when the circles touch each other internally. In that case angs. DAB, EAC are coincident.

5. Euclid III. 22.

(Fig. 30) Join AD.

$\because$ $ABCD$ is a quadrilateral inscribed in the circle $ABCDEF$
$\therefore$ angs. ABC, ADC together=2 rt. angs. (III. 22.).

Similarly angs. AFE, EDA together = 2 rt. angs. $\therefore$ angs. at B, D, F are together equal to 4 rt. angs.

In the same manner it may be shown that angs. at A, E, C are together equal to 4 rt. angs.

$\therefore$ angs. $A+E+C$=angs. $B+D+F$.

6. (*a*) Euclid IV. 4.

(*b*) (Fig. 31.) Let AB, CD and AE be three given str. lines. two of which AB, CD are prll. Bisect angs. BAE, DEA by AF, EF respectively. From F draw FG, FH, FK perp. to CD, AB, AE respectively.

Now, in trs. HAF, AFK ang. H = ang. K (ax. 11) and ang. HAF = ang. FAK (constr.) and AF is common $\therefore$ $HF = KF$ (I. 26).

Similarly, $FG = FK$, $\therefore$ HF, FK, FG are all equal to one another. Hence from the centre F with distance of any of them describe a circle. This circle will pass through the other two and will touch the three given straight lines AB CD, AE in the points H, G, and K respectively (III 16, cor.).

1886-87.

WEDNESDAY, 19TH NOVEMBER.

J. T. HATHORNTHWAITE, M. A.; JOHN JACK

Arithmetic and Algebra.

1 Explain carefully the meaning of *Prime Number*, *Factor*, *Divisor*, *Measure*, *Multiple*. 10

Resolve 5005 into its prime factors. Add together as decimals

$$8{\cdot}1\dot{3}\dot{8},\ 14{\cdot}\dot{6}565\dot{1},\ {\cdot}2050896\dot{3}$$

2. The circumference of the fore wheels of a carriage is $6\frac{7}{8}$ feet and that of the hind wheel is $12\frac{5}{6}$ feet. How many feet must the carriage pass over before the wheels shall have made a complete number of revolutions. 4

3. A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture be $\frac{1}{2}$ water and $\frac{1}{2}$ syrup? 8

4. The surface of a cube is $308{\cdot}\dot{1}\dot{6}$ sq. ft.; find the length of its edge. 11

Extract the cube root of $45{\cdot}\dot{6}9\dot{8}$ to 4 places of decimals.

5. If the price of gold be £3 10*s*. $10\frac{1}{2}$*d*. an oz. and a cubic inch of gold weigh 10 oz., what is the price of gold that would be required to gild a dome whose surface is 5,000 sq. ft., the thickness of the gold gilding being ·0002 of an inch. 7

6. A person invests in 4 per cent. Government paper so as to receive 4 % clear when the income tax is 5 pies in the rupee: what percentage will be received if the tax be increased to 7 pies in the rupee. 6

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7. Simplify—

$$24\{x-\tfrac{1}{2}(x-1)\}\{x-\tfrac{2}{3}(x-2)\}\{x-\tfrac{3}{4}(x-1\tfrac{1}{3})\}.$$ 6

and substract the result from $\dfrac{(x^2+7x+12)(x^2-x-6)}{(x-3)}$.

8. Reduce to its simplest form— 8

$$\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}.$$

9. Resolve into factors $4(ad-bc)^2-(a^2+d^2-b^2-c^2)^2$. 10

If $x+\frac{1}{x}=p$, express $x^3+\frac{1}{x^3}$ in terms of p.

10. (i) Find the G. C. M. and L. C. M. of—
$x^5+x^4-4x^3+2x^2+6x-9$, x^4-x^3+6x-9 and 8
$x^4+2x^3-5x^2-6x+9$.

(ii) Extract the square root of $\frac{x^3}{16}-\frac{x^2}{6}-\frac{x^{\frac{3}{2}}}{4}+\frac{x}{9}+\frac{x^{\frac{1}{2}}}{3}+\frac{1}{4}$; and the cube root of $8x^6-36x^5+66x^4-63x^3+33x^2-9x+1$.

11. Solve the equations— 10

(i) $\sqrt{\{(x-a)^2+2ab+b^2\}}=x-a+b$.

(ii) $$\begin{cases}\frac{15}{x}-\frac{1}{y}-4\frac{1}{2}=1\\ \frac{9}{x}+\frac{2}{y}=4\end{cases}$$

12. A man walks from the University towards Malabar Hill at the rate of 3 miles an hour, runs part of the way back at the rate of $8\frac{1}{3}$ miles an hour, and then walks the remainder in 1 hour 5 minutes. He was out 2 hours 44 minutes: find how far he had gone. 8

Euclid.

1. All the exterior angles of any rectilineal figure, made by producing the sides successively in the same direction, are together equal to four right angles. 8

Show that the angle of a regular pentagon is to the angle of a regular decagon as 3 to 4.

2. The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another. 8

3. P and Q are any two points, and A and B are any other two points on opposite sides of the straight line PQ. The triangle APQ is equal to the triangle BPQ and PQ, or PQ produced, cuts AB in C: prove that AC is equal to CB. 10

4. If a straight line be divided into two equal and also into two unequal parts the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section. 17

If two chords in a circle cut each other at right angles the sum of the squares on their segments is equal to the square on the diameter.

5. To draw a straight line from a given point without the circumference, which shall touch a given circle. 18

A quadrilateral circumscribing a circle has two of its sides parallel: show that each of the other two sides subtends a right angle at the centre.

6. Prove that the lines bisecting the angles of a regular pentagon meet in a point. 7

7. About a given circle to describe a triangle equiangular to a given triangle. 7

SOLUTIONS.

Arithmetic and Algebra.

1. A number which cannot be separated into factors, which are respectively greater than unity, is called a *prime number*.

When a number is made up by multiplying two or more numbers, each of the latter is called a *factor* of the former. In multiplication both the multiplicand and the multiplier

are called *factors*, because they are factors or makers of the product.

Divisor of any number is a number which is contained in that number, called the dividend, any number of times.

A *measure* of any given number is a number which will divide the given number exactly.

A *multiple* of any given number is a number which contains it an exact number of times.

$$\begin{array}{r} 5)\underline{\ 5005} \\ 7)\underline{\ 1001} \\ 11)\underline{\ \ 143} \\ 13 \end{array} \qquad\qquad \begin{array}{r} 8\cdot 1383838383838 \\ 14\cdot 6565165165165 \\ \cdot 2050896350896 \\ \hline 22\cdot 9999899899899 \end{array}$$

5, 7, 11, 13. *Ans.* $\qquad$ $22\cdot 98\dot{9}9\dot{8}$. *Ans.*

2. The circumference of the fore-wheel is $6\frac{7}{8}$ feet, therefore, the first revolution will be completed when the carriage has passed over $6\frac{7}{8}$ feet; the second revolution will be completed when the carriage has passed over $6\frac{7}{8}+6\frac{7}{8}$ feet: the third when the carriage has passed over $6\frac{7}{8}+6\frac{7}{8}+6\frac{7}{8}$ feet; and generally it will make a complete number of revolutions when the carriage has passed over a distance which is any multiple of $6\frac{7}{8}$ feet. Similarly, the hind-wheel will make a complete number of revolutions when the carriage has passed over a distance which is any multiple of $12\frac{5}{6}$ feet.

$\therefore$ the two wheels will simultaneously make a complete number of revolutions when the carriage has passed over a distance which is any multiple of $6\frac{7}{8}$ feet and $12\frac{5}{6}$ feet.

$\therefore$ the two wheels will simultaneously make a complete number of revolutions for the first time when the distance passed over is the L. C. M. of $6\frac{7}{8}$ feet and $12\frac{5}{6}$ feet, *i.e.*, of $\frac{55}{8}$ and $\frac{77}{6}$.

$$\text{The required L. C. M.} = \frac{\text{L. C. M. of 55 and 77}}{\text{G. C. M. of 8 and 6}}$$

$$= \frac{385}{2} \text{ feet} = 192\tfrac{1}{2} \text{ feet.} \quad \textit{Ans.}$$

3. The cask contains 5 parts syrup and 3 parts water. In order that the resulting mixture may be half and half, *i.e.*, may contain 4 parts syrup and 4 parts water, we must draw off 1 part out of 5 parts of syrup, *i. e.*, $\frac{1}{5}$ of the syrup must be drawn off.

Parts 5 : 1 :: 1 the whole mixture $= \frac{1}{5}$ of the whole mixture. *Ans.*

Or thus—

There are 8 parts in all, $\therefore$ $\frac{5}{8}$ of the mixture is syrup and $\frac{3}{8}$ of the mixture is water.

In the second case $\frac{1}{2}$ of the mixture should be syrup and $\frac{1}{2}$ water.

$\therefore$ $\frac{1}{2}-\frac{3}{8}=\frac{1}{8}$ of the mixture is the excess of water.

Now the question is : "How much of the mixture must be drawn off and replaced by water so that we may introduce into the mixture a quantity of water equal to $\frac{1}{8}$ of the mixture?" If the whole mixture be drawn off and replaced by an equal quantity of water, we would introduce a quantity of water equal to $\frac{5}{8}$ of the mixture.

Hence, to introduce a quantity of water equal to $\frac{1}{8}$ of the mixture, we should draw off $\frac{8}{5}\times\frac{1}{8}=\frac{1}{5}$ of the mixture. *Ans.*

4. (i) $308\frac{1}{6}\frac{5}{0} = 308\frac{1}{6}=\frac{1,849}{6}$ sq. ft.

There are 6 equal sides of the cube, $\therefore$ one side of the cube contains $\frac{1,849}{6}\times\frac{1}{6} = \frac{1,849}{36}$ sq. ft.

$\therefore$ the length of its edge $=\sqrt{\frac{1849}{36}}=\frac{43}{6}$ ft. $=7\frac{1}{6}$ feet. *Ans.*

$\sqrt[3]{45 \cdot 698}$

$3\times30^2=$	2700	45·698989898989 (3·5752
		27
$3\times30\times5=$	450	18698
$5^2=$	25	15875
	3175	2823989
$3\times350^2=$	367500	
$3\times350\times7=$	7350	
$7^2=$	49	
	374899	2624293
$3\times3570^2=$	38234700	199696898
$3\times3570\times5=$	53550	
$5^2=$	25	
	38288275	19144175
		8255523989
$3\times35750^2=$	3834187500	
$3\times3570\times2=$	214500	
$2^2=$	4	
	3834402004	7668804008

3·5752. *Ans.*

Or thus:—

95	2700	45·698989898989 (3·5752
10	475	27
1057	3175	18698
14	25	15875
10715	367500	2823989
10	7399	2624293
107252	374899	199696898
	49	191441375
	38234700	8255523989
	53575	7668804008
	38288275	
	25	
	3834187500	
	214504	
	3834402004	

5. $5,000 \times 144$ sq. in. = surface of the dome.

$$5,000 \times 144 \times \frac{2}{10000} = 144 \text{ cubic inches.}$$

1 c. in : 144 :: 10 oz. = 1,440 oz., weight of gold

$$£3\ 10s.\ 10\tfrac{1}{2}d. = £\frac{567}{160}$$

$$1 \text{ oz.} : 1,440 :: £\frac{567}{160} = £9 \times 567 = £\ 5,103. \quad \textit{Ans.}$$

6. When 5 pies are deducted from Re. 1, the net income is Re. $\frac{187}{192}$.

When 7 pies are deducted from Re. 1 the net income is Re. $\frac{185}{192}$

$1 : 4 ::$ Rs. $\frac{187}{192}$, = Rs. $\frac{187}{48}$ net income when 5 pies are deducted.

$1 : 4 ::$ Rs. $\frac{185}{192}$ = Rs. $\frac{185}{48}$ net income when 7 pies are deducted;

$$\frac{187}{48} : \frac{185}{48} :: 4 \text{ per cent.} = \frac{185}{48} \times \frac{48}{187} \times \frac{4}{1}$$

$$= \frac{740}{187} \text{ per cent.} = 3\tfrac{179}{187}\ \%. \quad \textit{Ans.}$$

7. (i) $x - \frac{x-1}{2} = \frac{x+1}{2}$, $x - \frac{2x-4}{3} = \frac{x+4}{3}$,

$$x - \frac{3x}{4} + 1 = \frac{x+4}{4}.$$

$$\therefore \text{ the expression } = 24 \times \frac{x+1}{2} \times \frac{x+4}{3} \times \frac{x+4}{4}$$

$$= (x+1)(x+4)(x+4) \quad = x^3 + 9x^2 + 24x + 16.$$

Now, we are to subtract this result from

$$\frac{(x^2+7x+12)(x^2-x-6)}{x-3},$$

$$i.\,e., \frac{(x+4)(x+3)(x-3)(x+2)}{x-3}$$

$i.\,e.$, $(x+4)(x+3)(x+2)$, $i.\,e.$, $x^3+9x^2+26x+24$

$\therefore x^3+9x+26x+24-(x^3+9x^2+24x+16)$

$= 2x+8 = 2(x+4)$. *Ans.*

(ii). The expression

$$= -\frac{a^2}{(a-b)(c-a)} - \frac{b^2}{(b-c)(a-b)} - \frac{c^2}{(c-a)(b-c)}$$

$$= -\frac{a^2(b-c)+b^2(c-a)+c^2(a-b)}{(a-b)(b-c)(c-a)}$$

Now, $a^2(b-c)+b^2(c-a)+c^2(a-b)$

$= a^2(b-c)-b^2a+ac^2+b^2c-bc^2$

$= a^2(b-c)-a(b^2-c^2)+bc(b-c)$

$= (b-c)(a^2+bc-ab-ac)=(b-c)(a-b)(a-c)$

$= -(a-b)(b-c)(c-a)$

$\therefore$ The expression

$$\frac{-(a-b)(b-c)(c-a)}{-(a-b)(b-c)(c-a)}=1. \quad \textit{Ans.}$$

8. (i) $4(ad-bc)^2-(a^2+d^2-b^2-c^2)^2$

$= \{2(ad-bc)\}^2-(a^2+d^2-b^2-c^2)^2$

$= (2ad-2bc+a^2+d^2-b^2-c^2)$ and

$(2ad-2bc-a^2-d^2+b^2+c^2)$

$= \{(a^2+2ad+d^2)-(b^2+2bc+c^2)\}$ and

$\{(b^2-2bc+c^2)-(a^2-2ad+d^2)\}$

$= \{(a+d)^2-(b+c)^2\}\{(b-c)^2-(a-d)^2\}$

$= (a+d-b-c)(a+d+b+c)(b-c-a+d)(b-c+a+d)$. *Ans*

(ii) $a^3+b^3=(a+b)^3-3ab(a+b)$

$$\therefore \quad x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right), \quad \text{but } x+\frac{1}{x}=p;$$

$\therefore$ the value of the expression $=p^3-3p$.

Or thus—

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$$

$$=\left(x+\frac{1}{x}\right)\left(x^2+2+\frac{1}{x^2}-3\right)$$

$$=\left(x+\frac{1}{x}\right)\left\{\left(x+\frac{1}{x}\right)^2-3\right\}; \quad \text{but } x+\frac{1}{x}=p$$

$\therefore = p(p^2-3) = p^3-3p$.

9. $x^5+x^4-4x^3+2x^2+6x-9$

$=x^5+x^4-3x^3-x^3-x^2+3x^2+3x+3x-9$

$=x^5+x^4-3x^3-x^3-x^2+3x+3x^2+3x-9$

$=x^3(x^2+x-3)-x(x^2+x-3)+3(x^2+x-3)$

$=(x^3-x+3)(x^2+x-3)$

$x^4-x^2+6x-9=x^4-(x^2-6x+9)=x^4-(x-3)^2$

$=(x^2+x-3)(x^2-x+3)$

$x^4+2x^3-5x^2-6x+9$

$=x^4+x^3+x^3-3x^2-3x^2+x^2-3x-3x+9$

$=x^4+x^3-3x^2+x^3+x^2-3x-3x^2-3x+9$

$=x^2(x^2+x-3)+x(x^2+x-3)-3(x^2+x-3)$

$=(x^2+x-3)(x^2+x-3)$

$\therefore$ The G. G. M. and the L. C. M. of $(x^3-x+3)(x^2+x-3)$, $(x^2+x-3)(x^2-x+3)$ and $(x^2+x-3)^2$ $=(x^2+x-3)$ and $(x^2+x-3)^2(x^3-x+3)(x^2-x+3)$ respectively. *Ans.*

10.

$$
\begin{array}{l|l|l}
 & \frac{x^3}{16}-\frac{x^2}{6}-\frac{x^{\frac{3}{2}}}{4}+\frac{x}{9}+\frac{x^{\frac{1}{2}}}{3}+\frac{1}{4} & \frac{x^{\frac{3}{2}}}{4}-\frac{x^{\frac{1}{2}}}{3}-\frac{1}{2} \\
 & \frac{x^3}{16} & \\
\hline
\frac{x^{\frac{3}{2}}}{2}-\frac{x^{\frac{1}{2}}}{3} & -\frac{x^2}{6}+\frac{x}{9} & \\
 & -\frac{x^2}{6}+\frac{x}{9} & \\
\hline
\frac{x^{\frac{3}{2}}}{2}-\frac{2x^{\frac{1}{2}}}{3}-\frac{1}{2} & -\frac{x^{\frac{3}{2}}}{4}+\frac{x^{\frac{1}{2}}}{3}+\frac{1}{4} & \\
 & -\frac{x^{\frac{3}{2}}}{4}+\frac{x^{\frac{1}{2}}}{3}+\frac{1}{4} & \\
\hline
\end{array}
$$

$\therefore \frac{x^{\frac{3}{2}}}{4}-\frac{x^{\frac{1}{2}}}{3}-\frac{1}{2}$. *Ans.*

$$\begin{array}{rr}
3(2x^2)^2 = & 12x^4 \\
3\times 2x^2\times -3x = & +18x^3 \\
(-3x)^2 = & +9x^2 \\
\hline
& 12x^4+18x^3+9x^2 \\
3(2x^2-3x)^2 = & 12x^4-36x^3+27x^2 \\
3(2x^2-3x)\times 1 = & +6x^2-9x \\
1^2 = & +1 \\
\hline
& 12x^4-36x^3+33x^2-9x+1
\end{array}$$

$$\begin{array}{l|l}
8x^6-36x^5+66x^4-63x^3+33x^2-9x+1 & \underline{2x^2-3x+1.} \\
8x^6 & \\
\hline
\quad -36x^5+66x^4-63x^3 & \\
\quad -36x^5+54x^4-27x^3 & \\
\hline
\qquad 12x^4-36x^3+33x^2-9x+1 & \\
\qquad 12x^4-36x^3+33x^2-9x+1 & \\
\hline
\end{array}$$

$\therefore 2x^2-3x+1.$ *Ans.*

11. (i) $\sqrt{\{(x-a)^2+2ab+b^2\}}=x-a+b$

By squaring both sides, we have $(x-a)^2+2ab+b^2=(x-a+b)^2$

$\therefore \quad (x-a)^2+2ab+b^2=(x-a)^2+2b(x-a)+b^2$

$\therefore \quad 2ab=2b(x-a) \quad \therefore a=x-a \;\therefore x=2a.$ *Ans.*

(ii) Multiplying the 1st by 2 and adding the two together, we get

$$\left.\begin{array}{l}\dfrac{30}{x}-\dfrac{2}{y}=9 \\[2ex] \dfrac{9}{x}+\dfrac{2}{y}=4\end{array}\right\} \qquad \left.\begin{array}{r}\therefore \dfrac{39}{x}=13 \;\therefore x=3 \\[2ex] \text{Hence } y=2\end{array}\right\} \textit{Ans.}$$

12. Let x be the number of miles gone over by him : so he takes $\frac{x}{3}$ hours to walk from the University towards Malabar Hill. He walks part of the distance in 1 hour 5 minutes, *i. e.*, $1\frac{1}{12}$ hours.

$\therefore$ 1 hr. : $1\frac{1}{12}$ hr. :: 3 miles $= 1\frac{1}{4}$ miles are gone over in 1 hour 5 min.

$\therefore$ remaining distance $= \left(x - \frac{13}{4}\right)$ miles, which he goes over at the rate of $8\frac{1}{3}$ miles; $\therefore$ he takes $\left(x-\frac{13}{4}\right) \times \frac{3}{25}$ or $\frac{12x-39}{100}$ hours. But the total time taken $=$ 2 hours 44 min. $= 2\frac{11}{15}$ hours.

$\therefore \ \frac{x}{3} + 1\frac{1}{12} + \frac{12x-39}{100} = \frac{41}{15}$, whence $x = 4\frac{1}{2}$ $\therefore$ he had gone over $4\frac{1}{2}$ miles. *Ans.*

Euclid.

1. 2nd Corollary to I. 32.

Each angle of a regular polygon of n sides contain $\frac{2(n-2)}{n}$ rt. ang. $\therefore$ each ang. of a regular pentagon $=$ $\frac{2(5-2)}{5}$, *i. e.*, $\frac{6}{5}$ rt. angs.

Similarly, each ang. of a regular decagon contains $\frac{2(10-2)}{10} = \frac{8}{5}$ rt. angs. $\therefore$ one angle of a regular polygon is to one angle of a regular decagon as $\frac{6}{5}$ is to $\frac{8}{5}$, *i. e.*, as 6 to 8, *i. e.*, as 3 to 4.

2. Euc. I. 43.

(Fig. 32.) From A and B draw AD, BE perp. to PQ Area of tr. $APQ = \frac{1}{2}AD, PQ$ and area of tr. $BPQ = \frac{1}{2}BE, PQ$; but trs. APQ, BPQ are equal (hyp.); $\therefore \frac{1}{2}AD, PQ = \frac{1}{2}BE, PQ$ $\therefore AD = BE$.

Again, in trs. ADC, CBE, ang. $ADC =$ ang. CEB (ax. 11), ang. $ACD =$ ang. ECB (I. 15) and $AD = BE$ $\therefore AC = CE$ (I. 26).

4. Euc. II. 9.

(Fig. 33.) Let the chords AB, CD intersect at rt. angles in E. Draw the diameter AF and join AC, AD, CF, DB. Then

ang. ACF in a semi-circle = a rt. ang. (III. 31) and = ang. AED; also ang. ADC = ang. AFC (III. 21). Hence in ADE, AFC, there are two angles in the one respectively equal to two angles in the other, ∴ ang. CAF = ang. DAB (I. 32). ∴ arc DB = arc CF (III. 26) ; ∴ chord DB = chord CF (III. 29).

Again, ∵ AEC is a rt. angled triangle, ∴ $AE^2+EC^2=AC^2$ (I. 47)

Similarly, $DE^2+EB^2=DB^2$ ∴ $AE^2+EC^2+DE^2+EB^2=AC^2+DB^2$; but DB was proved equal to CF and $AC^2+FC^2=AF^2$ (I.47).

∴ $AE^2+EC^2+DE^2+EB^2=AF^2$, *i. e.*, the square on AF, the diameter of the circle.

5. Euc. III. 17.

(*Vide* Fig. 31.) FH and GF are perp. to A and CD respectively (III. 18); but AB and CD are prll. (hyp.) ∴ FH, GF are in one straight line. Again, AF, FE bisect HFK, KFG respectively (III. 17, cor.) ∴ ang. AFE is half of four angs. at F, *i. e.*, $\frac{1}{2}$ of two rt. angs. (I. 13); ∴ ang. AFE = one rt. ang. Similarly, it may be shown that BD subtends one right angle at the centre.

6. Euc. IV. 13., Cor.

7. Euc. IV. 3.

1887-88.

Arithmetic and Algebra.

TUESDAY 22ND NOVEMBER.

GOVIND VITHAL KURKURAY, B.A.

KAVASJI JAMSHEDJI SANJANA, M.A.

1. Simplify— **8**

$$\frac{\cdot\dot{1}4285\dot{7}\times\cdot\dot{0}7692\dot{3}}{\cdot\dot{0}1098\dot{9}}+\frac{2\cdot7\dot{5}+11\cdot25}{6\cdot2}.$$

2. If 9 lbs. of rice cost as much as 4 lbs. of sugar, and 14 lbs. of sugar are worth as much as $1\frac{1}{2}$ lbs. of tea, **8**

and 2 lbs. of tea are worth 5 lbs. of coffee, find the cost of 11 lbs. of coffee if $2\frac{1}{2}$ lbs. of rice cost $6\frac{1}{4}$d.

3. If Rs. 165 14 as. $1\frac{7}{17}$ ps. be the discount of a debt of Rs. 2,820, simple interest being at the rate of $3\frac{3}{4}$ per cent., how many months before due was the debt paid?

4. The price of gold is £3 17s. $10\frac{1}{2}$d. per oz.; a composition of gold and silver weighing 18 lbs. is worth £637 7s., but if the proportion of gold and silver were interchanged, it would be worth only £259 1s. Find the proportion of gold and silver in the composition, and the price of silver per oz. 12

5. By selling four dozen mangoes for Rs. 13 it was found that $\frac{3}{10}$ths of the outlay was gained; what ought the retail price per mango to have been in order to have gained 60 per cent.? 8

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6 If $a+b=c+d$, prove that either of them is equal to $\dfrac{abcd}{ab+cd}\left\{\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}\right\}$; and if $x+\dfrac{1}{y}=1$ and $y+\dfrac{1}{z}=1$, prove that $z+\dfrac{1}{x}=1$ and $xyz+1=0$. 11

7. Simplify— 10

(i) $\dfrac{(b+c)(x^2+a^2)}{(c-a)(a-b)}+\dfrac{(c+a)(x^2+b^2)}{(a-b)(b-c)}+\dfrac{(a+b)(x^2+c^2)}{(b-c)(c-a)}$.

(ii) $\left(\dfrac{x}{y}+\dfrac{y}{z}+\dfrac{z}{x}\right)\left(\dfrac{x}{z}+\dfrac{z}{y}+\dfrac{y}{x}\right)-\left(\dfrac{x}{y}+\dfrac{y}{z}\right)\left(\dfrac{y}{z}+\dfrac{z}{x}\right)\left(\dfrac{z}{x}+\dfrac{x}{y}\right)$.

8. Shew that $(ax+by+cz)^3+(cx-by+az)^3$ is divisible by $(a+c)(x+z)$; find the three factors of $x^3-2x^2-23x+60$. 10

9. Extract the square root of— 7

$$(a-b)^2\{(a-b)^2-2(a^2+b^2)\}+2(a^4+b^4).$$

10. Solve the equation— 7

$$\frac{x-8}{x-10}-\frac{x-5}{x-7}=\frac{x-7}{x-9}-\frac{x-4}{x-6}.$$

11. A number consists of three digits, the right hand one being zero. If the left hand and middle digits be interchanged the number is diminished by 180; if the left hand digit be halved and the middle and right hand digits be interchanged, the number is diminished by 336; find the number. 11

Euclid.

1. The straight line which joins the middle points of two sides of a triangle is parallel to and half of the third side: prove this with the help of First Book only. 9

2. Describe a parallelogram that shall be equal to a given triangle and have one of its angles equal to a given rectilineal angle. 7

3. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which when produced the perpendicular falls and the straight line intercepted without the triangle between the perpendicular and the obtuse angle. 7

4. *ABC* is an equilateral triangle; in *BC* produced *D* is taken so that the rectangle *BD.DC* is equal to the square on *BC*: prove that the square on *BD* is equal to twice the square on *AC*. 8

5. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle. 8

6. If from any point without a circle, there be drawn two straight lines one of which cuts the circle and the other meets it and if the rectangle contained by the whole line which cuts the circle and the part of it without the circle, be equal to the square on the line 8

which meets the circle, the line which meets the circle shall touch it.

7. From an external point O, OP is drawn to touch a circle and OQR to cut it; it is found that OP is twice the radius, and that OR is twice OQ: prove that QR subtends a right angle at the centre. 9

8. Inscribe a circle in a given triangle. 8

9. If a circle be inscribed in a right angled triangle the excess of the sides containing the right angle over the hypotenuse is equal to the diameter of the circle. 10

SOLUTIONS.

1. $\frac{142857}{999999}$ or $\frac{1}{7} \times \frac{76923}{999999} \times \frac{999999}{10989} = 1$

$2\frac{68}{90} \times 11\frac{25}{100} \div 6\frac{2}{10}$

$\frac{124}{45} \times \frac{45}{4} \times \frac{5}{31} = 5 \ \therefore \ 5+1=6.$ *Ans.* (See Appendix.)

2. 4 lbs. of sugar : 14 :: 9 lbs. of rice $=\frac{63}{2}$ lbs. of rice

$1\frac{1}{2}$ lbs. of tea : 2 :: $\frac{63}{2}$ lbs. of rice = 42 lbs. of rice

5 lbs. of coffee : 11 :: 42 $=\frac{462}{5}$ lbs. of rice.

$2\frac{1}{2}$ lbs. of rice : $\frac{462}{5}$ :: $6\frac{1}{4}d.$ = 19*s.* 3*d.*, cost of 11 lbs. of coffee. *Ans.*

3. Rs. 165 as. 14 pies $1\frac{1}{17}$ = Rs. $\frac{2820}{17}$

Rs. $2{,}820 -$ Rs. $\frac{2820}{17} =$ Rs. $2820\left(1-\frac{1}{17}\right) =$ Rs. $2{,}820 \times \frac{16}{17}$ P.W.

$\frac{2820 \times 16}{17}$ P. W. : 100 :: Rs. $\frac{2820}{17}$ Int. = Rs. $\frac{25}{4}$ Interest.

Rs. $3\frac{3}{4}$: $\frac{25}{4}$ Int. :: 12 months = 20 months. *Ans.*

4. If to the mass of gold and silver weighing 18 lbs. we add another mass of gold and silver weighing 18 lbs. in which the proportion of gold and silver is interchanged, the resulting 36 lbs. will contain 18 lbs. of gold and 18 lbs. of silver.

$\therefore$ £637 7*s.* + £259 1*s.*, *i.e.*, £896 8*s.* is the price of the mass containing 18 lbs. of gold and 18 lbs. of silver.

But the price of 18 lbs. of gold is equal to—
(1 oz : 12×18 :: £3 17*s.* $10\frac{1}{2}d.$) £841 1*s.*

$\therefore$ the price of 18 lbs. of silver = £896 8*s.*—£841 1*s.*, *i. e.*, £55 7*s.*; $\therefore$ the price of one ounce of silver $= £55\frac{7}{20} \times \frac{1}{18 \times 12} = 5s.\ 1\frac{1}{2}d.$ *Ans.*

The price of 18 lbs. of the given mixture = £637 7*s.*

$\therefore$ the price of 1 oz. of the mixture $= \frac{12747}{20} \times \frac{1}{18 \times 12}$

$= \frac{4249}{1440} = £2\ 19s.\ 0\frac{1}{6}d.$

$\therefore$ the proportion of gold and silver in the mixture $= (£2\ 19s.\ 0\frac{1}{6}d. - 5s.\ 1\frac{1}{2}d.) : (£3\ 17s.\ 10\frac{1}{2}d - £2\ 19s.\ 0\frac{1}{6}d.)$ $= £2\ 13s.\ 10\frac{2}{3}d. : 18s.\ 10\frac{1}{3}d. = \frac{1940}{3}d. : \frac{679}{3}d.$, *i.e.*, the proportion is 20 to 7.

5. Let the cost price be Rs. 100 $\therefore$ the gain $= \frac{3}{10} \times 100 =$ Rs. 30.

130 S. P. : 13 :: 100 = Rs. 10, cost price of 4×12 mangoes.
Rs. 100 C. P. : 10 :: 160 S. P. = 16 S. P. of 48 mangoes.

$\therefore$ the retail price of one mango $= \frac{16}{48} =$ Re. $\frac{1}{3}$ = 5as. 4ps. *Ans.*

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6. (i) $$= \frac{abcd}{ab+cd}\left\{\frac{bcd+acd+abd+abc}{abcd}\right\}$$
$$= \frac{abcd}{ab+cd}\left\{\frac{cd(b+a)+ab(c+d)}{abcd}\right\}.$$ But $a+b=c+d$

Therefore the whole expression is either equal to $a+b$ or $c+d$.

(ii) $x+\frac{1}{y}=1, \therefore x=1-\frac{1}{y}=\frac{y-1}{y}, \therefore \frac{1}{x}=\frac{y}{y-1}$,

$y+\frac{1}{z}=1 \therefore \frac{1}{z}=1-y \therefore z=\frac{1}{1-y}$.

Hence $z+\frac{1}{x}=\frac{1}{1-y}+\frac{y}{y-1}=\frac{1}{1-y}-\frac{y}{1-y}=\frac{1-y}{1-y}=1$, which was to be proved.

$x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$. By multiplying one by the other, we get $xy+1+\frac{x}{z}+\frac{1}{yz}=1 \therefore xy+\frac{x}{z}+1+\frac{1}{yz}=1$

$\therefore x\left(y+\frac{1}{z}\right)+1+\frac{1}{yz}=1$. But $y+\frac{1}{z}=1$;

$\therefore x+1+\frac{1}{yz}=1 \therefore x+\frac{1}{yz}=0 \therefore xyz+1=0$, which was to be shewn.

7. (i) The expression=

$$\frac{(b-c)(b+c)(x^2+a^2)+(c-a)(c+a)(x^2+b^2)+(a-b)(a+b)(x^2+c^2)}{(a-b)(b-c)(c-a)}$$

$$=\frac{(b^2-c^2)(x^2+a^2)+(c^2-a^2)(x^2+b^2)+(a^2-b^2)(x^2+c^2)}{(a-b)(b-c)(c-a)}$$

The numerator when simplified $=0$

$\therefore \frac{0}{(a-b)(b-c)(c-a)}=0$. *Ans.*

(ii) The expression=

$$\frac{x^2}{yz}+\frac{xy}{z^2}+1+\frac{xz}{y^2}+1+\frac{z^2}{xy}+1+\frac{y^2}{xz}+\frac{zy}{x^2}-$$

$$\left(1+\frac{y^2}{xz}+\frac{z^2}{xy}+\frac{yz}{x^2}+\frac{x^2}{yz}+\frac{xy}{z^2}+\frac{xz}{y^2}+1\right)=1. \quad \textit{Ans.}$$

8. $(ax+by+cz)^3+(cx-by+az)^3$

$$=(ax+by+cz+cx-by+az)\{(ax+by+cz)^2$$
$$-(ax+by+cz)(cx-by+az)+(cx-by+az)^2\}$$
$$=(ax+cx+az+cz)\times \text{ the other factor,}$$
$$=(a+c)(x+z)\times \text{ the other factor.}$$

i.e., the expression is divisible by $(a+c)(x+z)$, which was to be shown.

$x^3-2x^2-23x+60$
$=x^3-3x^2+x^2-3x-20x+60$
$=x^2(x-3)+x(x-3)-20(x-3)$
$=(x-3)(x^2+x-20)$

Now $x^2+x-20=x^2+5x-4x-20=x(x+5)-4(x+5)$
$=(x+5)(x-4)$

$\therefore$ the three factors are $(x-3)(x-4)(x+5)$. *Ans.*

9. The expression

$= (a-b)^2(a^2-2ab+b^2-2a^2-2b^2)+2(a^4+b^4)$
$= (a-b)^2\{-(a^2+2ab+b^2)\}+2(a^4+b^4)$
$= -(a-b)^2(a+b)^2+2(a^4+b^4)$
$= -(a^2-b^2)^2+2(a^4+b^4)$
$= -a^4+2a^2b^2-b^4+2a^4+2b^4=a^4+2a^2b^2+b^4$
$= (a^2+b^2)^2$ $\therefore$ the square root$=a^2+b^2$. *Ans.*

10. $1+\dfrac{2}{x-10}-1-\dfrac{2}{x-7}=1+\dfrac{2}{x-9}-1-\dfrac{2}{x-6}$

$\therefore \dfrac{1}{x-10}-\dfrac{1}{x-7}=\dfrac{1}{x-9}-\dfrac{1}{x-6}$

$\therefore \dfrac{x-7-(x-10)}{(x-10)(x-7)}=\dfrac{x-6-(x-9)}{(x-9)(x-6)}$

$\therefore \dfrac{3}{(x-10)(x-7)}=\dfrac{3}{(x-9)(x-6)}$

$\therefore (x-10)(x-7)=(x-9)(x-6)$
$x^2-17x+70=x^2-15x+54$
$\therefore -2x=-16$, whence $x=8$. *Ans.*

11. Let x be the digit in the hundreds' place and y the digit in the tens' place, the number formed$=100x+10y$.

If the left-hand and middle digits be interchanged, he number$=100y+10x$

$\therefore 100y+10x=100x+10y-180$

$\therefore -90x+90y=-180$ $\therefore x-y=2$ ....(i)

If the left-hand digit be halved, *i.e.*, $=\frac{x}{2}$, and the middle and right-hand digits be interchanged the number $= 100\times\frac{x}{2}+10\times 0+y$, *i.e.*, $50x+y$ $\therefore 50x+y=100x+10y-336$

$\therefore\ 50x+9y=336$ ....(ii)

Multiplying the 1st equation by 9 and adding the two together we get—

$9x-9y=18$ .... (i)

$50x+9y=336$ .... (ii)

$\therefore 59x=354 \therefore x=6$; whence $y=4$

$\therefore$ the number $=640$. *Ans.*

Euclid.

(Fig. 34.) Let D, E be the middle points of AB, AC. Produce DE to F, making $EF=DE$ (I. 3). Join CF.

Then in trs. ADE, CEF, $AE=EC$ (hyp.), $DE=EF$ (constr.), ang. $AEF=$ ang. CEF (I. 15) $\therefore$ $AD=CF$ (I. 4), and ang. $DAE=$ ang. ECF, but these are alternate angles $\therefore$ AD is prll. to CF (I. 27), but $CF=AD=BD$ (hyp.) $\therefore$ CF is equal and prll. to BD $\therefore$ DF is equal and prll. to BC (I. 33) and $DE=EF$ (constr.) $\therefore$ $DE=2BC$ and prll. to it.

2. Euclid I. 42

3. Euclid II. 12

4. (Fig. 35). Bisect BC in E (I. 10). Join AE.

Then in trs. ABE, AEC, $BE=EC$ (constr.), AE is common, $AB=AC$ (hyp.) $\therefore$ ang. $AEB=$ ang. AEC (I. 8) $\therefore$ each of them is a rt. ang. (I. def. 7).

$\therefore$ AD^2 is greater than AC^2+CD^2 by 2 rect. $EC.CD$ 12); but $BC=2EC$.

$\therefore$ 2 rect. $EC.CD$=rect. $BC.CD$ $\therefore$ AD^2 is greater than AC^2+CD^2 by rect. $BC.CD$; *i.e.*, $AD^2=AC^2+CD^2+$rect. $BC.CD$; but rect. $BD \cdot DC$=rect. $BC.CD+CD^2$ (II. 3). $\therefore$ $AD^2=AC^2+$rect. $BD \cdot DC$, but rect. $BD.DC=BC^2$ (hyp.) $\therefore AD^2=AC^2+BC^2$; but $AC=BC$ (hyp.) $\therefore$ $AD^2=2AC^2$.

5. Euclid III. 17.

6. Euclid III. 37.

7. (Fig. 36.) Find K the centre (III. 1). Join KR, KQ. Then QKR shall be a rt. ang.

$OP=2$ radius $\therefore$ $OP^2=4$ (radius)^2 and rect. $OR.OQ$ = rect. $RQ.QO+QO^2$ (II. 3); but $OR=2OQ$, $\therefore$ $OR.OQ$ $=2OQ^2=2QR^2$. Now $OP^2=OR.OQ$ (III. 36), $\therefore$ $2QR^2$ $=4$ (radius)^2 $\therefore$ $QR^2=2$ (radius)^2. But $KQ^2+KR^2=$ 2(radius)^2 $\therefore$ $QR^2=KQ^2+KR^2$ $\therefore$ ang. QKR is a rt. ang. (I. 48), *i.e.*, QR subtends a rt. ang. at the centre.

8. Euclid IV. 4.

9. Let ABC be a tr. right-angled at A. Inscribe cir. DEF (IV. 4) touching AB, BC, CA in D, E, F respectively. Let O be the centre of the circle. Join OD, OE, OF. Then OD, OE, OF are perp. to the tangents AB, BC, CA respectively (III. 18), $\because$ ang. ADO is a rt. ang. and ang. DAF is a rt. ang. (hyp.) $\therefore$ AF and DO are prll. (I. 28) $\therefore$ also AD and OF are prll. $\therefore$ $ADOF$ is a plm. $\therefore$ AD $=OF$ (I. 34) and $DO=AF$ (I. 34) $\therefore$ $AD+AF=OD+OF$, *i.e.*, the diameter of the inscribed cir.

Again, $BD=BE$ and $CE=CF$ (III. 17 Cor.) $\therefore$ $BD+BC$ $+CF=2\,BC$. $\therefore$ the perimeter $AB+BC+CF$ is greater than the diameter, *i.e.*, $AD+AF$, by $BD+BC+CF$, *i.e.*, 2 BC, *i.e.*, twice the hypotenuse (*i.e.*, the excess of AB and BC over AC the hypotenuse=diameter of cir.)

1888.

TUESDAY, 20TH NOVEMBER.

Arithmetic and Algebra.

GOVIND VITHAL KURKARAY, B.A.
KRISHNAJI BALVANT WAGLE, M.A.
VINAYAK NARAYAN NENE, Esq.

1. One clerk has $24{\cdot}\dot{4}2857\dot{1}$ and a second clerk 7
has $38\frac{1}{7}$ sheets to copy: they call in a third clerk and agree to divide the work equally among the three and to pay the third clerk at the rate of $\cdot 2430\dot{5}$ shillings per sheet. How much will he receive from each of them?

2. If the manufacturer makes a profit of 20 per 9
cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 40 per cent.; what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

3. If 15 men eat 28 shillings' worth of bread in 14 8
days when wheat is at 52 shillings per quarter; what must be the price of wheat per quarter that 18 shillings' worth may provide bread for 13 men for 5 days?

4. Find the value of— 9

$$\sqrt{9025250801{\cdot}7424} - \sqrt[3]{347740371{\cdot}036161}.$$

5. If the discount on £678 8*s.*, which is due at the 6
end of a year and a half be £38 8*s.*, what is the rate per cent. of simple interest?

---

6. Find the factors of the following expressions:— 11

(i) $2y^2z^2+2z^2x^2+2x^2y^2-x^4-y^4-z^4$.

(ii) $x^4-(p^2+2)x^2y^2+y^4$.

7. Given the relation 8

$$\frac{1-2bx+b^2}{1-b^2}=\frac{1-b^2}{1+2by+b^2};$$

prove that

$$\frac{x-y}{1-xy}=\frac{2b}{1+b^2}.$$

8. Divide $1+a+a^2+a^3+a^4+a^6+a^7+a^8+a^9+a^{15}$ by $1-a^5+a^6$. 8

9. Simplify the fraction 9

$$\frac{(y-z)(y+z)^3+(z-x)(z+x)^3+(x-y)(x+y)^3}{(y+z)(y-z)^3+(z+x)(z-x)^3+(x+y)(x-y)^3}$$

10. Solve the equation 8

$$\frac{a-b}{x}+\frac{a+b}{y}=\frac{2(a^2+b^2)}{a^2-b^2};\quad \frac{a+b}{x}+\frac{a-b}{y}=2.$$

11. A number consists of three digits whose sum is 10. The middle digit is equal to the sum of the other two; and the number will be increased by 99 if its digits be reversed. Find the number. 9

12. If 19 lbs. of gold weigh 18 lbs. in water, and 10 lbs. of silver weigh 9 lbs. in water; find the quantity of gold and silver in a mass of gold and silver weighing 106 lbs. in air and 99 lbs. in water. 8

Euclid.

1. Prove that any two sides of a triangle are together greater than the third side. 5

Shew that any two sides of a triangle are together greater than twice the straight line drawn from the vertex to the middle point of the base. 10

2. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram shall be double of the triangle. 6

P is any point within a parallelogram $ABCD$. Shew that the triangles PAB and PCD are together equal to half the parallelogram. 7

3. If a straight line be divided into any two parts, the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part. 8

4. Prove that the angle at the centre of a circle is double of an angle at the circumference, standing on the same arc. 8

5. Prove that the angle in a semicircle is a right angle. 7

A, B, C, D are the angular points of a square inscribed in a circle. *P* is any point on the circumference. Shew that the squares on the straight lines *PA, PB, PC, PD* are together equal to twice the square on the diameter. 13

6. Inscribe a regular hexagon in a given circle. 11

SOLUTIONS.

Arithmetic and Algebra.

1. $$24{\cdot}\dot{4}2857\dot{1} = 24\tfrac{3}{7}$$

$$24\tfrac{3}{7} + 38\tfrac{4}{7} = 63$$

The work is divided equally among the three ; $\therefore$ each has to copy $\frac{63}{3}$ = 21 sheets

$$24\tfrac{3}{7} - 21 = 3\tfrac{3}{7}$$
$$38\tfrac{4}{7} - 21 = 17\tfrac{4}{7}$$

$1 : 3\frac{3}{7} :: {\cdot}2430\dot{5}s. = \frac{21875}{90000} \times \frac{24}{7} = \frac{35}{144} \times \frac{24}{7} = \frac{5}{6}s. = 10d.$ the 3rd clerk gets from the 1st. *Ans.*

$1 : 17\frac{4}{7} :: {\cdot}2430\dot{5}s. = \frac{21875}{90000} \times \frac{123}{7} = \frac{35}{144} \times \frac{123}{7} = \frac{615}{144}s.$ $= 4s.\ 3\frac{1}{4}d.$ the 3rd clerk gets from the 2nd clerk. *Ans.*

2. Let the cost of the manufacture of the article be £1 $\therefore$ the manufacturer sells it for £ $\frac{120}{100}$ $\therefore$ the wholesale dealer sells it for

$$\left(100 : \frac{120}{100} :: 121\right) = £\,\frac{120}{100} \times \frac{125}{100}$$

and the shopkeeper sells it for $\left(100 : \frac{120}{100} \times \frac{125}{100} :: 140\right) =$ $£\,\frac{120}{100} \times \frac{125}{100} \times \frac{140}{100}$, *i.e.*, for £ $\frac{21}{10}$.

Now when the retail price of the article is £ $\frac{21}{10}$ the cost is £ 1; $\therefore$ the retail price being 17*s.* 6*d.* the cost of the manufacture of the article will be

$$17\tfrac{1}{2}\times\frac{10}{21} \text{ or } \frac{35}{2}\times\frac{10}{21}=\frac{25}{3}=8s.\ 4d. \quad \textit{Ans.}$$

$$\text{3. } \left.\begin{array}{lll} \text{Inverse} & 13 \text{ men} & : 15 \text{ men} \\ \quad ,, & 5d. & : 14d. \\ \text{Direct} & 28s. & : 18s. \end{array}\right\} :: 52s. = 108s. \quad \textit{Ans.}$$

4.

$$\begin{array}{r|l|l}
 & 9025250801{\cdot}7424 & 95001{\cdot}32 \\
 & 81 & \\
185 & \quad 925 & \\
 & \quad 925 & \\
190001 & \qquad 250801 & \\
 & \qquad 190001 & \\
1900023 & \qquad\; 6080074 & \\
 & \qquad\; 5700069 & \\
19000262 & \qquad\;\; 38000524 & \\
 & \qquad\;\; 38000524 & \\
\end{array}$$

$$\begin{array}{rr|ll}
 & 3\times70^2 & 347740371{\cdot}686161 & 703{\cdot}21 \\
 & & 343 & \\
3\times700^2= & 14760000 & \quad 4740371 & \\
3\times700\times3= & 6300 & & \\
3^2= & 9 & & \\
 & 1476309 & \quad 4428927 & \\
 & & \quad\; 311444686 & \\
3\times7030^2= & 148262700 & & \\
3\times7030\times2= & 42180 & \quad\; 296609768 & \\
2^2= & 4 & \qquad 14834918161 & \\
 & 148304884 & & \\
3\times70320^2= & 14834707200 & & \\
3\times70320\times1= & 210960 & & \\
1^2= & 1 & & \\
 & 14834918161 & \qquad 14834918161 & \\
\end{array}$$

$\therefore$ 95001·32 − 703·21 = 94298·11. *Ans.*

Or thus :—

2103	1470000	347740371·686161	703·21
6	6309	343	
21092	1476309	4740371	
4	9	4428927	
210961	148262700	311444686	
	42184	296609768	
	148304884	14834918161	
	4	14834918161	
	14834707200		
	210961		
	14834918161		

$\therefore$ 95001·32 − 703·21 = 94298·11. *Ans.*

5. £678 8*s*. Amount £640 : £100 :: £38$\frac{2}{5}$ = £6 Int.

£ 38 8*s*. Discount 1$\frac{1}{2}$ yrs. : 1 :: £6 Int. = 4. *Ans.*

£640 Present Value

6. (i) The expression is equal to—

$$2y^2z^2+2y^2z^2-2y^2z^2+2z^2x^2+2x^2y^2-x^4-y^4-z^4$$
$$=4y^2z^2-(x^4+y^4+z^4-2x^2y^2-2x^2z^2+2y^2z^2)$$
$$=4y^2z^2-(x^2-y^2-z^2)^2$$
$$=(2yz)^2-(x^2-y^2-z^2)^2$$
$$=(2yz+x^2-y^2-z^2)(2yz-x^2+y^2+z^2)$$
$$=\{x^2-(y^2-2yz+z^2)\}\{(y^2+2yz+z^2)-x^2\}$$
$$=\{x^2-(y-z)^2\}\{(y+z)^2-x^2\}$$
$$=(x-y+z)(x+y-z)(y+z+x)(y+z-x)$$
$$=(x+y+z)(x-y+z)(x+y-z)(y+z-x). \quad \textit{Ans.}$$

Or, the expression is equal to—

$$-\{x^4+y^4+z^4-2y^2z^2-2z^2x^2-2x^2y^2\}$$
$$=-\{x^4-2x^2(y^2+z^2)+y^4+z^4-2y^2z^2\}$$
$$=-\{x^4-2x^2(y^2+z^2)+(y^2+z^2)^2-(y^2+z^2)^2+y^4+z^4$$
$$-2y^2z^2\}$$
$$=-\{x^2-(y^2+z^2)\}^2-(y^2+z^2)^2+y^4+z^4-2y^2z^2$$
$$=-\{x^2-(y^2+z^2)\}^2-4y^2z^2$$
$$=-(x^2-y^2-z^2-2yz)(x^2-y^2-z^2+2yz)$$
$$=-\{x^2-(y+z)^2\}\{x^2-(y-z)^2\}$$
$$=-(x+y+z)(x-y-z)(x+y-z)(x-y+z)$$
$$=(x+y+z)(y+z-x)(x-y+z)(x+y-z). \quad \textit{Ans.}$$

(ii) $x^4-(p^2+2)x^2y^2+y^4$

$$=x^4-p^2x^2y^2-2x^2y^2+y^4$$
$$=(x^4-2x^2y^2+y^4)-p^2x^2y^2$$
$$=(x^2-y^2)^2-(pxy)^2=(x^2-y^2+pxy)(x^2-y^2-pxy). \quad \textit{Ans.}$$

7. $$\frac{1-2bx+b^2}{1-b^2}=\frac{1-b^2}{1+2by+b^2}$$

$$\therefore (1+b^2-2bx)(1+b^2+2by)=(1-b^2)^2$$
$$\therefore (1+b^2)^2-2bx(1+b^2)+2by(1+b^2)-4b^2xy=(1-b^2).$$
$$\therefore (1+b^2)^2-(1-b^2)^2-4b^2xy=2bx(1+b^2)-2by(1+b^2)$$
$$\therefore 4b^2-4b^2xy=2bx(1+b^2)-2by(1+b^2)$$
$$\therefore 4b^2(1-xy)=2b\{x(1+b^2)-y(1+b^2)\}$$
$$=2b(x-y)(1+b^2)$$
$$\therefore 2b(1-xy)=(x-y)(1+b^2)$$

$\therefore \dfrac{x-y}{1-xy}=\dfrac{2b}{1+b^2}$, which was to be proved. *Ans.*

1. $1-a^5+a^6)\,1+a+a^2+a^3+a^4+a^6+a^7+a^8+a^9+a^{15}\,(1+a+a^2+a^3+a^4+a^5+a^6+a^7+a^8+a^9.$

$$
\begin{array}{l}
\underline{1-a^5+a^6} \\
a+a^2+a^3+a^4+a^5+a^7 \\
\underline{a-a^6+a^7} \\
a^2+a^3+a^4+a^5+a^6+a^8 \\
\underline{a^2-a^7+a^8} \\
a^3+a^4+a^5+a^6+a^7+a^9 \\
\underline{a^3-a^8+a^9} \\
a^4+a^5+a^6+a^7+a^8+a^{15} \\
\underline{a^4-a^9+a^{10}} \\
a^5+a^6+a^7+a^8+a^9-a^{10}+a^{15} \\
\underline{a^5-a^{10}+a^{11}} \\
a^6+a^7+a^8+a^9-a^{11}+a^{15} \\
\underline{a^6-a^{11}+a^{12}} \\
a^7+a^8+a^9-a^{12}+a^{15} \\
\underline{a^7-a^{12}+a^{13}} \\
a^8+a^9-a^{13}+a^{15} \\
\underline{a^8-a^{13}+a^{14}} \\
a^9-a^{14}+a^{15} \\
\underline{a^9-a^{14}+a^{15}}
\end{array}
$$

$1+a+a^2+a^3+a^4+a^5+a^6+a^7+a^8+a^9.$ *Ans.*

9. The fraction is equal to

$$\frac{(y^2-z^2)(y+z)^2+(z^2-x^2)(z+x)^2+(x^2-y^2)(x+y)^2}{(y^2-z^2)(y-z)^2+(z^2-x^2)(z-x)^2+(x^2-y^2)(x-y)^2}$$

The numerator

$$=(y^2-z^2)\{(y^2+z^2)+2yz\}+(z^2-x^2)\{(z^2+x^2)+2zx\}$$
$$+(x^2-y^2)\{(x^2+y^2)+2xy\}$$
$$=y^4-z^4+2yz(y^2-z^2)+z^4-x^4+2zx(z^2-x^2)+x^4-y^4$$
$$+2xy(x^2-y^2)$$
$$=2yz(y^2-z^2)+2zx(z^2-x^2)+2xy(x^2-y^2)$$

The denominator

$$=(y^2-z^2)\{(y^2+z^2)-2yz\}+(z^2-x^2)\{(z^2+x^2)-2zx\}$$
$$+(x^2-y^2)\{(x^2+y^2)-2xy\}$$
$$=y^4-z^4-2yz(y^2-z^2)+z^4-x^4-2zx(z^2-x^2)$$
$$+x^4-y^4-2xy(x^2-y^2)$$
$$=-2yz(y^2-z^2)-2zx(z^2-x^2)-2xy(x^2-y^2)$$
$$=-\{2yz(y^2-z^2)+2zx(z^2-x^2)+2xy(x^2-y^2)\}$$

$\therefore$ the fraction

$$=\frac{2yz(y^2-z^2)+2zx(z^2-x^2)+2xy(x^2-y^2)}{-\{2yz(y^2-z^2)+2zx(z^2-x^2)+2xy(x^2-y^2)\}}$$
$$=-1. \quad \textit{Ans.}$$

10. Multiply the first equation by $(a+b)$ and the second by $(a-b)$

$$\therefore \text{1st}=\frac{a^2-b^2}{x}+\frac{(a+b)^2}{y}=\frac{2(a^2+b^2)(a+b)}{(a-b)(a+b)}, \text{ i.e., } \frac{2(a^2+b^2)}{a-b} \quad (1)$$

$$\text{2nd}=\frac{a^2-b^2}{x}+\frac{(a-b)^2}{y}=2(a-b) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Subtracting the second from the first, we have

$$\frac{1}{y}\left\{(a+b)^2-(a-b)^2\right\}=2\left\{\frac{a^2+b^2}{a-b}-(a-b)\right\}$$
$$\frac{2(a^2+b^2)-2(a-b)^2}{a-b}$$

$$\therefore \quad \frac{1}{y}\times 4ab=\frac{4ab}{a-b} \quad \therefore \frac{1}{y}=\frac{1}{a-b}, \; y=a-b.$$

Substituting the value of y in the second equation, we get—

$$\frac{a+b}{x}+\frac{a-b}{a-b}=2 \therefore \frac{a+b}{x}+1=2 \therefore x = a+b.$$

Hence $x = a+b$, $y = a-b$. *Ans.*

11. Let x be the digit in the hundreds' place

„ y „ „ „ tens' „

„ z „ „ „ units' „

$\therefore x+y+z=10$ ....(i)

$y=x+z$ ....(ii)

The number$=100x+10y+z$. Now, if the digits are reversed the number so formed will be represented by $100z+10y+x$.

$\therefore 100x+10y+z+99 = 100z+10y+x$,

i.e., $99x-99z = -99 \therefore x-z = -1$ ....(iii)

From (i) and (ii), putting the value of $x+z$ in the first, we get $2y= 10$, *i. e.*, $y=5 \quad \therefore \quad x+z = 5$.

Now, $x+z=5$ and $x-z = -1 \therefore 2x = 4$ and $x=2$ $\therefore z = 3 \quad \therefore$ the number required $= 253$. *Ans.*

12. In a mass of gold and silver weighing 106 lbs. let there be x lbs. of gold, then there are $(106-x)$ lbs. of silver.

19 lbs. of gold weigh 18 lbs. in water, $\therefore x$ lbs. will weigh $\frac{18x}{19}$ lbs. in water.

10 lbs. of silver weigh 9 lbs. in water, $\therefore (106-x)$ lb. will weigh $\frac{9(106-x)}{10}$ lbs. in water.

But the whole mass weighs 99 lbs. in water.

$$\therefore \frac{18x}{19}+\frac{9(106-x)}{10}=99$$

$$\therefore 180x+171(106-x) = 99\times 190$$

$$\therefore 180x-171x = 99\times 190-171\times 106$$

$\therefore\ 9x = 9\times11\times19\times10-9\times19\times106$

$\therefore\ 9x = (9\times19)(110-106) = 9\times19\times4$

$\therefore\ x = \dfrac{9\times19\times4}{9} = 76$ $\therefore$ there are 76 lbs. of gold and $106-76$ or 30 lbs. of silver. *Ans.*

Euclid.

1. Euclid I. 20.

(Fig. 37.) Let ABC be a tr. Let AD bisect BC in D. Produce AD to E (post. 2), making $DE=AD$ (I. 3). Join EC (post. 1). Then in trs. ABD, DCE, $\because BD=DC$ (hyp.), $AD=DE$ (consr.), ang. $ADB=$ang. EDC (I. 15) $\therefore AB=CE$ (I. 4). But $AC+CE$ is greater than AE (I. 20), *i. e.*, $AC+AB$ is greater than AE; but $AE=2AD$ (constr.) $\therefore AB+AC$ is greater than $2AD$.

2. Euclid I. 41.

(Fig. 38.) Through P draw EF prll. to AB or CD (I. 31), meeting AD, BC in E, F respectively. Then because tr. PAB and plm. $AEFB$ stand on the same base AB and between the same parallels, $\therefore$ tr. $PAB = \frac{1}{2}AEFB$ (I. 41). Similarly tr. $PCD=\frac{1}{2}EDCF$ (I. 41). $\therefore$ trs. PAB, PCD $= \frac{1}{2}$ plms. $AEFB+EDCF=\frac{1}{2}ABCD$.

3. Euclid II. 7.

4. Euclid III. 20.

5. (Fig. 39). Join AC, BD intersecting at F. Then F is the centre of cir. (IV. 5 cor.)

Now, ang. APC in a semi-circle is a rt. ang. (III. 31). $\therefore AP^2+PC^2=AC^2$ (I. 47). Also ang. BPD is a rt. ang. (III. 31) $\therefore BP^2+PD^2=BD^2$ (I. 47) $\therefore PA^2+PB^2+PC^2+PD^2 = AC^2+BD^2$, *i.e.*, $2AC^2$, or $2BD^2$ $\because AC=BD$.

Or thus :—

Join AC, BD. Find F the center and join PF. Then because in any triangle the sum of the squares on two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side, $\therefore PA^2+PC^2=2PF^2+2AF^2$. But $PF=AF$, being radii of the same circle $\therefore$ $PA^2+PC^2=4$ (radius)^2 $=(2$ radius$)^2=$(diameter)^2.

In the same way in tr. BPD, $PB^2+PD^2=2PF^2+2BF^2$, but $PF=BF$ $\therefore PB^2+PD^2=4$ (radius)$^2=$(diameter)^2. $\therefore$ $PA^2+PB^2+PC^2+PD^2=2$ (diameter)^2.

6. Euclid IV. 15.

1889.

TUESDAY, 19TH NOVEMBER.

Arithmetic and Algebra.

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1. Simplify :— 5

$$\frac{5\frac{1}{3} \text{ of } \cdot\dot{2} \text{ of } 2\cdot\dot{5}7142\dot{8}-1\div(\frac{1}{3}+\cdot 5)}{1-\frac{3}{14} \text{ of } \left\{\cdot 5+\frac{1}{2} \text{ of } \frac{\cdot 05}{\cdot\dot{1}4285\dot{7} \text{ of } 1\frac{1}{7}}\right\}}.$$

2. A rectangular cistern, whose length is equal to its breadth, is $5\frac{5}{7}$ feet deep and contains 5 tons of water. If a cubic foot of water weighs 1,000 ounces, find the dimensions of the cistern. 8

3. A, B, and C can walk at the rate of 3, 4, 5 miles an hour; they start from Poona at 1, 2, 3 o'clock, respectively; when B catches A, B sends him back with message to C; when will C get the message? 7

4. If I borrow money at 3 per cent. per annum, interest payable yearly and lend it immediately at 5 per cent. per annum, interest payable half-yearly (receiving compound interest for the second half year), and gain thereby at the end of the year Rs. 660 ; what was the sum of money which I borrowed ? 8

5. A person buys tea at 6 annas per seer and also some at 4 annas per seer. In what proportions must he mix them so that by selling the mixture at $5\frac{1}{4}$ annas per seer he may gain 20 per cent. on each seer sold ? 10

---

6. Find the divisor when $(4x^2+7xy+5y^2)^2$ is the dividend, $8(x+2y)^2$ the quotient, and $y^2(9x+11y)^2$ the remainder. 7

7. Find the value of the expression $x(y+2)+\frac{x}{y}+\frac{y}{x}$ in terms of a when $x=\frac{y}{y+1}$ and $y=\frac{a-2}{2}$ 8

8. Simplify :—

$$\frac{\left(p+\frac{1}{q}\right)^m\left(p-\frac{1}{q}\right)^n}{\left(q+\frac{1}{p}\right)^n\left(q-\frac{1}{p}\right)^n}.$$ 7

9. If $a+b=1$, prove that $(a^2-b^2)^2=a^3+b^3-ab$.

10. Solve the following equations :—

(i) $\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{a^2-ax+x^2}=\frac{2a^4}{x(a^4+a^2x^2+x^4)}$. 6

(ii) $\frac{2x+3}{x+1}=\frac{4x+5}{4x+4}+\frac{3x+3}{3x+1}$. 5

11. Show that if a number of two digits is four times the sum of its digits, the number formed by interchanging the digits is seven times their sum. 10

12. A certain resolution was carried in a debating society by a majority which was equal to one-third of the number of votes given on the losing side; but if 10

with the same number of votes 10; more votes had been given to the losing side, the resolution would only have been carried by a majority of one. Find the number of votes given on each side.

Euclid.

1. If one acute angle at the base of a triangle be double the other angle at the base, and a perpendicular be drawn from the vertex upon the base, show that the difference between the segments of the base is equal to the smaller side. 13

2. At a given point in a given straight line to make a rectilineal angle equal to a given rectilineal angle. 5

A is a given point; and *B* is a given point in a given straight line. It is required to draw from *A* to the given straight line a straight line *AP*, such that the sum of *AP* and *PB* may be equal to a given length, greater than the distance from *A* to *B*. 8

3. In any right-angled triangle the square which is described upon the side subtending the right angle is equal to the squares described upon the sides which contain the right angle. 9

If any point *P* be joined to *A*, *B*, *C*, *D*, the angular points of a rectangle *ABCD*, the squares on *PA* and *PC* are together equal to the squares on *PB* and *PD*, the angles *A* and *C* being opposite to each other. 10

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts. 7

In a right-angled triangle, if a perpendicular be drawn from the right angle to the hypotenuse, the square on this perpendicular is equal to the rectangle contained by the segments of the hypotenuse. 9

5. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle. 8

6. In a given circle, to inscribe a triangle equiangular to a given triangle. 6

SOLUTIONS.

Arithmetic and Algebra.

1. $$\frac{\frac{11}{2}\times\frac{2}{9}\times2\frac{4}{7}-1\div\left(\frac{1}{5}+\frac{1}{2}\right)}{1-\frac{3}{14}\left(\frac{1}{2}+\frac{1}{2}\text{ of }\frac{\frac{1}{20}}{\frac{1}{7}\times\frac{21}{20}}\right)}=\frac{\frac{22}{7}-\frac{10}{7}}{1-\frac{3}{14}\times\frac{2}{3}}=\frac{\frac{1}{7}}{\frac{6}{7}}=2.\quad \textit{Ans.}$$

2. 5 tons $=5\times20\times4\times28\times16=179{,}200$ ozs.

1,000 oz. : 179,200 oz. :: 1 cub. ft. $=\frac{896}{5}$ cub. ft. $=$ contents of the cistern.

The cistern is $5\frac{5}{7}$ feet deep $\therefore$ the area of the bottom of the cistern $=$ cub. ft. $\frac{896}{5}\div$ ft. $\frac{40}{7}=\frac{784}{25}$ square feet.

The length is equal to the breadth, $\therefore$ the length or the breadth $=\sqrt{\frac{784}{25}}=\frac{28}{5}$ feet $=5\frac{3}{5}$ feet. *Ans.*

3. *B* walks 1 mile per hour faster than *A*, $\therefore$ he will catch *A* in 3 hours, *i.e.*, when each has travelled a distance of 12 miles.

Now *C* has gone 10 miles from 3 to 5 o'clock, *i. e.*, in 2 hours, walking at the rate of 5 miles per hour.

B sends *A* with a message to *C*, $\therefore$ as *A* has walked 12 and *C* only 10 miles, *A* and *C* have $12-10$ or 2 miles to go between them.

But *A* and *C* go $3+5$ or 8 miles in 1 hour $\therefore$ they will walk 2 miles in $\frac{1}{4}$ hour or 15 minutes.

$\therefore$ *C* will get the message at 15 mins. past 5 o'clock. *Ans.*

4. The interest for the first half-year in the second case is Rs. $\frac{5}{2}$ = Rs. $2\frac{1}{2}$ $\therefore$ the interest for the second half-year is Rs. 100 : Rs. $102\frac{1}{2}$ :: Rs. $2\frac{1}{2}$ = Rs. $\frac{41}{16}$.

$\therefore$ the total interest $= \frac{41}{16} + \frac{5}{2} =$ Rs. $\frac{81}{16}$ and the yearly interest in the first case = Rs. 3.

$\therefore$ the difference = Rs. $\frac{81}{16}$ − Rs. 3 = Rs. $\frac{33}{16}$ gain.

But the total gain = Rs. 660.

Rs. $\frac{33}{16}$: Rs. 660 :: Rs. 100 = Rs. 32,000. *Ans.*

5. The cost price of 1 lb. of the mixture

$= \frac{100}{120} \times 5\frac{1}{4}$ as. $= \frac{5}{6} \times \frac{21}{4}$ as. $= \frac{35}{8}$ as. $= 4\frac{3}{8}$ as.

6 as. − $4\frac{3}{8}$ as. = $1\frac{5}{8}$ an., *i.e.*, each lb. of the dearer tea brings a loss of $1\frac{5}{8}$ an. when the mixture is sold at $4\frac{3}{8}$ as. per lb.

$4\frac{3}{8}$ as. −4 as. = $\frac{3}{8}$ an., *i.e.*, each lb. of the cheaper tea brings a gain of $\frac{3}{8}$ an. when the mixture is sold at $4\frac{3}{8}$ as. per lb.

Now for the loss in one case to be made up by the gain in the other, for every $\frac{3}{8}$ lb. of the dearer, we must take $1\frac{5}{8}$ lbs. of the cheaper, *i.e.*, the proportion must be as $\frac{3}{8} : \frac{13}{8}$, *i.e.*, as 3 : 13, *i.e.*, the teas must be mixed in the inverse ratio of the differences of the two prices and the mean price.

---

6. The divisor $= \dfrac{\text{dividend } \textit{minus} \text{ remainder}}{\text{quotient}}$

$$\therefore = \frac{(4x^2+7xy+5y^2)^2 - y^2(9x+11y)^2}{8(x+2y)^2}$$

The numerator being of the form $a^2 - b^2$ we have—

$$\frac{\{(4x^2+7xy+5y^2)+y(9x+11y)\}\{(4x^2+7xy+5y^2)-y(9x+11y)\}}{8(x+2y)^2}$$

$$= \frac{(4x^2+16xy+16y^2)(4x^2-2xy-6y^2)}{8(x+2y)^2}$$

$$= \frac{8(x^2+4xy+4y^2)(2x^2-xy-3y^2)}{8(x+2y)^2}$$

$$= \frac{8(x+2y)^2\,(2x^2-xy-3y^2)}{8(x+2y)^2} = 2x^2 - xy - 3y^2. \quad \textit{Ans.}$$

7. $x = \dfrac{y}{y+1}$ and $y = \dfrac{a-2}{2}$ $\therefore$ $x = \dfrac{a-2}{a}$

$\therefore \dfrac{x}{y} = \dfrac{2}{a}$ and $\dfrac{y}{x} = \dfrac{a}{2}$

$\therefore$ the expression $= \left(\dfrac{a-2}{a}\right)\left(\dfrac{a+2}{2}\right) + \dfrac{2}{a} + \dfrac{a}{2}$

$= \dfrac{a^2-4}{2a} + \dfrac{a^2+4}{2a} = \dfrac{a^2-4+a^2+4}{2a} = \dfrac{2a^2}{2a} = a.$ *Ans.*

8. The whole fraction

$$= \frac{\dfrac{(pq+1)^m}{q^m} \times \dfrac{(pq-1)^n}{q^n}}{\dfrac{(pq+1)^m}{p^m} \times \dfrac{(pq-1)^n}{p^n}} = \frac{p^m}{q^m} \times \frac{p^n}{q^n} = \frac{p^{m+n}}{q^{m+n}}$$

$$= \left(\frac{p}{q}\right)^{m+n}.$$ *Ans.*

9. $(a^2-b^2)^2 = (a+b)(a-b)(a+b)(a-b) = (a+b)^2(a-b)^2$
$= (a-b)^2$ $\because$ $a+b=1$
$(a-b)^2 = a^2+b^2-2ab = (a^2+b^2-ab)-ab = (a+b)$
$(a^2+b^2-ab)-ab = a^3+b^3-ab$, which was to be proved.

10. (i) Clearing off fractions, we have—

$x(a+x)(a^2-ax+x^2)+x(a-x)(a^2+ax+x^2) = 2a^4$

$\therefore$ $x(a^3+x^3)+x(a^3-x^3) = 2a^4$

$\therefore 2a^3x = 2a^4$ $\therefore$ $x = a.$ *Ans.*

(ii) $\dfrac{2x+3}{x+1} = 2+\dfrac{1}{x+1}$, $\dfrac{4x+5}{4x+4} = 1+\dfrac{1}{4x+4}$

$\dfrac{3x+3}{3x+1} = 1+\dfrac{2}{3x+1}$

$\therefore 2+\dfrac{1}{x+1} = 1+\dfrac{1}{4(x+1)}+1+\dfrac{2}{3x+1}$

$\therefore \dfrac{1}{x+1}-\dfrac{1}{4(x+1)} = \dfrac{2}{3x+1}$ $\therefore$ $\dfrac{3}{4(x+1)} = \dfrac{2}{3x+1}$

$\therefore 8x+8 = 9x+3$ $\therefore$ $x=5.$ *Ans.*

11. Let x be the digit in the tens' place and y be the digit in the units' place; then the number formed $= 10x+y$, and the number formed by interchanging the digits $=10y+x$

$\therefore 10x+y=4(x+y) \therefore 6x=3y$

$\therefore 10y+x=7y+3y+x=7y+6x+x$

$= 7(x+y)$, which was to be proved.

12. Let x be the number of votes on the side of the majority, and y the number of votes on the losing side. As the majority is $\frac{1}{3}$ of the number of votes on the losing side $x-y = \frac{1}{3}y$ .... (i)

Again, with the same number of votes if 10 more votes be given to the losing side, the votes on the losing side would be $y+10$, and then the resolution would be carried by a majority of one.

$\therefore (x-10)-(y+10) = 1$, *i. e.*, $x-y=21$.........(ii)

$\therefore$ from (i) and (ii) we have $\frac{1}{3}y=21$

$\therefore y = 63 \therefore x=84$

$\therefore$ 84 votes for the resolution, 63 against it. *Ans.*

Or—

Let x be the number of votes on the losing side; then $x+\frac{1}{3}x$ would be the number of votes on the side of the majority.

$\therefore$ as above, we have $(x+\frac{1}{3}x-10)-(x+10)=1$

$\therefore 3x+x-30-3x-30=3$

$\therefore x=63 \therefore x+\frac{1}{3}x=63+21=84$.

84 votes on the gaining side; 63 votes on the losing side. *Ans.*

Euclid.

1. (Fig. 40). Let ABC be a tr.; let ang. $B=2C$. From A draw AD perp. to BC (I. 12); make $DE=DB$ (I. 3). Join AE. In trs. $ABD, ADE \because DB=DE$ (constr.), AD is common, ang. ADB = ang. ADE (ax. 11), $\therefore AB$

$= AE$ $\therefore$ ang. $AED = ABD$ (I. 4)$=$ twice ang. C (hyp.) $=$ ang. C and ang. CAE (I. 32) $\therefore$ ang. $ACE=$ ang. CAE $\therefore$ $EC=AE$ (I. 6), but $EC=CD-ED$ or $CD-BD$ ($\because$ $ED=BD$) and $AE=AB$ $\therefore$ $AB=$ difference of CD, BD, *i.e.*, difference between the segments of the base.

2. Euclid I., 23.

Let B be the point in the str. line CD. Let K be the given length. From CD cut off $BC=K$ (I. 3). Join AC. At A in AC make an ang. $CAP=$ang. ACP (I. 23), meeting CD in P.

Then $\because$ ang. $CAP=$ang. ACP (constr.) $\therefore$ $AP=PC$ (I. 6). Add PB to each of the equals $\therefore$ $AP+PB=PC+PB=CB=K$ (constr.) greater than AB ($\because$ $AP+PB$ are greater than AB) (I. 20).

3. Euclid I. 47.

(Fig. 41.) Let the diagonals AC, BD meet in E. Join PE. Then in trs. ABD, ABC $\because$ $AD=BC$ (I. 34) AB is common, ang. $DAB=$ ang. ABC (ax. 11) $\because$ they are rt. angs. $\therefore$ $AC=BD$ (I. 4), and as the diagonals of a plm. bisect each other $AE=DE$.

Again, in any triangle the sum of the squares on two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.

$\therefore$ in tr. APC, $PA^2+PC^2=2(AE^2+PE^2)$ and in tr. BPD, $PB^2+PD^2=2(DE^2+PE^2)$; but $AE=DE$ $\therefore$ $PA^2+PC^2=PB^2+PD^2$.

4. Euclid II. 4.

See Question 4 of 1880.

5. Euclid III. 32.

6. Euclid IV. 2.

3. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle. 7

4. If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section. 8

5. If two chords of a circle intersect at right angles the sum of the squares on the four segments of the chords is equal to the square on the diameter of the circle. 8

6. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle. 15

Construct a triangle, having given the base, the verticle angle, and the point in the base on which the perpendicular falls from the verticle angle.

7. Describe a circle about a given triangle. 7

8. From the angular points A, B, C of a triangle perpendiculars are drawn on the opposite sides and terminated at the points D, E, F on the circumference of the circumscribing circle; if L be the point of intersection of the perpendiculars, shew that LD, LE, LF are bisected by the sides of the triangle. 9

SOLUTIONS.

1. £6 7*s.* 8*d.* price of 1 cwt.

33

		£	s.	d.			
		210	13	0	,,	,,	33 cwts.
2 qrs.	$\frac{1}{2}$ of 1 cwt.	3	3	10	,,	,,	2 qrs.
1 qr.	$\frac{1}{2}$,, 2 qrs.	1	11	11	,,	,,	1 qr.
7 lbs.	$\frac{1}{4}$,, 1 qr.	0	7	$11\frac{3}{4}$	,,	,,	7 lbs.
$3\frac{1}{2}$ lbs.	$\frac{1}{2}$,, 7 lbs	0	3	$11\frac{7}{8}$	,,	,,	$3\frac{1}{2}$ lbs.
1 lb.	$\frac{1}{7}$,, 7 lbs.	0	1	$1\frac{19}{28}$	,,	,,	1 lb.
		216	1	$10\frac{17}{56}$	price of		33 cwt. 3 qr $11\frac{1}{2}$ lb

2. 3 % is the annual increase; but $\frac{1}{2}$ % is carried off, so there is an actual increase of $2\frac{1}{2}$ % $\therefore$ on 100 the increase at the end of the first year would be $100+2\frac{1}{2}=102\frac{1}{2}$ $=102\cdot5$

100 : 102·5 :: 102·5=(102·5)(1·025), increase at the end of the second year.

100 : 102·5 :: (102·5)(1·025)=(1·025)(1·025)(102·5), increase at the end of the 3rd year = 107·6890625 $\therefore$ the increase per cent.=107·6890625−100=7·6890625. *Ans.*

3. On Re. 1 the net income is 192−5 or 187 pies.
Re. 1 : Rs. 4 :: Re. $\frac{187}{192}$ net income=Rs. $\frac{187}{48}$
Rs. 100= monthly income $\therefore$ yearly income=Rs. 1,200
$\frac{187}{48}$: 1,200 :: $102\frac{1}{2}$=Rs. $31,572\frac{36}{187}$. *Ans.*

4. If 100 is the cost price, the selling price is 175.
Re. 1 : Rs. 175 :: Re. $\frac{1}{4}$=Rs. $43\frac{3}{4}$;
so $100-43\frac{3}{4}=56\frac{1}{4}$% loss. *Ans.*

5. If the area of the rice crop be represented by 1 bigha;

then	,, ,,	wheat ,,	,, ,, ,,	2 ,,
,,	,, ,,	maize ,,	,, ,, ,,	3 ,,

Bighas

$$\left.\begin{array}{l} 6:30::1=\ \ 5 \text{ bighas of rice} \\ 6:30::2=10 \quad ,, \quad ,, \text{ wheat} \\ 6:30::3=15 \quad ,, \quad ,, \text{ maize} \end{array}\right\} = 30 \text{ bighas.}$$

The prices are proportional to 9, 6, 5, and the total crop is worth Rs. 1,200

$$\left.\begin{array}{l} 1:\ 5::\text{Rs. }9=\text{Rs. }45 \\ 1:10::\text{Rs. }6=\text{Rs. }60 \\ 1:15::\text{Rs. }5=\text{Rs. }75 \end{array}\right\} = \text{Rs. }180.$$

Rs. 180 : Rs. 1,200 :: Rs. 9 = Rs. 60 value of 1 bigha of rice. *Ans.*

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6. The fractions, when $x=a+b$, are equal to

$$\frac{(a+b)^3-3ab(a+b)-2b^3}{(a+b)^2-ab}+\frac{(a+b)^2-4ab}{(a+b)-2a}$$

Now, $a^3+b^3=(a+b)^3-3ab(a+b)$

$$\therefore = \frac{a^3+b^3-2b^3}{a^2+ab+b^2}+\frac{a^2-2ab+b^2}{b-a}$$

$$= \frac{a^3-b^3}{a^2+ab+b^2}+\frac{(a-b)^2}{b-a}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}-\frac{(a-b)^2}{a-b}$$

$$= (a-b)-(a-b)=0. \quad \textit{Ans.}$$

7. (*a*) $m=a^x$ $\therefore$ raising both sides to the yth power, we have $m^y=a^{xy}$ (i) : in the same way, $n^x=a^{xy}$ (ii). Multiplying (i) and (ii) we get $m^y n^x=a^{2xy}$. Raise both sides to the zth power

$\therefore (m^y n^x)^z = a^{2xyz}$. But, according to the question $a^2 = (m^y n^x)^z$ $\therefore$ $a^{2xyz}=a^2$

$\therefore$ $2xyz=2$ $\therefore$ $xyz=1$.

(*b*) The expression $= \dfrac{(3y+4z-2x)(3y-4z+2x)}{(2x+3y+4z)(2x+3y-4z)}$

$$+\frac{(4z+2x-3y)(4z-2x+3y)}{(3y+4z+2x)(3y+4z-2x)}+\frac{(2x+3y-4z)(2x-3y+4z)}{(4z+2x+3y)(4z+2x-3y)}$$

$$= \frac{3y+4z-2x}{2x+3y+4z}+\frac{4z+2x-3y}{2x+3y+4z}+\frac{2x+3y-4z}{2x+3y+4z}$$

$$= \frac{2x+3y+4z}{2x+3y+4z}= 1. \quad \textit{Ans.}$$

8. The expression $= \left(\dfrac{x^2}{a^2}+\dfrac{a^2}{x^2}+2\right) - \left(\dfrac{2x}{a}+\dfrac{2a}{x}\right)+1$

$$= \left\{ \left(\frac{x}{a}\right)^2+\left(\frac{a}{x}\right)^2+2\left(\frac{x}{a}\right)\left(\frac{a}{x}\right) \right\}-2\left(\frac{x}{a}+\frac{a}{x}\right)+1$$

$$= \left(\frac{x}{a}+\frac{a}{x}\right)^2-2\left(\frac{x}{a}+\frac{a}{x}\right)+1= \left\{ \left(\frac{x}{a}+\frac{a}{x}\right)-1 \right\}^2$$

$\therefore$ the square root $= \dfrac{x}{a}+\dfrac{a}{x}-1$. *Ans.*

9. $x^3-3x^2+3x-1=(x-1)^3$

$x^3-x^2-x+1=x^2(x-1)-(x-1)$

$=(x^2-1)(x-1)=(x-1)^2(x+1)$

$x^4-2x^3+2x-1=(x^4-1)-2x(x^2-1)$

$=(x^2-1)(x^2+1-2x)$

$=(x^2-1)(x-1)^2=(x-1)^3(x+1)$

$\therefore$ the L. C. M. of $(x-1)^3$, $(x-1)^2(x+1)$, $(x-1)^3(x+1)$

$=(x-1)^3(x+1)$. *Ans.*

10. (i) $$\frac{1}{ab-ax}=\frac{1}{ac-ax}-\frac{1}{bc-bx}$$

$$\therefore \frac{1}{a(b-x)}=\frac{1}{a(c-x)}-\frac{1}{b(c-x)}$$

$$\therefore \frac{1}{a(b-x)}=\frac{b-a}{ab(c-x)}\ \therefore \frac{1}{b-x}=\frac{b-a}{b(c-x)}$$

$$\therefore bc-bx=b^2-bx-ab+ax$$

$$\therefore bc-b^2+ab=ax\ \therefore b(a-b+c)=ax$$

$$\therefore x=\frac{b(a-b+c)}{a}.\quad \textit{Ans.}$$

(ii) Multiplying (i) by 3, and adding (ii) to it, we get

$$\frac{3}{x}+\frac{3}{y}+\frac{3}{z}=18$$

$$\frac{2}{x}-\frac{3}{y}+\frac{4}{z}=8$$

$$\frac{5}{x}+\frac{7}{z}=26 \text{.................................(iv)}$$

And multiplying (i) by 4, and adding (iii) to it, we have

$$\frac{4}{x}+\frac{4}{y}+\frac{4}{z}=24.$$

$$\frac{3}{x}-\frac{4}{y}+\frac{5}{z}=10$$

$$\frac{7}{x}+\frac{9}{z}=34 \text{.................................(v)}$$

Again, multiplying (iv) by 7 and (v) by 5, and subtracting one from the other, we get—

$$\frac{35}{x}+\frac{49}{z}=182$$
$$\frac{35}{x}+\frac{45}{z}=170$$
$$\frac{4}{z}=12 \qquad \therefore\ z=\tfrac{1}{3}.$$

Putting the value of z in (iv), we have $\frac{5}{x}+21=26$

$\therefore x=1$. Again, substituting the value of x and z in (i), we have $1+\frac{1}{y}+3=6\ \therefore\ y=\frac{1}{2}$

$\therefore\ x=1,\ y=\frac{1}{2},\ z=\frac{1}{3}$. *Ans.*

11. Let x gallons be drawn off from the first, and y gallons from the second vessel.

$\therefore\ x+y=15$ .... (i)

In the first vessel, there are 2 parts wine and 1 part water $\therefore$ from it $\frac{2x}{3}$ gallons of wine and $\frac{x}{3}$ gallons of water are drawn.

In the second there are 3 parts water and 1 part wine, and $\therefore \frac{3y}{4}$ gallons of water and $\frac{y}{4}$ gallons of wine are drawn from it.

Now, the quantities of wine and water drawn are equal.

$\therefore\ \frac{2x}{3}+\frac{y}{4}=\frac{x}{3}+\frac{3y}{4}$ or $4x=6y\ \therefore\ 2x=3y\ \therefore\ x=\frac{3y}{2}$

Substituting the value of x in (i), we get $\frac{3y}{2}+y=15$ or

$3y+2y=30$ or $5y=30\ \therefore\ y=6$ and $\therefore\ x=\frac{3\times 6}{2}=9$.

$\therefore$ 9 gallons are drawn from the first, and 6 gallons from the second vessel. *Ans.*

Euclid.

1. Euclid I. 21.

(Fig. 42.) Let ABC be a tr. and let D be a point within it. Join DA, DB, DC.

Then $\because$ $DA+DB$ is less than $CA+CB$ (I. 21), similarly $DB+DC$ is less than $AB+AC$; $DC+DA$ is less than $BC+BA$ $\therefore$ 2 $(DA+DB+DC)$ is less than $(2AB+BC+CA)$ $\therefore$ $DA+DB+DC$ is less than $AB+BC+CA$.

2. (Fig. 43.) Join AE, EF, AF. Through E draw EG prll. to AB or DC (I. 31); through F draw FKH prll. to AD or BC, meeting EG in K. Join AK.

Then $\because$ 2 tr. $AKF=KD$ (I. 41); 2 tr. $AEK=BK$ (I. 41); 2 trs. $KEF=KC$ (I. 41) $\therefore$ 2 tr. $AEF=$the gnomon BFG. Now HG is the rectangle contained by HK,KG; but $HK=BE$ (I. 34) and $KG=DF$ (I. 34), *i. e.*, the rect. BE, DF $\therefore$ 2 tr. $AEF+$rect. $BE.DF=ABCD$.

3. Euclid I. 45.

4. Euclid II. 9.

5. See Question 4 of 1886 with Answer.

6. Euclid III. 33.

Let AB be the given base, C the point in it on which the perp. falls, and let D be the given angle. On AB describe a segment of cir. containing an angle$=$the given vertical angle D (III. 33). From C draw CK at right angles to AB, meeting cir. in K. Join KA, KB. Then KAB is the required tr.

For ang. AKB being in the segment $AKB=$given vertical ang. D and stands on the base AB and the perp. CK meets AB in C.

7. Euclid IV. 5.

(Fig. 44.) Let the three perps. from the angular points cut the sides AB, BC, CA in G, H, K respectively. Join BF.

The perps. from the angular points of a tr. on the opposite sides meet in the same point.

Now in trs. BGL, LKC, ang. BGL=ang. LKC (ax. 11) $\because$ they are rt. angs. Again, ang. GLB=ang. KLC (I. 15) $\therefore$ ang. $GBL=KCL$ (I. 32). But ang. ACF=ang. ABF (III. 21), $\therefore$ ang. ABF=ang. ABL (ax. 1). Again, in trs. FGB, GBL, $\because$ ang. FBG=ang. GBL (proved), and ang. $FGB=LGB$ ($\because$ they are rt. angs.) and BG is common, $\therefore$ $FG=GL$ (I. 26). Similarly, by joining CE and CD, it may be shewn that $LK=KE$, and $LH=HD$ respectively.

1891.

TUESDAY, 17TH NOVEMBER.

Arithmetic and Algebra.

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1. Simplify —

(i) $$\frac{\frac{7}{13} \text{ of } \frac{91}{126} + \frac{5}{9} \text{ of } \frac{13}{35}}{5\frac{2}{77} \text{ of } \frac{132}{153} - \frac{299}{323} \text{ of } \frac{19}{23}}.$$ 3

(ii) $$\frac{3\cdot6\dot{4}2857\dot{1} - (\cdot009923 + \cdot0102 - \cdot000123)\frac{\cdot145}{\cdot0056}}{\sqrt{34\cdot5744} - \sqrt[3]{9\cdot663597}}$$ 9

2. Two passengers have together 5 cwt. of luggage and are charged for the excess above the weights allowed 5s. 2*d.* and 9*s.* 10*d.* respectively; but if the luggage had all belonged to one of them he would have been charged 19*s.* 2*d.* How much luggage is each passenger allowed to carry free of charge, and how much luggage had each passenger? 8

3. Two clocks, A and B, whose rates are uniform, at noon yesterday indicated 11hrs. 55 min. A.M. and 0hr. 2min. P.M. respectively. A indicated the correct time at 9 P.M. yesterday and B at 6 P. M. this morning. When did A and B last agree and what time did they then indicate? 10

4. A person borrows two equal sums of money at the same time at 5 per cent. and $3\frac{3}{4}$ per cent. simple interest respectively, and finds that if he repays the former sum with interest on a certain date a year before the latter, he will have to pay in each case the same amount, *viz.*, Rs. 736. Find the amounts borrowed. 10

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5. Simplify by using factors— 4

(i) $\dfrac{x^2-7xy+12y^2}{x^2+5xy+6y^2} \div \dfrac{x^2-5xy+4y^2}{x^2+xy-2y^2}$.

(ii) $\dfrac{(x^2-xy+y^2)^3+(x^2+xy+y^2)^3}{2(x^2+y^2)}$. 4

6. (i) If $a^2-b^2=b^2-c^2=c^2-a^2$, shew that— 8

$$\frac{ab-c^2}{a-b}+\frac{bc-a^2}{b-c}+\frac{ca-b^2}{c-a}=0.$$

(ii) Simplify :— 6

$$\frac{\left(p^2-\frac{1}{q}\right)^p\left(p-\frac{1}{q}\right)^{q-p}}{\left(q^2-\frac{1}{p^2}\right)^q\left(q+\frac{1}{p}\right)^{p-q}}.$$

7. Find the G.C.M. and the L.C.M. of—

$$x^3-2x^2-19x+20,\ x^3+2x^2-23x-60,$$
$$x^4+7x^3-4x^2-52x+48.$$

8. Solve :— 5

$$\cfrac{6}{7-\cfrac{6}{7-\cfrac{6}{7-x}}}=1.$$

9. What value of a will make the product of $3-8a$ and $3a+4$ equal to the product of $6a+11$ and $3-4a$? 5

10. The gross income of a certain man was £40 more in the second of two particular years than in the first, but in consequence of the income-tax rising from 4*d*. in the pound to 6*d*. in the pound in the second year, his net income after paying the tax was unaltered. Find his income in each year. 10

11. The sum of the ages of a man and his wife are six times the sum of the ages of their children. Two years ago the sum of their ages was ten times the sum of the ages of the children, and six years hence the sum of their ages will be three times the sum of the ages of the children. How many children have they. 10

Euclid.

1. If at a point in a straight line, two straight lines on opposite sides of it make the adjacent angles together equal to two right angles they are in the same straight line. 3

ABCD is a rhombus. *AC* is bisected at *O*. If *O* is joined to the angular points *B* and *D*, show that *OB* and *OD* are in one straight line. 5

2. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram is double of the triangle. 4

If two equal straight lines intersect each other any where at right angles the quadrilatral formed by joining their extremities is equal to half the square on either straight line. 5

3. Divide a straight line into two parts so that the ectangle contained by the whole and one part shall be equal to the square on the other part. 9

If a straight line AB be divided into any two parts in point H, such that the rectangle contained by AB, BH be equal to the square on AH, show that the sum of the squares on AB, BH is three times the square on AH. 8

4. In equal circles, angles either at the centres or at the circumferences which stand on equal arcs are equal. 7

CD is a chord of a circle at right angles to the diameter AB; E is any point in the arc BC; AE cuts CD in F: prove that the angle DFE, ACE are equal. 10

5. Show that the perimeter of a right-angled triangle exceeds the diameter of the inscribed circle by twice the hypotenuse. 12

6. Describe an isosceles triangle having each of the angles at the base double of the third angle. 12

SOLUTIONS.

1. (i) $\frac{7}{18}+\frac{13}{63}=\frac{75}{126}$

$\frac{516}{119}-\frac{13}{17}=\frac{425}{119}\therefore\frac{75}{126}\times\frac{119}{425}=\frac{1}{6}.$ *Ans.*

(ii) $\cdot009923+\cdot0102-\cdot000123=\cdot02=\frac{1}{50}$

$3{\cdot}64\dot{2}8571=3+\frac{1}{10}\times6\frac{3}{7}=3\frac{9}{14}=\frac{51}{14}$

$\frac{\cdot145}{\cdot0056}=\frac{145}{1000}\times\frac{10000}{56}=\frac{725}{28}$

$\sqrt{34{\cdot}5744}=5{\cdot}88$; thus—

```
        34·5744 | 5·88
        25
 108    957
        864
1168    9344
        9344
```

$\sqrt[3]{9{\cdot}663597}=2{\cdot}13$; thus—

$$\begin{array}{rr|l}
3\times 20^2= & 1200 & 9{\cdot}663597\,\underline{|2{\cdot}13} \\
3\times 20\times 1= & 60 & 8 \\
1^2= & 1 & 1663 \\
\hline
 & 1261 & 1261 \\
3\times 210^2= & 132300 & 402597 \\
3\times 210\times 3= & 1890 & \\
3^2= & 9 & 402597 \\
\hline
 & 134199 &
\end{array}$$

$\therefore\ 5{\cdot}88-2{\cdot}13=3{\cdot}75=3\frac{3}{4}=\frac{15}{4}$

$\therefore$ the whole expression $=\dfrac{\frac{61}{14}-\frac{1}{20}\times\frac{728}{25}}{\frac{15}{4}}=\dfrac{\frac{61}{14}-\frac{20}{56}}{\frac{15}{4}}=\frac{5}{6}={\cdot}83$. *Ans*

2. The charge on 5 cwts of luggage *minus* twice the free allowance $=5s.\ 2d.+9s.\ 10d.=15s.$

Again, the charge on 5 cwts. *minus* the free allowance $=19s.\ 2d.$

$\therefore$ the charge on free allowance $=19s.\ 2d.-15s.=4s.\ 2d.$

$\therefore$ the charge on 5 cwts. $=19s.\ 2d.+4s.\ 2d.=23s.\ 4d.$

$\therefore$ $23s.\ 4d. : 4s.\ 2d.\ :: 5\times 112$ lbs. $=100$ lbs., *i. e.*, 100 lbs are allowed free of charge to each passenger.

Again, the first paid $5s.\ 2d.$

$\therefore$ $23s.\ 4d : 5s.\ 2d.\ ::$ 5 cwt $=\frac{31}{28}$ cwts.

$\therefore$ cwts. $\frac{31}{28}$ + cwt. $\frac{100}{112}$ allowed free $=2$ cwts. of luggage the first passenger had with him.

$\therefore$ the second passenger had with him $5-2=3$ cwts. of luggage. *Ans.*

3. At noon *A* was (12 hrs.$-$11 hrs. 55 mins.) $=5$ mins. behind the correct time and *B* was (12 hrs. 2 mins.$-$12 hrs.) $=2$ mins. in advance of the correct time. *A* showed correct time at 9 P.M.

$\therefore$ *A* gains 5 mins. in 9 hrs., *i. e.*, $\frac{5}{9}$ min. in 1 hr. *B* showed correct time at 6 A.M., that morning.

$\therefore$ B loses 2 mins. in 18 hrs., *i.e.*, $\frac{1}{9}$ min. in 1 hr. $\therefore$ in 1 hr. A and B approach each other by $\frac{5}{9}$ min. $+\frac{1}{9}$ min. $=\frac{2}{3}$ min.

But at noon yesterday, the difference between the indications of A and B was 5 min. $+2$ min. $=7$ min.

$\therefore$ they shall indicate the same time or approach each other 7 mins. in ($\frac{2}{3} : 7 :: 1$ hr.) $\frac{21}{2}$ hrs., *i. e.*, 10 hrs. 30 mins.

$\therefore$ both the clocks last agreed at 10 hrs. 30 mins. P.M.

Now in one hr. A gains $\frac{5}{9}$ min., so in $10\frac{1}{2}$ hrs. it gains $\frac{35}{6}=$ $5\frac{5}{6}$ mins. $\therefore$ the time indicated by A was 11 hrs. 55 mins. $+$ 10 hrs. 30 mins. $+ 5\frac{5}{6}$ mins. $=22$ hrs. $30\frac{5}{6}$ mins., *i.e.*, 10 hrs. 30 mins. 50 sec., which is also the time indicated by the second. *Ans.*

4. The rates of interest are as $5 : 3\frac{3}{4}$ $\therefore$ the number of years for which the sums are borrowed are as $3\frac{3}{4} : 5$, and $\therefore$ the difference $= 5-3\frac{3}{4}=1\frac{1}{4}$. But this is equal to 1 year by the question.

$\therefore$ $1\frac{1}{4} : 5 :: 1$ year $=4$ years $\therefore$ $1\frac{1}{4} : 3\frac{3}{4} :: 1$ year $=3$ years

$\therefore$ the number of years for which the sums are borrowed are 3 and 4.

Now we are to find the principal which will amount to Rs. 736 in 3 years at 5 per cent. or in 4 years at $3\frac{3}{4}$ per cent.

1 year : 3 :: 5 $=$ Rs. 15 interest.

Rs. 115 : Rs. 736 :: Rs. 100 sum $=$ Rs. 640. *Ans.*

---

5. (i) $x^2-7xy+12y^2=x^2-3xy-4xy+12y^2$

$=x(x-3y)-4y(x-3y)=(x-3y)(x-4y)$

$x^2+5xy+6y^2=x^2+2xy+3xy+6y^2$

$=x(x+2y)+3y(x+2y)=(x+2y)(x+3y)$

$x^2-5xy+4y^2=x^2-xy-4xy+4y^2$

$=x(x-y)-4y(x-y)=(x-y)(x-4y)$

$x^2+xy-2y^2=x^2+2xy-xy-2y^2$

$=x(x+2y)-y(x+2y)=(x+2y)(x-y)$

$\therefore$ the expression$=\dfrac{(x-3y)(x-4y)}{(x+2y)(x+3y)}\div\dfrac{(x-y)(x-4y)}{(x+2y)(x-y)}$

$=\dfrac{(x-3y)}{(x+3y)}\times\dfrac{(x-4y)}{(x-4y)}\times\dfrac{(x+2y)}{(x+2y)}\times\dfrac{(x-y)}{(x-y)}=\dfrac{x-3y}{x+3y}$. *Ans.*

(ii) The two factors of the numerator are—

$x^2 - xy + y^2 + x^2 + xy + y^2$, *i.e.*, $2(x^2+y^2)$ and

$(x^2-xy+y^2)^2 - (x^2-xy+y^2)(x^2+xy+y^2) + (x^2+xy+y^2)^2$

i. e. $(x^2+y^2)^2 - 2xy(x^2+y^2) + x^2y^2 - (x^4+x^2y^2+y^4)$

$+ (x^2+y^2)^2 + 2xy(x^2+y^2) + x^2y^2$,

i. e., $2(x^2+y^2)^2 + 2x^2y^2 - x^4 - x^2y^2 - y^4$

i. e., $x^4 + 5x^2y^2 + y^4$

$\therefore$ the expression $= \dfrac{2(x^2+y^2)(x^4+5x^2y^2+y^4)}{2(x^2+y^2)}$

$$= x^4 + 5x^2y^2 + y^4. \quad \textit{Ans.}$$

6. (i) The expression

$$= \frac{(a+b)(ab-c^2)}{(a+b)(a-b)} + \frac{(b+c)(bc-a^2)}{(b+c)(b-c)} + \frac{(c+a)(ca-b^2)}{(c+a)(c-a)}$$

But by the question, the denominators are all equal

$$\therefore = \frac{(a+b)(ab-c^2)+(b+c)(bc-a^2)+(c+a)(ca-b^2)}{(a^2-b^2) \text{ or } (b^2-c^2) \text{ or } (c^2-a^2)}$$

$$= \frac{0}{a^2-b^2} = 0.$$

(ii) The expression

$$= \frac{(p^2q^2-1)^p}{q^{2p}} \times \frac{(pq-1)^{q-p}}{q^{q-p}} \times \frac{p^{2q}}{(p^2q^2-1)^q} \times \frac{p^{p-q}}{(pq+1)^{p-q}}$$

$$= \frac{(p^2q^2-1)^p \times (pq-1)^{q-p}}{(p^2q^2-1)^q \times (pq+1)^{p-q}} \times \frac{p^{2q} \times p^{p-q}}{q^{2p} \times q^{q-p}}$$

$$= \frac{(p^2q^2-1)^{p-q} \times (pq-1)^{q-p}}{(pq+1)^{p-q}} \times \frac{p^{p+q}}{q^{p+q}}$$

$$= \frac{(pq+1)^{p-q} \times (pq-1)^{p-q} \times (pq-1)^{p-q}}{(pq+1)^{p-q}} + \frac{p^{p+q}}{q^{p+q}}$$

$$= (pq-1)^{p-q} \times (pq-1)^{q-p} \times \frac{p^{p+q}}{q^{p+q}}$$

$$= (pq-1)^0 \times \frac{p^{p+q}}{q^{p+q}} = 1 \times \frac{p^{p+q}}{q^{p+}} = \left(\frac{p}{q}\right)^{p+q} \quad \textit{Ans.}$$

7. $x^3-2x^2-19x+20$

$=(x^3+4x^2)-(6x^2+24x)+(5x+20)$

$=x^2(x+4)-6x(x+4)+5(x+4)$

$=(x^2-6x+5)(x+4)=(x-1)(x-5)(x+4)$

$x^3+2x^2-23x-60=x^3+4x^2-2x^2-8x-15x-60$

$=x^2(x+4)-2x(x+4)-15(x+4)$

$=(x^2-2x-15)(x+4)=(x-5)(x+3)(x+4)$

$x^4+7x^3-4x^2-52x+48$

$=x^4+4x^3+3x^3+12x^2-16x^2-64x+12x+48$

$=x^3(x+4)+3x^2(x+4)-16x(x+4)+12(x+4)$

$=(x^3+3x^2-16x+12)(x+4)$; and $x^3+3x^2-16x+12$

$=x^3-x^2+4x^2-4x-12x+12$

$=x^2(x-1)+4x(x-1)-12(x-1)$

$=(x^2+4x-12)(x-1)=(x-2)(x+6)(x-1)$

$\therefore$ the whole expression $=(x-2)(x+6)(x-1)(x+4)$.

Now we are to find the H. C. F. and L. C. M. of

$(x-1)(x-5)(x+4)$,

$(x+3)(x-5)(x+4)$,

and $(x-1)(x-2)(x+4)(x+6)$

$\therefore$ the H. C. F. $=x+4$; and L.C.M. $=$

$(x-1)(x-2)(x+3)(x+4)(x-5)(x+6)$. *Ans.*

8. $$\frac{6}{7-\cfrac{6}{7-\cfrac{6}{7-x}}}=1$$

Multiplying crosswise, we have

$$7-\frac{6}{7-\cfrac{6}{7-x}}=6$$

$$\therefore -\frac{6}{7-\cfrac{6}{7-x}}=6-7=-1$$

$$\therefore \frac{6}{7-\cfrac{6}{7-x}}=1.$$ Again, multiplying crosswise we get

$$7-\frac{6}{7-x}=6 \quad \therefore \ -\frac{6}{7-x}=6-7=-1$$

$$\therefore \ \frac{6}{7-x}=1 \ \therefore \ 7-x=6 \ \therefore \ x=1. \quad \textit{Ans.}$$

9. $(3-8x)(3x+4)=(6x+11)(3-4x)$

$\therefore \ 9x-24x^2+12-32x=18x+33-24x^2-44x$

$\therefore \ 9x-32x-18x+44x=33-12$

$\therefore \ 3x=21$ and $\therefore \ x=7.$ *Ans.*

10. Let £ x be the gross income in the 1st year; then £$(40+x)$ =income in the 2nd year. Now on the 1st income he pays 4*d.* in the pound $\therefore$ his net income during that year

$$= £1 : £x :: \frac{236}{240} = £\frac{59x}{60}$$

Again, on the 2nd income, he pays an income-tax of 6*d.* in the pound $\therefore$ his net income during that year

$$= £\,1 : £\,(40+x) :: £\frac{234}{240} = £\,(40+x)\frac{117}{120}$$

$$\therefore \frac{59x}{60}=\frac{117}{120}(40+x) \ \therefore \frac{59x}{60}=\frac{117}{3}+\frac{117x}{120}$$

$\therefore 118x=4{,}680+117x$

$\therefore x=$ £4,680 income during the 1st year

$\therefore$ £4,680 + £40, or £4,720, income during the 2nd year. *Ans.*

11. Let x be the number of children, and y the sum of their ages $\therefore$ the sum of the ages of the man and his wife $=6y$.

Two years ago the total age of the children $=y-2x$ and that of the man and his wife $=6y-4$.

Six years hence the sum of the ages of the children $=y+6x$, and that of the man and his wife $=6y+12$

$\therefore \ 6y-4=10(y-2x)$ and $6y+12=3(y+6x)$

$\therefore \ 5x-y=1$ .... (i)

$6x-y=4$ .... (ii)

$\therefore \ x=3.$ 3 children. *Ans.*

Euclid.

1. Euclid I. 14.

(Fig. 45.) The opposite angles of a rhombus are bisected by the diagonal which joins them ∴ ang. $BAO=DAO$. ∴ in trs. BAO, DAO, $DA=AB$ (hyp.), AO is common, ang. $DAO=$ang. BAO ∴ trs. are equal in all respects (I. 4) ∴ ang. $DOA=$ang. AOB.

Again in trs. AOB, COB, $AB=BC$ (hyp.), BO is common, $AO=OC$ (hyp.) ∴ trs. are equal in all respects (I. 8), ∴ ang. $AOB=$ang. COB; but these are adjacent angles ∴ each of them is a right angle ∴ ang. DOA, AOB also together$=$2rt. angs ∴ OB, OD are in one st. line (I. 14).

2. Euclid I. 41.

(Fig. 46.) Let the two equal st. lines AB, CD cut each other at right angles in O. Join AC, CB, BD, DA. Through C and D draw EF, GH prll. to AB (I. 31), and through A, B, draw EG, FH, prll. to CD, (I 31). Now $ECDG$ is a plm. ∴ $EG=CD$ (I. 34). Also $FH=CD$ (I. 34), $EF=AB=GH$ (I. 34); but $AB=CD$ (hyp.) ∴ $EFHG$ is equilateral.

Again ∵ $AECO$ is plm. ∴ ang. $AOC=AEC$ (I. 34); but AOC is a rt. ang. (hyp.) ∴ AEC is a rt. ang. Similarly the angs. at F, H, G are rt. angs. ∴ the figure is rectangular; and it is shewn to be equilateral ∴ it is a square described on EF or FH, *i.e.*, AB or CD.

Again, tr. $ACD=\frac{1}{2}$ plm. $ECDG$ (I. 41), and tr. $BCD=\frac{1}{2}$ plm. $CFHD$ (I. 41).

∴ $ABCD=\frac{1}{2}$ $EFHG$, *i.e.*, half the square on AB or CD.

3. Euclid II. 11.

Rect. $AB.BH=AH^2$ (hyp.), and $AB^2+BH^2=2$ rect. $AB.BH+AH^2$ (II. 7)$=2AH^2+AH^2=3AH^2$.

4. Euclid III. 27.

(Fig. 47). Let the diameter AB cut CD at right angles in the point G. Join CA, AD, DE, CE.

In trs. AGC, AGD $\because$ ang. AGC=ang. AGD $\because$ they are rt. angs. $CG=GD$ (III. 3), and AG is common, $\therefore$ the st. line $AC=AD$ (I. 4) $\therefore$ the arc AC=arc AD $\therefore$ ang. ACD=ang. AEC. Add ang. FCE to each of the equals. $\therefore$ angs. $ACD+FCE$=angs. $AEC+FCE$. $\therefore$ ang. ACE=ang. FEC, and ang. FCE=ang. DFE (I. 32).

5. See Question 9 of 1887.

6. Euclid IV. 10.

1892.

TUESDAY, 22ND NOVEMBER.

Arithmetic and Algebra.

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1. What decimal of a rupee is ·964 pie? Find the value of ·97625 rupee. 8

Simplify :—

$$\frac{\frac{5}{14}-\frac{3}{7}\text{ of }\frac{1}{2}}{\frac{5}{16}+\frac{7}{12}\text{ of }3\frac{1}{4}-\left(\frac{7}{8}\text{ of }\frac{3}{2}\frac{1}{2}-\frac{1}{3}\right)}\div\frac{\frac{1}{8}\text{ of }\frac{1}{2}+\frac{2}{3}\text{ of }5}{9\frac{1}{3}-1\frac{2}{3}}.$$

2. How long will two examiners, working 8 hours a day, take to look over the answers to this paper, if four examiners, working 5 hours a day, can do it in 8 days? 6

3. On a river, B is intermediate to and equidistant from A and C; a boat can go from A to B, and back, in 5 hours 15 minutes, and from A to C in 7 hours; how long would it take to go from C to A? 8

4. What income will a retired officer obtain in England, from one lakh of rupees, Indian Government [illegible] per cent. bonds, when, for drawing and remitting it, gents in India charge him 3 per cent., and exchange 1s. $2\frac{1}{4}d$. for the rupee? 10

5. Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass being as follows: In the first glass as 2 : 3, in the second 3 : 4, and in the third 4 : 5. The contents of the three glasses are poured into a single vessel: what is the proportion of spirits to water in it? 8

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6. Define an 'algebraical expression' and the 'degree of an expression.' What is a homogeneous expression? 8

Find the numerical value of—

$$(xyz+x+y+z)^2-(xy+yz+zx+1)^2,$$

when $x=2$, $y=3$, and $z=4$.

7. Find the G. C. M. of— 9

$$2x^3-x^2-x-3 \text{ and } x^5-x^3-4x^2-3x-2;$$

and the L. C. M. of—

$$9x^2-4,\ 4x^2-36,\ 3x^2-7x-6, \text{ and } 3x^2+7x-6.$$

8. Simplify :— 10

$$\text{(i)}\ \sqrt{\left\{\frac{(a-b)^6+8ab(a-b)^4+16a^2b^2(a-b)^2}{a^6b^2-2a^4b^4+a^2b^6}\right\}}$$

and

$$\text{(ii)}\ x+1-\cfrac{x}{x+2-\cfrac{x+1}{x+\cfrac{1}{x+2}}}.$$

9. Solve :— 12

$$\text{(i)}\ \tfrac{1}{2}(x-2)-\frac{x-5}{9}+\frac{5(x-1)}{6}=0.$$

$$\text{(ii)}\ \frac{3}{x}+\frac{6}{y}=4;\ \frac{9}{x}-\frac{2}{y}=2.$$

$$\text{(iii)}\ \frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}.$$

10. Find the fraction which becomes equal to a half when the numerator is increased by one, and equal to a third when the denominator is increased by one. 9

11. In a mile race between a bicycle and a tricycle, their rates were proportional to 5 and 4. The tricycle had a minute's start, but was beaten by 176 yards. Find the rates of each. 12

WEDNESDAY, 23RD NOVEMBER.

Euclid.

1. Any two angles of a triangle are together less than two right angles. 4

State the axiom on which Euclid founds his theory of parallel straight lines; shew that the theorem in question may be regarded as the converse of this axiom. 10

2. If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a right angle. 7

3. The sides AB, AC of a triangle are bisected in F and E, and the lines BE, CF are drawn and produced to M and N, so that BM and CN are respectively double of BE and CF: prove that MA and AN are in a straight line. 10

4. In every triangle, the square on the side subtending an acute angle, is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides and the straight line intercepted between the perpendicular let fall upon it from the opposite angle, and the acute angle. 9

If from one of the extremities of the base of an isosceles triangle a perpendicular is drawn to the opposite side, then twice the rectangle contained by that side and the segment adjacent to the base is equal to the square on the base. 10

5. If a straight line drawn through the centre of a cle bisect a straight line in it which does not pass 7

through the centre, it shall cut it at right angles; and if it cut it at right angles, it shall bisect it.

6. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles. 8

The straight lines which bisect any angle of a quadrilateral figure inscribed in a circle and the opposite exterior angle meet on the circumference. 11

7. Inscribe an equilateral and equiangular hexagon in a given circle. 10

8. Two men desire to dig a well equidistant from each of two intersecting straight roads, and also equidistant from their two houses on one of the roads. Shew how to find the position of the well. 14

SOLUTIONS.

1. $\frac{\text{pie } \cdot 964}{\text{pies } 192} = \cdot 0050208\dot{3}$. *Ans.*

Re. $\cdot 97625$ = As. $\cdot 97625 \times 16$ = As. $15 \cdot 62 = 15 + \cdot 62 \times 12$ pies = As. 15 $7 \cdot 44$ pies = As. 15 $7\frac{11}{25}$ pies. *Ans.*

$\frac{5}{14} - \frac{3}{7} \times \frac{1}{2} = \frac{1}{7}$. $\frac{7}{6} \times \frac{37}{21} - \frac{1}{3} = \frac{29}{24}$ $\therefore$ $\frac{5}{16} + \frac{7}{12} \times \frac{13}{4} - \frac{29}{24} = 1$.
$\therefore$ $\frac{1}{7} \div 1 = \frac{1}{7}$. Now $\frac{1}{3} \times \frac{1}{2} + \frac{3}{2} \times 5 = \frac{23}{3}$.
And $\frac{28}{3} - \frac{5}{3} = \frac{23}{3}$ $\therefore$ $\frac{23}{3} \div \frac{23}{3} = 1$ $\therefore$ $\frac{1}{7} \div 1 = \frac{1}{7}$. *Ans.*

2. Four examiners working 5 hrs. a day can look over $\frac{1}{5}$ in one day.

$\therefore$ 4 examiners can look over $\frac{1}{40}$ in one hour.
$\therefore$ 1 ,, ,, ,, $\frac{1}{160}$,, ,,
$\therefore$ 2 ,, ,, ,, $\frac{2}{160}$,, ,,
$\therefore$ 2 examiners, working 8 hrs., can look over $\frac{8 \times 2}{160}$ of the answers in a day.

i.e., 2 examiners, working 8 hrs., can do $\frac{1}{10}$ in a day, *i.e.*, they take 10 days to examine the answers. *Ans.*

3. The boat goes from A to C in 7 hrs., and as B is equidistant from A and C, it can go from A to B in $3\frac{1}{2}$ hrs. $\therefore$ the boat can go from B to A in $5\frac{1}{4}-3\frac{1}{2}=1\frac{3}{4}$ hrs and the distances from B to C and B to A are equal $\therefore$ the boat can go from C to A in $1\frac{3}{4}\times 2$, i.e., in $3\frac{1}{2}$hrs. *Ans.*

4. Inc. from Rs. 100 of bonds = Rs. $4\frac{1}{2}$

„ „ one lakh „ „ $= \dfrac{100000}{100}\times 4\frac{1}{2} =$ Rs. 4500.

3 per cent. of Rs. 4500 $= 4500\times\frac{3}{100}=$Rs. 135

$\therefore$ the actual sum remitted$=4500-135=$Rs. 4365

Rc. 1 = 1*s*. $2\frac{1}{4}$*d*. = £$\frac{57}{960}$ $\therefore$ Rs. 4365

$= 4365\times$ £$\frac{57}{960}$ = £259 3*s*. $5\frac{1}{4}$*d*. *Ans.*

5. There are altogether 5 parts in the first vessel, of which 2 parts are wine and 3 water, *i. e.*, $\frac{2}{5}$ths wine and $\frac{3}{5}$ths water. In the same way the 2nd has $\frac{3}{7}$ths wine and $\frac{4}{7}$ths water and the third has $\frac{4}{9}$ths wine and $\frac{5}{9}$ths water.

$\therefore$ the ratio required $= \frac{2}{5}+\frac{3}{7}+\frac{4}{9} : \frac{3}{5}+\frac{4}{7}+\frac{5}{9}$,

i.e., spirit : water :: $\frac{401}{315} : \frac{544}{315}$. 401 : 544. *Ans.*

6. Any collection of algebraical symbols which are the letters and signs used in Algebra is called an *algebraical expression.*

The number of the letters which occur as factors of an algebraical product is called the *degree* of the product thus, x^3y is said to be of the fourth *degree.*

An expression is said to be *homogeneous*, when all its terms are of the same dimensions: thus, $a^3+a^2b+ab^2+b^3$ is homogeneous, for each of its terms is of four dimensions.

$x=2$, $y=3$, $z=4$ $\therefore$ the expression

$= (24+2+3+4)^2-(6+12+8+1)^2=(33)^2-(27)^2$

$= (33+27)(33-27)=60\times 6=360$. *Ans.*

7.

$$x^5-x^3-4x^2-3x-2$$
$$2$$
$$2x^3-x^2-x-3\ )\ 2x^5-2x^3-8x^2-6x-4\ (\ x^2+x-1$$
$$2x^5-\ x^3-3x^2-x^4$$
$$x^4-x^3-5x^2-6x-4$$
$$2$$
$$2x^4-2x^3-10x^2-12x-8$$
$$2x^4-\ x^3-\ \ x^2-\ 3x$$
$$-x^3-9x^2-9x-8$$
$$2$$
$$-2x^3-18x^2-18x-16$$
$$-2x^3+\ x^2+\ x+3$$
$$-19x^2-19x-19$$

$$-19)\ x^2+x+1\ )\ 2x^3-x^2-x-3\ (\ 2x-3.$$
$$2x^3+2x^2+2x$$
$$-3x^2-3x-3$$
$$-3x^2-3x-3$$

$\therefore$ the G. C. M. $=x^2+x+1$. *Ans.*

$9x^2-4=(3x+2)(3x-2)$

$4x^2-36=4(x^2-9)=4(x+3)(x-3)$.

$3x^2-7x-6=3x^2-9x+2x-6$

$=3x(x-3)-2(x-3)=(3x-2)(x-3)$.

$3x^2+7x-6=3x^2+9x-2x-6$

$=3x(x+3)-2(x+3)=(3x-2)(x+3)$

$\therefore$ the L. C. M. of $(3x+2)(3x-2)$, $4(x+3)(x-3)$, $(3x-2)(x-3)$, and $(3x-2)(x+3)$

$=4(3x+2)(3x-2)(x+3)(x-2)$. *Ans.*

8. (i) $$\sqrt{\frac{(a-b)^2}{a^2b^2}}\cdot\sqrt{\frac{(a-b)^4+8ab(a-b)^2+16a^2b^2}{a^4-2a^2b^2+b^4}}$$

$$=\frac{a-b}{ab}\cdot\sqrt{\frac{\{(a-b)^2+4ab\}^2}{(a^2-b^2)^2}}$$

$$=\frac{a-b}{ab}\cdot\frac{(a-b)^2+4ab}{a^2-b^2}=\frac{a-b}{ab}\times\frac{(a+b)^2}{a^2-b^2}=\frac{a+b}{ab}.\quad \textit{Ans.}$$

(ii) $x+\frac{1}{x+2}=\frac{x^2+2x+1}{x+2}=\frac{(x+1)^2}{x+2}$

$$\frac{x+1}{\frac{(x+1)^2}{x+2}}=\frac{x+2}{x+1};\ x+2-\frac{x+2}{x+1}=\frac{x(x+2)}{x+1}$$

$$x\div\frac{x(x+2)}{x+1}=\frac{x+1}{x+2};\ x+1-\frac{x+1}{x+2}=\frac{x^2+2x+1}{x+2}$$

$\therefore$ the whole expression$=\frac{x^2+2x+1}{x+2}=\frac{(x+1)^2}{x+2}$. *Ans.*

9. Clearing off fractions, we get—

(i) $9x-18-2x+10+15x-15=0$

$$\therefore 22x=23\ \therefore x=1\tfrac{1}{22}.\quad \textit{Ans.}$$

(ii) Multiply the second equation by 3 and add the two equations : thus—

$$\left.\begin{array}{l}\frac{3}{x}+\frac{6}{y}=4\\ \frac{27}{x}-\frac{6}{y}=6\end{array}\right\}\therefore\ \frac{30}{x}=10\ \therefore\frac{3}{x}=1\ \therefore x=3$$

Substitute the value of x in the first equation

$$\therefore\ \frac{3}{3}+\frac{6}{y}=4\ \therefore\frac{6}{y}=3\ \therefore\ y=2.\quad \textit{Ans.}$$

(iii) The equation can be put thus—

$$1+\frac{1}{x-2}-1-\frac{1}{x-3}=1+\frac{1}{x-6}-1-\frac{1}{x-7}$$

$$\therefore\ \frac{1}{x-2}-\frac{1}{x-3}=\frac{1}{x-6}-\frac{1}{x-7}$$

$$\therefore\ \frac{(x-3)-(x-2)}{(x-2)(x-3)}=\frac{(x-7)-(x-6)}{(x-6)(x-7)}$$

$$\therefore\ \frac{-1}{(x-2)(x-3)}=\frac{-1}{(x-6)(x-7)}$$

$$\therefore\ (x-2)(x-3)=(x-6)(x-7)$$

$$\therefore\ x^2-5x+6=x^2-13x+42$$

$$\therefore\ 8x=36\ \therefore x=4\tfrac{1}{2}.\quad \textit{Ans.}$$

10. Let x be the numerator, and y be the denominator of the fraction.

$\therefore\ \frac{x+1}{y}=\frac{1}{2}$ ....(i) and $\frac{x}{y+1}=\frac{1}{3}$ ...(ii)

$2x-y=-2$................. (i) and $3x-y=1$............(ii)

Subtracting one from the other, we get $x=3$ $\therefore$ $y=8$

$\therefore$ the fraction$=\frac{3}{8}$ *Ans.*

11. Let the rate of the bicycle be $5x$ miles per hour, therefore the rate of the tricycle would be $4x$ miles per hour.

$\frac{1}{5x}$ hrs. $\therefore$ is taken by the bicycle to travel a mile's distance.

The tricycle takes 1 minute, *i. e.*, $\frac{1}{60}$ of an hour more.

$\therefore$ it takes $\frac{1}{5x}+\frac{1}{60}$ hr. $\therefore$ the rate being $4x$ miles per hour, it goes over $\left(\frac{1}{5x}+\frac{1}{60}\right)4x$ miles and still is beaten by 176 yards, *i.e.*, $\frac{176}{1760}$ or $\frac{1}{10}$ of a mile $\therefore 1-4x\left(\frac{1}{5x}+\frac{1}{60}\right)=\frac{1}{10}$

$\therefore\ 1-\frac{4}{5}-\frac{x}{15}=\frac{1}{10}\ \therefore\ \frac{x}{15}=\frac{1}{10}\ \therefore\ 2x=3.\ \ \therefore\ 4x=6$

$\therefore$ 6 miles per hour is the rate of the tricycle, and $5x$, *i. e.*, $5\times\frac{3}{2}$ or $7\frac{1}{2}$ miles per hour, the rate of the bicycle. *Ans.*

Euclid.

1. Euclid I. 17.

The 12th axiom. I. 17 is the converse of axiom 12, and it may be enunciated thus :—If two straight lines meet at a point if produced, and are cut by another straight line, then the interior angles on the same side of this line are together less than two right angles.

2. Euclid I. 48.

3. (Fig. 48.) In trs. ANF, PFC, $\because$ $AF=FB$ (hyp.) $CF=FN$ (constr.) and ang. $AFN=$ ang. BFC (I. 15) $\therefore$ ang. $NAF=$ ang. ABC (I. 4). Similarly ang. $MAE=$ ang. ACB (I. 4); but angs. $ABC+BCA+CAB=2$rt. angs. (I. 32), $\therefore$ angs. $NAF+BAC+CAM=2$ rt. angs. $\therefore$ AN, AM are in one str. line (I. 14).

4. Euclid II. 13.

Let ABC be an isosceles tr. and from the extremity B of the base BC let BD be drawn perp. to AC.

Then $AC^2+CD^2=2$ rect. $AC.CD+AD^2$ (II. 7). Add BD^2 to each of the equals ; $\therefore$ $AC^2+BD^2+CD^2=2$ rect. $AC.CD+AD^2+BD^2$; but $BD^2+CD^2=BC^2$ (I. 47) and $AD^2+BD^2=AB^2$, $\therefore$ $AC^2+BC^2=2$ rect. $AC.CD+AB^2$; but $AC^2=AB^2$ $\because AB=AC$ (hyp.) $\therefore$ $BC^2=2$ rect. $AC.CD$.

5. Euclid III. 3.

6. Euclid III. 22.

(Fig. 49) Let $ABCD$ be inscribed in cir. Let ang. BAD be bisected by AF and the exterior angle BCE by CF. Then the two lines will meet on the circumference.

Again, ang. $DCF=$ angs. $DCB+BCF$ and ang. $BCF=$ ang. BAF (III. 21), $\therefore$ ang. $DCF=$ angs. $DCB+BAF$; add ang. DAF to each of the equals. $\therefore$ angs. $DCF+DAF=$ angs. $DCB+BAF+DAF$, *i. e.*, angs. $DCB+BAD$; but angs. $DCB+BAD=2$ rt. angs. (III. 22) $\therefore$ angs. $DCF+DAF=2$ rt. angs., *i. e.*, $ADCF$ can be inscribed in the circle (converse of III. 22) $\therefore$ AF, CF meet on the circumference.

7. Euclid IV. 15.

(Fig. 50). Let AB, CD intersect each other at the point O Let E, F be the two houses on AB, on one side of the roads. It is required to find a point equidistant from AB, CD and also from E and F.

Bisect EF in G (I. 10). Through G draw GH at right angles to AB (I. 11), meeting OH, which bisects the ang. AOD (I. 9), in H. Through H draw HK perp. to OD (I. 12). Then H shall be the required point.

In trs. GHO, HOK, $\because$ ang. $GOH=$ ang. HOK (constr.), ang. $HGO=$ang. HKO, $\because$ they are rt. angs., and HO, which is opposite to one of the equal angles, is common to both, $\therefore$ $HG=HK$ (I. 26), *i. e.*, the point H is equidistant from AB and CD.

Join EH, FH. In trs. EHG, GHF, $\because$ $EG=GF$ (constr.), GH is common to both, and ang. $EGH=$ang. FGH (ax. 11) $\because$ they are rt. angs. $\therefore$ $EH=HF$ (I. 4), *i. e.*, the point H is equidistant from the given points E and F. $\therefore$ H is equidistant from AB, CD and E and F.

1893-94.

(Set in the Mofussil.)

TUESDAY, 21st NOVEMBER,

Arithmetic and Algebra.

JAMSHADJI A. DALAL, M.A., L.L.B.

KAVASJI J. SANJANA, M.A.

RAGHUNATH NARAYAN APTE, M.A., L.L.B.

VINAYAK NARAYAN NENE, ESQ.

1. Divide each of the numbers 2,572,125 and 4,061,250 by 125; and express as a decimal the first quotient divided by the second. 5

2. Find, by practice, the value of 5 yards $22\frac{1}{2}$ in. at £2 1*s.* 2*d.* a yard. 4

3. If the carriage of 2 cwt. 1 qr. and 18 lbs. of goods, for 56 miles, be £1 1*s.*, what weight can be carried at the same rate, 200 miles, for £4 3*s.* 4*d.* ? 5

4. A man invests £3,000 in 5 per cents. If, after deducting an income tax of 8*d.* in the pound, the man's clear income is £174, what is the price of the 5 per cents. ?

5. A cistern is filled by two taps A and B in 4 and 6 hours respectively, and is emptied by a waste pipe C in 3 hours. When the cistern is half full A and B are closed, and C is opened; after one hour, B is turned on; and after half an hour more, A is turned on. In what time after C is first opened, does the cistern become full? 18

6. A person buys two kinds of tea, at 5*s.* a lb. and 6*s.* a lb. respectively; and after mixing them he sells the mixture at 6*s.* 6*d.* a lb., thereby gaining 17 per cent. In what proportion does he mix them? 8

---

7. Simplify :—

(*a*) $\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{a^2-ax+x^2}+\frac{2x^3}{a^4+a^2x^2+x^4}$. 4

(*b*) $\frac{6x^2y^2}{m+n}\div\left[\frac{3(m-n)x}{7(r+s)}\div\left\{\frac{4(r-s)}{21xy^2}\div\frac{r^2-s^2}{4(m^2-n^2)}\right\}\right]$. 4

8. If $a=y+z-2x$, $b=z+x-2y$, $c=x+y-2z$, find $b^2+c^2-a^2+2bc$ in terms of x, y, z. 8

9. (*a*) Find the factors of— 4

$$(a+b)^2+(a+c)^2-(b+d)^2-(c+d)^2.$$

(*b*) Find the L. C. M. of— 8

x^4+7x^2+16, x^2+3x+4 and x^2+3x-4.

10. Find the square root of—

(*a*) $x^4+x^2yz+\frac{y^2z^2}{4}-2x^2z^2-yz^3+z^4$. 4

(*b*) Find the cube root of— 7

$$\frac{a^3}{b^3}-\frac{b^3}{a^3}-3\left\{\frac{a^2}{b^2}+\frac{b^2}{a^2}\right\}+5.$$

11. Solve :—

(*a*) $(a+b-x)(a-b+x)+(a+x)(b+x)-a^2=0$. 6

(*b*) $3x-2y+2=5x-3y+1\frac{5}{6}=6x-y-4\frac{1}{2}$. 6

12. The denominator of a fraction exceeds the numerator by 4, and if 5 is taken from each, the sum of the ·iprocal of the new fraction and four times the original ..ction is 5. Find the original fraction. 9

1893-94.

Set at Bombay.

THURSDAY, 23RD NOVEMBER.

Arithmetic.

1. Reduce to their simplest forms:— 5

$$\text{(i) } \frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{6}}\ ;\ \text{(ii) } \frac{2}{3+\frac{4}{5-\frac{6}{7}}}.$$

2. Find, by practice, the value of 9 cwts. 3 qrs. 24 lbs. at £3 5*s*. 8*d*. per cwt. 5

3. If 40 men, 60 women or 80 children can do a piece of work in 6 months, in what time will 10 men, 10 women and 10 children do one-third of the work? 5

4. A person invested £1,000 in the 3 per cents at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$, he sold out, and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{8}$: find the increase in his income. 9

5. A cistern can be filled by two pipes, *A* and *B*, in 12 minutes and 14 minutes respectively and can be emptied by a third, *C*, in 8 minutes. If all the taps are turned on at the same moment what part of the cistern will remain unfilled at the end of 7 minutes? 8

6. Two clocks point to 2 o'clock at the same instant on the afternoon of 25th April: one loses 7 seconds and the other gains 8 seconds, in 24 hours; when will one be half an hour before the other, and what time will each clock then shew? 8

Algebra.

7. Simplify:— 4

$$\text{(i) } \left(1-\frac{1}{1+x}\right)\left(x+\frac{1}{2+x}\right)\frac{\frac{1}{x^2}-x}{1+\frac{1}{x}}\div\left(1+x+\frac{1}{x}\right);$$

(ii) $\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x+1}-\frac{x}{x^2-1}+\frac{3}{x(x^2-1)}$. 4

8. If $b^2=ac$, $x=\frac{1}{2}(a+b)$, and $y=\frac{1}{2}(b+c)$, prove 8

that $\frac{a}{x}+\frac{c}{y}=2$.

9. (*a*) Find the *H.C.M.* of— 5

$x^3+6x^2+11x+x$ and $x^3+9x^2+27x+27$; and

(*b*) the *L.C.M.* of xy, $x-y$, and y^3-x^2y. 4

10. (*a*) Find the square root of— 4

$\frac{x^2}{y^2}+\frac{y^2}{x^2}-\frac{x}{y}+\frac{y}{x}-1\frac{3}{4}$; and

(*b*) The cube root of— 8

$x^3+\frac{8}{x^3}-12x^2-\frac{48}{x^2}+54x+\frac{108}{x}-112$.

11. Solve—

(i) $\frac{1-x^2}{1-4x^2}-\frac{x}{2x-1}+\frac{1}{4}=0$ 6

(ii) $\left.\begin{aligned}\frac{2}{x}-\frac{3}{2y}&=\frac{41}{35}\\ \frac{2\frac{1}{2}}{2x}+\frac{3\frac{1}{2}}{y}&=-\frac{73}{70}\end{aligned}\right\}$ 9

12. A labourer is engaged for 30 days, on condition that he receives 2*s.* 6*d.* for each day he works and loses 1*s.* for each day he is idle; he receives £2 7*s.* in all; for how many days did he work? 8

Euclid.

WEDNESDAY, 22ND NOVEMBER.

1. Draw a straight line at right angles to a given straight line, from a given point in the same. 6

2. Triangles upon the same base and between the same parallels are equal. 6

If through the angular points of a triangle *ABC*, 12
re be drawn three parallel straight lines, *AD*, *BE*,
F to meet the opposite sides, or this produced, in

D, E, and F respectively; prove that the area of the triangle DEF is double of the area of the triangle ABC.

3. If a straight line is divided into any two parts, the square on the whole line is equal to the sum of the squares on the two parts, together with twice the rectangle contained by the two parts. 8

In a triangle ABC, C is a right angle: prove that the area of the square on AB, increased by four times the area of the triangle, is equal to the area of the square on the line made up of AC and CB. 9

4. No parallelograms but such as are rectangular can be inscribed in a circle. 8

5. In equal circles the arcs which subtend equal angles, whether at the centres or at the circumferences, are equal. 7

AOB and COD are diameters of a circle at right angles to each other, E is a point in the arc AD, and EFG is a chord, meeting COD in F, and drawn in such a direction that EF is equal to the radius: shew that the arc BG is equal to three times the arc AE. 14

6. If a straight line touch a circle, and from the point of contact a chord be drawn the angles which this chord makes with the tangent are equal to the angles in the alternate segments of the circle. 8

C is the centre of a circle; CA, CB are two radii, at right angles to each other; from B any chord BP is drawn cutting CA at N; if the circle around the triangle ANP be drawn, shew that it will be touched by BA. 12

7. Inscribe a circle in a given regular pentagon. 10

SOLUTIONS.

Set in the Mofussil.

1. $\dfrac{2572125}{125}=20577,\ \dfrac{4061250}{125}=32490.\ \dfrac{20577}{32490}=\cdot 6\dot{3}$ *Ans.*

2.

			£	s.	d.	
			2	1	2	value of 1 yd.
					5	
			10	5	10	value of 5 yds.
15 ins.		$\frac{1}{12}$ of 5 yds.		17	$1\frac{5}{6}$	„ „ 15 ins.
$7\frac{1}{2}$ „		$\frac{1}{2}$ of 15 ins.		8	$6\frac{11}{12}$	„ „ $7\frac{1}{2}$ „
			11	11	$6\frac{3}{4}$	value of 5 yds. $22\frac{1}{2}$ ins. *Ans.*

3. $\left.\begin{array}{l}(\textit{Inverse})\ 200 \text{ miles} : 56 \text{ miles}\\ (\textit{Direct})\ 252d. : 1{,}000d.\end{array}\right\} :: 270 \text{ lbs.}$

$=300$lbs. $=2$cwts. 2qrs. 20lbs. *Ans.*

4. $240d.-8d.=232d.$, net income of £1 $=£\dfrac{232}{240}=£\dfrac{29}{30}$

gross income £1 : £5 :: net income £$\frac{29}{30}$ $=£\frac{29}{6}$.

£174 is the net income obtained by investing £3,000,

$\therefore$ £$\frac{29}{6}$ being the net income, $\frac{29}{6}\times\frac{3000}{174}=\frac{250}{3}$

$=£83\frac{1}{3}$ is the price of the 5 per cents. *Ans.*

5. $\frac{1}{4}$ of the cistern will be filled by A in one hour.

$\frac{1}{6}$ „ „ „ „ B „ „

$\frac{1}{3}$ „ „ emptied „ C „ „

When the cistern is half-full A and B are closed and C is opened for 1 hr. $\therefore\ \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ of the cistern is filled. Now B is opened, and B and C work together for half an hour.

In one hour $\frac{1}{3}-\frac{1}{6}$, *i.e.*, $\frac{1}{6}$ is emptied when B and C are set in motion $\therefore$ in half an hour, $\frac{1}{12}$ of the cistern is emptied; hence $\frac{1}{6}-\frac{1}{12}=\frac{1}{12}$ is filled when A is opened $\therefore\ 1-\frac{1}{12}=\frac{11}{12}$ remains to be filled. Now all the taps are set in motion, $\therefore\ \frac{1}{4}+\frac{1}{6}-\frac{1}{3}=\frac{1}{12}$ is filled in one hour, $\therefore\ \frac{11}{12}$ will be filled in 1 hours after the time when A is opened; therefore from

the time when C is first opened $1+\frac{1}{2}+11=12\frac{1}{2}$ hrs. are wanted to fill the cistern. *Ans.*

6. The cost price per lb. of the mixture=

$$117s. : 6\tfrac{1}{2}s. :: 100s.=\frac{50}{9}s.= 5s.\ 6\tfrac{2}{3}d.$$

When the mixture is sold at $5s.\ 6\frac{2}{3}d.$, each lb. of the cheaper tea brings a gain of $5s.\ 6\frac{2}{3}d.-5s.=6\frac{2}{3}d.$, and each lb. of the dearer tea brings a loss of $6s.-5s.\ 6\frac{2}{3}d.=5\frac{1}{3}d.$

Hence in order that the loss in one case may be compensated by a gain in the other, the teas ought to be mixed in the proportions of $5\frac{1}{3}$ lbs. of the former to $6\frac{2}{3}$ lbs. of the latter, and thus the mixture will cost the person just $5s.\ 6\frac{2}{3}d.$

$\therefore$ the proportion$=5\frac{1}{3} : 6\frac{2}{3}$, *i. e.*, $\frac{16}{3} : \frac{20}{3}=4 : 5$. *Ans.*

Or thus :—

The cost price per lb. of the mixture is $5s.\ 6\frac{2}{3}d.$ The gain per lb. of the 1st sort$=5s.\ 6\frac{2}{3}d.-5s.=6\frac{2}{3}d.$ And the loss per lb. of the 2nd sort$=6s.-5s.\ 6\frac{2}{3}d.=5\frac{1}{3}d.$

$\therefore$ for 1 lb. of the second sort he must sell as many lbs. of the first as would give a profit of $5\frac{1}{3}d.$

Now $6\frac{2}{3}d.$ is gained by selling 1 lb. of the first sort.

$\therefore$ $5\frac{1}{3}d.$,, ,, $\frac{3}{20}\times\frac{16}{3}=\frac{4}{5}$ lb. of the first, therefore the teas must be mixed in the proportion of $\frac{4}{5}$ to 1, *i.e.*, 4: 5. *Ans.*

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7. (i) $a^4+a^2x^2+x^4=(a^2+ax+x^2)(a^2-ax+x^2)$.

$\therefore$ the expression$=$

$$\frac{(a+x)(a^2-ax+x^2)+(a-x)(a^2+ax+x^2)+2x^3}{a^4+a^2x^2+x^4}$$

$$=\frac{(a^3+x^3)+(a^3-x^3)+2x^3}{a^4+a^2x^2+x^4}=\frac{2(a^3+x^3)}{a^4+a^2x^2+x^4}$$

$$=\frac{2(a+x)(a^2-ax+x^2)}{(a^2+ax+x^2)(a^2-ax+x^2)}=\frac{2(a+x)}{a^2+ax+x^2}.\quad \textit{Ans.}$$

(ii) $$\frac{4(r-s)}{21xy^2}\times\frac{4(m^2-n^2)}{(r+s)(r-s)}=\frac{16(m^2-n^2)}{21xy^2(r+s)}$$

$$\frac{3(m-n)x}{7(r+s)}\times\frac{21xy^2(r+s)}{16(m-n)(m+n)}=\frac{9x^2y^2}{16(m+n)}.$$

Now the expression $=$

$$\frac{6x^2y^2}{m+n}+\frac{16(m+n)}{9x^2y^2}=\frac{32}{3}=10\tfrac{2}{3}. \quad \textit{Ans.}$$

8. $b^2+c^2-a^2+2bc=b^2+2bc+c^2-a^2$

$$=(b+c)^2-a^2=(b+c+a)(b+c-a).$$

Now $a+b+c=y+z-2x+z+x-2y+x+y-2z=0.$

$\therefore$ $(b+c+a)(b+c-a)=0.$ *Ans.*

9. (i) The expression $=$

$$(a+b)^2-(c+d)^2+(a+c)^2-(b+d)^2.$$

$$(a+b)^2-(c+d)^2=(a+b+c+d)(a+b-c-d)$$

$$(a+c)^2-(b+d)^2=(a+c+b+d)(a+c-b-d).$$

Hence the expression

$$=(a+b+c+d)(a+b-c-d)+(a+b+c+d)(a+c-b-d)$$

$$=(a+b+c+d)\{(a+b-c-d)+(a+c-b-d)\}$$

$$=(a+b+c+d)(2a-2d)=2(a+b+c+d)(a-d). \quad \textit{Ans.}$$

(ii) $x^4+7x^2+16=x^4+8x^2+16-x^2=(x^2+4)^2-x^2$

$$=(x^2+x+4)(x^2-x+4)$$

$$x^3+3x+4=x^3+x^2-x^2-x+4x+4$$

$$=x^2(x+1)-x(x+1)+4(x+1)=(x^2-x+4)(x+1)$$

$$x^3+3x-4=x^3-x^2+x^2-x+4x-4$$

$$=x^2(x-1)+x(x-1)+4(x-1)=(x^2+x+4)(x-1).$$

$\therefore$ the L. C. M. of $(x^2+x+4)(x^2-x+4)$,

$(x^2-x+4)(x+1)$ and $(x^2+x+4)(x-1)$

$$=(x^2-x+4)(x^2+x+4)(x+1)(x-1). \quad \textit{Ans.}$$

10. (i) The expression

$$=x^4+\left(x^2yz-2x^2z^2\right)+\left(\frac{y^2z^2}{4}-yz^3+z^4\right)$$

$$=x^4+2x^2z\left(\frac{y}{2}-z\right)+\left\{z\left(\frac{y}{2}-z\right)\right\}^2$$

$$=\left\{x^2+z\left(\frac{y}{2}-z\right)\right\}^2.$$

$\therefore$ the square root of the expression $=x^2+\dfrac{yz}{2}-z^2.$ *Ans.*

(ii) $3\left(\frac{a}{b}\right)^2 = \frac{3a^2}{b^2}$

$3 \times \frac{a}{b} \times -1 = -\frac{3a}{b}$

$1^2 = +1$

$$\frac{3a^2}{b^2} - \frac{3a}{b} + 1$$

$3\left(\frac{a}{b}-1\right)^2 = \frac{3a^2}{b^2} - \frac{6a}{b} + 3$

$3\left(\frac{a}{b}-1\right) \times -\frac{b}{a} = -3 + \frac{3b}{a}$

$\left(-\frac{b}{a}\right)^2 = +\frac{b^2}{a^2}$

$$\frac{3a^2}{b^2} - \frac{6a}{b} + \frac{3b}{a} + \frac{b^2}{a^2}$$

$$\frac{a^3}{b^3} - \frac{3a^2}{b^2} + 5 - \frac{3b^2}{a^2} - \frac{b^3}{a^3} \;\Big|\; \frac{a}{b} - 1 - \frac{b}{a}$$

$$\frac{a^3}{b^3}$$

$$-\frac{3a^2}{b^2} + 5$$

$$-\frac{3a^2}{b^2} - 1 + \frac{3a}{b}$$

$$-\frac{3a}{b} + 6 - \frac{3b^2}{a^2} - \frac{b^3}{a^3}$$

$$-\frac{3a}{b} + 6 - \frac{3b^2}{a^2} - \frac{b^3}{a^3}$$

$$\frac{a}{b} - 1 - \frac{b}{a}. \quad \textit{Ans.}$$

11. (i) $(a+b-x)(a-b+x) + (a+x)(b+x) - a^2 = 0$

$\therefore a^2 - b^2 - x(a-b) + x(a+b) - x^2 + ab + x(a+b) + x^2 - a^2 = 0$

$\therefore \quad 2x(a+b) - x(a-b) = b^2 - ab$

$\therefore \quad x(2a+2b-a+b) = b(b-a)$

$\therefore \quad x(a+3b) = b(b-a) \quad \therefore x = \frac{b(b-a)}{a+3b}$

(ii) $3x-2y+2=5x-3y+1\frac{5}{6}$ ....(i)

$3x-2y+2=6x-y-4\frac{1}{2}$ ....(ii)

$\therefore \quad 2x-y=\frac{1}{6}$ ....(i); $3x+y=6\frac{1}{2}$ ......(ii)

Add (i) and (ii)

$\therefore \quad 5x=6\frac{2}{3} \;\therefore\; x=\frac{4}{3}=1\frac{1}{3} \;\therefore\; y=2\frac{1}{2}$. *Ans.*

12. Let x be the numerator, then $x+4$ is the denominator $\therefore$ the fraction $=\dfrac{x}{x+4}$

(*N.B.*—Two quantities are said to be *reciprocals* of each other when their product is equal to unity).

If 5 is taken from each, the new fraction

$$=\frac{x-5}{x+4-5} \text{ or } \frac{x-5}{x-1}$$

$$\therefore \quad \frac{x-1}{x-5}+\frac{4x}{x+4}=5$$

$$\therefore \quad x^2+3x-4+4x^2-20x=5x^2-5x-100.$$

whence $x=8 \;\therefore\; \dfrac{x}{x+4}=\frac{8}{12}$. *Ans.*

SOLUTIONS.

1893-94.

(Set at Bombay)

1. (i) $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$; $\frac{1}{2}\times\frac{1}{3}\times\frac{1}{6}=\frac{1}{36} \;\therefore\; \frac{2}{3}\div\frac{1}{36}=24$. *Ans.*

(ii) $5-\frac{9}{7}=\frac{26}{7}$; $4\times\frac{7}{26}=\frac{28}{26}$: $3+\frac{28}{26}=\frac{106}{26}$,

$2\times\frac{26}{106}=\frac{52}{106}$. *Ans.*

2.

		£	s.	d.	
		3	5	8	price of 1 cwt.
				9	
		29	11	0	price of 9 cwt.
3 qr.	$\frac{1}{12}$ of 9cwts.	2	9	3	" " 3 qrs.
21 lbs.	$\frac{1}{4}$ of 3 qrs.	0	12	$3\frac{3}{4}$	" " 21 lbs.
3 lbs.	$\frac{1}{7}$ of 21 lbs.	0	1	$9\frac{3}{28}$	" " 3 lbs.
		32	14	$3\frac{6}{7}$	price of 9 cwt. 3qrs. 24 lbs. *Ans.*

3. 40 men : 10 men $::$ $\frac{1}{6}$ of the work in one month $= \frac{1}{24}$
60 ,, : 10 ,, $::$ $\frac{1}{6}$,, ,, ,, ,, $= \frac{1}{36}$
80 ,, : 10 ,, $::$ $\frac{1}{6}$,, ,, ,, ,, $= \frac{1}{48}$
$\frac{1}{24} + \frac{1}{36} + \frac{1}{48} = \frac{13}{144}$ work done by 10 men, 10 women and 10 children in one month.

$\frac{13}{144} : \frac{1}{3} :: 1$ month $= \frac{48}{13} = 3\frac{9}{13}$ months. *Ans.*

4. £$90\frac{5}{8}$: £1,000 $::$ £3 $=$ £$\frac{960}{29}$, income in the 1st case
£$90\frac{5}{8}$: £1,000 $::$ £100 $=$ £$\frac{32000}{29}$, amount of stock held in the 1st case.

£100 : £$\frac{32000}{29}$ $::$ £$91\frac{1}{4}$ $=$ £$\frac{29200}{29}$, cash realised by selling the stock.

£$97\frac{1}{3}$: £$\frac{29200}{29}$ $::$ £$\frac{7}{2}$ $=$ £$\frac{1050}{29}$, income in the 2nd case.

$\frac{1050}{29} - \frac{960}{29} = \frac{90}{29} =$ £3 $2\frac{2}{29}$*s.* *Ans.*

5. $\frac{1}{12}$ of the cistern will be filled by *A* in 1 min.
$\frac{1}{14}$,, ,, ,, ,, ,, *B* ,, ,,
$\frac{1}{8}$,, ,, ,, emptied ,, *C* ,, ,,
$\frac{1}{12} + \frac{1}{14} - \frac{1}{8} = \frac{5}{168}$ of the cistern filled in 1 min.

$\therefore$ at the end of 7 min. $\frac{5}{168} \times 7 = \frac{5}{24}$ of the cistern will be filled $\therefore 1 - \frac{5}{24} = \frac{19}{24}$ of the cistern is left unfilled. *Ans.*

6. The faster gains on the slower $8'' + 7'' = 15''$ in 24 hrs.
,, ,, ,, ,, ,, $30'$ in $\frac{24 \times 30 \times 60}{15}$
or 2880 hrs. $=$ 120 days. $\therefore$ the faster will be $30'$ ahead of the other in the afternoon of the 23rd of August (counting 120 days from the afternoon of the 25th of April). In 24 hrs., $7''$ are lost by the first $\therefore$ in 2880 hrs., $\frac{2880}{24} \times \frac{7}{60}$ or $14'$ are lost. In 24 hrs., $8''$ are gained by the second $\therefore$ in 2880 hrs., $\frac{2880}{24} \times \frac{8}{60}$ or $16'$ are gained.

$\therefore$ the 1st will show 2 o'clock *minus* $14' =$ 1-46 P.M.
and the 2nd will show 2 o'clock *plus* $16' =$ 2-16 P.M. } *Ans.*

7. (i) $\frac{x}{1+x}\times\frac{(x+1)^2}{x+2}\times\frac{1-x^3}{x^2}\times\frac{x}{x+1}\times\frac{x}{x^2+x+1}$

$=\frac{x(1-x)}{2+x}$. *Ans.*

(ii) The expression

$=\frac{x^2-1+x^3+x+x^2-1-x^3+3}{x(x^2-1)}=\frac{2(x^2+1)}{x(x^2-1)}$. *Ans.*

8. $x=\frac{a+b}{2},y=\frac{b+c}{2}$

$\therefore\quad \frac{a}{x}+\frac{c}{y}=\frac{2a}{a+b}+\frac{2c}{b+c}=2\left(\frac{a}{a+b}+\frac{c}{b+c}\right)$

$=2\left(\frac{ac+bc+ab+ac}{(a+b)(b+c)}\right)$; but $ac=b^2$

$\therefore\quad =2\left\{\frac{b^2+bc+ab+ac}{(a+b)(b+c)}\right\}=\frac{2(b+c)(a+b)}{(a+b)(b+c)}$

$=2$, which was to be proved.

9. (*a*) $x^3+6x^2+11x+6$

$= x^3+3x^2+3x^2+9x+2x+6$

$= x^2(x+3)+3x(x+3)+2(x+3)$

$= (x+3)(x^2+3x+2)=(x+3)(x+2)(x+1)$.

$x^3+9x^2+27x+27$

$= (x^3+27)+(9x^2+27x)$

$= (x^3+3^3)+9x(x+3)$

$= (x+3)\{(x^2-3x+9)+9x)\}$

$= (x+3)(x^2+6x+9)=(x+3)^3$

$\therefore$ the H. C. F. of—

$(x+3)(x+2)(x+1)$ and $(x+3)^3=x+3$. *Ans.*

(*b*) The L. C. M. of—

xy, $x-y$, and $y(y^2-x^2)=xy(x+y)(x-y)$. *Ans.*

10. (*a*) The expression

$=\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)-\left(\frac{x}{y}-\frac{y}{x}\right)+\frac{1}{4}=\left(\frac{x}{y}-\frac{y}{x}\right)^2-\left(\frac{x}{y}-\frac{y}{x}\right)$

$\left(\frac{x}{y}-\frac{y}{x}-\frac{1}{2}\right)^2$ $\therefore$ the square root$=\frac{x}{y}-\frac{y}{x}-\frac{1}{2}$. *Ans.*

10. (*b*)

$$\begin{array}{rcr}
 & & 3x^2 \\
3\times x\times -4 & = & -12x \\
(-4)^2 & = & +16 \\
\hline
 & & 3x^2-12x+16 \\
 & & \\
3(x-4)^2 & = & 3x^2-24x+48 \\
3(x-4)\times\frac{2}{x} & = & +6-\frac{24}{x} \\
\left(\frac{2}{x}\right)^2 & = & +\frac{4}{x^2} \\
\hline
 & & 3x^2-24x+54-\frac{24}{x}+\frac{4}{x^2}
\end{array}$$

$$\begin{array}{l|l}
x^3-12x^2+54x-112+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3} & x-4+\frac{2}{x} \\
\underline{x^3} & \\
\quad -12x^2+54x-112 & \\
\quad \underline{-12x^2+48x-64} & \\
\qquad 6x-48+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3} & \\
\qquad \underline{6x-48+\frac{108}{x}-\frac{48}{x^2}+\frac{8}{x^3}} & \\
\end{array}$$

$$x-4+\frac{2}{x}. \quad \textit{Ans.}$$

11. (i) The first equation can be written thus—

$$\frac{x^2-1}{4x^2-1}-\frac{x}{2x-1}+\frac{1}{4}=0 \therefore 4(x^2-1)-4x(2x+1)+4x^2-1=0.$$

$$\therefore 4x^2-4-8x^2-4x+4x^2-1=0 \therefore -4x=5 \therefore x=-\frac{5}{4}. \quad \textit{Ans.}$$

(ii) $\frac{2}{x}-\frac{3}{2y}=\frac{41}{35}$, $\frac{5}{4x}+\frac{7}{2y}=-\frac{73}{70}$. Multiply the first by 7 and the second by 3 and add the two together.

$$\left.\begin{aligned}\therefore\ \frac{14}{x}-\frac{21}{2y}&=\frac{41}{5}\\ \frac{15}{4x}+\frac{21}{2y}&=-\frac{219}{70}\end{aligned}\right\}\therefore\ \frac{71}{4x}=\frac{355}{70}\therefore x=\frac{71\times 70}{4\times 355}=\frac{7}{2}.$$

Hence $y=-\frac{5}{2}$. *Ans.*

12. Let x be the no. of days he worked, then he was idle for $(30-x)$ days.

$\therefore$ 1 day : x :: $\frac{5}{2}s.=\frac{5x}{2}s.$ wages of x days.

days 1 : $(30-x)$ :: $1s.=(30-x)s.$ he does not receive.

And £2 7*s.* = 47*s.*

$\therefore \frac{5x}{2}-(30-x)=47\therefore 5x-60+2x=94\therefore 7x=154\therefore x=22$

$\therefore$ he worked for 22 days. *Ans.*

Euclid.

1. Euclid I. 11.
2. Euclid I. 37.

(Fig. 51.) Let ABC be a tr. Through A draw AD meeting BC in D. Through B, C draw BE and CF prll. to AD (I. 31), meeting CA, BA produced in E and F respectively. Join EF, ED, FD.

Tr. EFB=tr. ECB (I. 37): take away from each of these equals the common tr. EAB $\therefore$ tr. EAF= tr. ABC (ax. 3); tr. AED = tr. ABD (I. 37); tr. ADF = tr. ADC (I. 37) $\therefore$ $DEAF$= tr. ABC; but tr. EAF=tr. ABC. $\therefore$ $DEAF$+ tr. EAF=2 tr. ABC $\therefore$ tr. DEF=2 tr. ABC.

3. Euclid II. 4.

Area of tr. $ABC=\frac{1}{2}$ rect. $AC\cdot CB$. The square on the line made up of AC and $CB=AC^2+CB^2+2$ rect. $AC\cdot CB$ (II. 4) $=AB^2$ (I. 47)$+2$ rect. $AC\cdot CB$.

Now 2 rect. $AC\cdot CB=4\ (\frac{1}{2}AC\cdot CB)$=four times the area of the triangle $\therefore$ AB^2+4 times the area of the tr.=the square on the line made up of AC and CB.

4. Let $ABCD$ be a plm. inscribed in a cir. $ABCD$; ang. BAD=ang. BCD (I. 34).

Angs. $BAD+BCD=2$ rt. angs. (III. 22) $\therefore$ each of the angs. BAD, BCD is a rt. ang. Similarly, each of the angls. ABC, ABD is a rt. ang. $\therefore$ plm $ABCD=$ a rectangle.

5. Euclid III. 26.

(Fig. 52.) Join EO, EB. Produce FE to meet BA produced in H.

FOH is a rt. angled tr. $\therefore$ ang. $FOH=$ angs. $FHO+HFO$ (I. 32); but $EO=EF$ (hyp.) $\therefore$ ang. $EFO=$ ang. EOF (I. 5) $\therefore$ ang. $EHO=$ ang. EOH.

Again, ang. $GEB=$ angs. $EHB+EBH$ (I. 32) $=$ angs. $EOH+ABE$; but ang. $EOH=2$ ang. ABE (III. 20.) $\therefore$ ang. $GEB=2$ ang. $ABE+$ ang. $ABE=3$ ang. ABE $\therefore$ the arc $GB=3$ the arc AE (III. 26).

6. Euclid III. 32.

(Fig. 53.) Ang. $APB=\frac{1}{2}$ ang. ACB (III. 20); but ACB is a right angle (hyp.) $\therefore$ ang. APB is half a right angle.

Again, angs. CAB, $CBA=$ a right angle (I. 32); but $CB=CA$ $\therefore$ ang. $CAB=$ ang. CBA. $\therefore$ ang. $CAB=\frac{1}{2}$ a right angle; but ang. $APN=$ half a right angle $\therefore$ ang. $CAB=$ ang. APN $\therefore BA$ touches the circle ANP (III. 32 converse).

7. Euclid IV. 13.

1894.

TUESDAY, 13TH NOVEMBER.

Arithmetic and Algebra.

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1. When the number representing the year is a multiple of four, it is a leap-year, consisting of 366 days, except when this number is a multiple of 100, in which case it is an ordinary year, consisting of 365 days, but when the number is a multiple of 400, it is 5

again a leap-year ; on this supposition, calculate the number of days from 1st January 1495 to 31st December 1894, both days inclusive.

2. A school of boys and girls consists of 453 children; the number representing the boys is $\cdot\dot{5}\dot{2}$ of the number of the girls. How many boys were there ? 5

3. Two-thirds of a certain number of poor persons received 1*s*. 6*d*. each, and the rest 2*s*. 6*d*. each ; the whole sum spent being £2 15*s*., how many poor persons were there ? 5

4. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days, find how long 13 men, 14 children, and 15 women will take to do it. 7

5. *A* sells a house to *B* for Rs. 4,860, thereby losing 19 per cent. ; *B* sells it to *C* at a price which would have given *A* 17 per cent. profit. Find *B*'s gain. 7

6. The compound interest on one rupee is one-quarter of a rupee at the end of three years ; find the rate per cent. per annum, correct to two places of decimals ; and calculate exactly the compound interest at the end of 9 years. 12

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7. Divide—

(i) $\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)^2$ by $\frac{y}{x}-\frac{x}{y}$. 4

(ii) $a_1 a_2 x^2+b_1 b_2 y^2+c_1 c_2 z^2+(a_1 b_2+a_2 b_1)xy$
$+(a_1 c_2+a_2 c_1)xz+(b_1 c_2+b_2 c_1)yz$ by $a_2 x+b_2 y+c_2 z$. 3

8. Find the H. C. F. of— 7

$$6x^4-2x^3+9x^2+9x-4 \text{ and } 9x^4+80x^2-9\ ;$$

and find such a value of x as will make both these expressions vanish.

9. Resolve into elementary factors the following expressions:— 9

(i) x^4+324;

(ii) $8x^3-5x+3$; and

(iii) $56x^2+5xy-99y^2$.

10. Prove that 6

$$(a-b)^3+(b-c)^3+(c-a)^3=3(a-b)(b-c)(c-a)$$

and hence or otherwise shew that— 7

$$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{a^3(b-c)^3+b^3(c-a)^3+c^3(a-b)^3}=\frac{(a+b)(b+c)(c+a)}{abc}.$$

11. Find the square root of— 7

$$a^3\left(\frac{a^3}{9}-\frac{10}{3b^3}\right)+b^3\left(2+\frac{9b^3}{a^6}\right)-\frac{30}{a^3}+\frac{25}{b^6}.$$

12. Solve:—

(i) $$\frac{2x+11}{x+5}-\frac{9x-9}{3x-4}=\frac{4x+13}{x+3}-\frac{15x-47}{3x-10}.$$ 5

(ii) $$\left.\begin{aligned} &b(a+b)x=a(a-b)y \\ &\frac{a-bx}{a^2}-\frac{b-ay}{b^2}=\frac{x}{a}+\frac{y}{b} \end{aligned}\right\}.$$ 6

13. A father's age was triple that of his son 5 years ago, while 5 years hence the father will be twice as old as his son; determine their respective ages. 5

1894.

WEDNESDAY 14TH NOVEMBER.

Euclid.

1. Equal triangles on the same base, and on the same side of it, are between the same parallels. 6

APB, *ADQ* are two straight lines, such that the triangles *PAQ* and *BAD* are equal. If the parallelogram *ABCD* be completed and *BQ* joined, cutting *CD* in *R*, shew that *CR* is equal to *AP*. 10

2. In any right-angled triangle, the square which is described on the side subtending the right angle, is equal to the squares described on the sides which contain the right angle. 9

3. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square on half the line. 7

4. In equal circles, the angles which stand on equal arcs are equal to one another, whether they be at the centres or circumferences. 6

P is a point in APB, an arc of a circle, such that the arc AP is twice as great as the arc PB. The tangent at P meets the chord AB produced in R, and AQ perpendicular to AB in Q. Prove that QR is bisected in P. 13

5. Given the base and the vertical angle, find the triangle whose area is a maximum. What is the locus of the vertices of such triangles? 12

6. In a circle, the angle in a semi-circle is a right angle; but the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle. 7

Circles are described on the sides of a quadrilateral as diameters; if three of them pass through a common point, shew that the fourth also passes through the same point. 11

7. Describe an equilateral and equiangular pentagon about a given circle. 10

8. Inscribe a regular hexagon in a given equilateral ·ngle. 9

SOLUTIONS.

Arithmetic and Algebra.

1. From the 1st Jan. 1495 to 31st Dec. 1894 there are 400 years; in these 400 years, the years 1500, 1700, and 1800 are not leap years, because the numbers must be a multiple of 400, $\therefore$ there are altogether $100-3=97$ leap years in 400 years.

$\therefore\quad 400-97=303$ are ordinary years

$\therefore\quad 97\times366+303\times365=35502+110595$
$=146097$ number of days. *Ans.*

2. Let the no. of girls be represented by $1 \therefore \frac{52}{99}$ represent no. of boys $\therefore\ 1+\frac{52}{99}=\frac{151}{99}$, total number, which is equal to 453 children.

$\frac{151}{99} : \frac{52}{99} :: 453=52\times3=156$ boys. *Ans.*

3. Out of 3 persons, 2 received 1*s.* 6*d.* each and one received 2*s.* 6*d.* $\therefore$ all the three received 3*s.*+2*s.* 6*d.*= 5*s.* 6*d.*; the total no. spent is £2 15*s.*=55*s.*

$5\frac{1}{2}s. : 55s. :: 3 \text{ persons}=\dfrac{55\times3\times2}{11}=30$ poor persons. *Ans.*

4. 3 men and 5 women do $\frac{1}{8}$ of the work in 1 day.

$\therefore$ 9 men and 15 women do $\frac{3}{8}$ „ „ „ (i)

Again, 2 men and 7 children do $\frac{1}{12}$ of the work in 1 day.

$\therefore$ 4 men and 14 children do $\frac{1}{6}$ „ „ (ii)

$\therefore$ adding (i) and (ii) 13 men, 15 women and 14 children do $\frac{3}{8}+\frac{1}{6}=\frac{13}{24}$ of the work in one day.

$\therefore$ they will finish the work in $\frac{24}{13}$, or $1\frac{11}{13}$ days. *Ans.*

5. £100 − £19 = £81, selling price.

£81 : £4860 :: £100 = £6000 = *A*'s cost price.

£100 : £6000 :: £117 = £7020 = *B*'s selling price or *C*'s cost price.

$\therefore$ 7020 − 4860 = £2160, *B*'s gain *Ans.*

6. The compound interest on one rupee is $\frac{1}{4}$ of a rupee at the end of 3 years.

$\therefore 1\frac{1}{4}$ is the amount of Re. 1 at the end of 3 years.

$\therefore \sqrt[3]{\frac{5}{4}}$ or $\sqrt[3]{1\cdot25}$ is the amount of Re. 1 at the end of one year.

3×100^2	$= 30000$	$1\cdot250000000000$	$\lvert 1\cdot0772$
$3 \times 100 \times 7$	$= 2100$	1	
7^2	$= 49$	250000	
	32149		
3×1070^2	$= 3434700$		
$3 \times 1070 \times 7$	$= 22470$	225043	
7^2	$= 49$	24957000	
	3457219		
3×10770^2	$= 347978700$	24200533	
$3 \times 10770 \times 2$	$= 64620$	756467000	
2^2	$= 4$		
	348043324	696086648	

$\therefore 1\cdot0772$ is the amount of Re. 1 for 1 year.

$\therefore$ Rs. $1\cdot0772 -$ Re. $1 =$ Re. $\cdot0772$ interest on Re. 1.

$\therefore$ the rate per cent. $= \cdot0772 \times 100 =$ Rs. $7\cdot72\ldots$ *Ans.*

As £$1\cdot25$ is the amount of Re. 1 at the end of 3 years,

$\therefore (1\cdot25)^3$ is the amount of Re. 1 at the end of 9 years.

$\therefore$ £$1\cdot953125 -$ Re. 1, *i.e.*, $\cdot953125$ is the interest on Re. 1.

$\therefore \cdot953125 \times 16 = 15\cdot25$ as. $= 15 + \cdot25 \times 12$ pies $= 15$ as. 3 pies. *Ans.*

---

7. (i) $\frac{x^2}{y^2}+\frac{y^2}{x^2}-2=\frac{y^2}{x^2}-2+\frac{x^2}{y^2}$

$$=\left(\frac{y}{x}\right)^2-2\left(\frac{y}{x}\right)\left(\frac{x}{y}\right)+\left(\frac{x}{y}\right)^2=\left(\frac{y}{x}-\frac{x}{y}\right)^2$$

$$\therefore \left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)^2=\left(\frac{y}{x}-\frac{x}{y}\right)^4$$

$$\left(\frac{y}{x}-\frac{x}{y}\right)^4\div\left(\frac{y}{x}-\frac{x}{y}\right)=\left(\frac{y}{x}-\frac{x}{y}\right)^3. \quad \textit{Ans.}$$

(ii) $(a_1a_2x^2+a_1b_2xy+a_1c_2xz)+(b_1b_2y^2+b_1a_2xy+b_1c_2yz)$
$+(c_1c_2z^2+c_1a_2xz+b_2c_1yz)$

$=a_1x(a_2x+b_2y+c_2z)+b_1y(b_2y+a_2x+c_2z)$
$+c_1z(c_2z+a_2x+b_2y)$

$=(a_1x+b_1y+c_1z)(a_2x+b_2y+c_2z) \therefore a_1x+b_1y+c_1z.$ *Ans.*

8. $6x^4-2x^3+9x^2+9x-4$

$= 6x^4-2x^3+9x^2-3x+12x-4$

$= 2x^3(3x-1)+3x(3x-1)+4(3x-1)$

$= (2x^3+3x+4)(3x-1)$

$9x^4+80x^2-9=9x^4+81x^2-x^2-9$

$= 9x^2(x^2+9)-(x^2+9)=(9x^2-1)(x^2+9)$

$= (3x+1)(3x-1)(x^2+9)$

$\therefore$ the H. C. F. of the given expressions $=3x-1$. *Ans.*

In order that both the expressions may vanish $3x-1$ must be equal to 0, *i.e*, $x=\frac{1}{3}$. *Ans.*

9. (i) $x^4+324=x^4+36x^2+324-36x^2$

$= (x^2+18)^2-(6x)^2=(x^2+6x+18)(x^2-6x+18)$. *Ans.*

(ii) $8x^3-5x+3=8x^3+8x^2-8x^2-8x+3x+3$

$=8x^2(x+1)-8x(x+1)+3(x+1)$

$=(8x^2-8x+3)(x+1)$. *Ans.*

(iii) $56x^2+5xy-99y^2$

$=(56x^2-72xy)+(77xy-99y^2)$

$=8x(7x-9y)+11y(7x-9y)=(8x+11y)(7x-9y)$. *Ans.*

10. (i) The expression $(a-b)^3+(b-c)^3+(c-a)^3-3(a-b)(b-c)(c-a)$ is of the type $x^3+y^3+z^3-3xyz$ and therefore it has a factor $(a-b)+(b-c)+(c-a)$ and is therefore $=0$; hence $(a-b)^3+(b-c)^3+(c-a)^3=3(a-b)(b-c)(c-a)$.

(ii) Applying the result obtained above, we have

$(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

$=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$ and

$a^3(b-c)^3+b^3(c-a)^3+c^3(a-b)^3$

$=3abc(b-c)(c-a)(a-b)$

Hence the expression

$$=\frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3abc(b-c)(c-a)(a-b)}=\frac{(a+b)(b+c)(c+a)}{abc},$$

which was to be proved.

11.

$$\frac{a^6}{9}-\frac{10a^3}{3b^3}+2b^3-\frac{30}{a^3}+\frac{9b^6}{a^6}+\frac{25}{b^6}\ \Big|\ \frac{a^3}{3}-\frac{5}{b^3}+\frac{3b^3}{a^3}$$

$$\frac{a^6}{9}$$

$$\frac{2a^3}{3}-\frac{5}{b^3}\quad\Big|\quad -\frac{10a^3}{3b^3}+\frac{25}{b^6}$$

$$-\frac{10a^3}{3b^3}+\frac{25}{b^6}$$

$$\frac{2a^3}{3}-\frac{10}{b^3}+\frac{3b^3}{a^3}\quad\Big|\quad +2b^3-\frac{30}{a^3}+\frac{9b^6}{a^6}$$

$$+2b^3-\frac{30}{a^3}+\frac{9b^6}{a^6}$$

$$\frac{a^3}{3}-\frac{5}{b^3}+\frac{3b^3}{a^3}.\qquad \textit{Ans.}$$

12. (i) $2+\dfrac{1}{x+5}-3-\dfrac{3}{3x-4}=4+\dfrac{1}{x+3}-5-\dfrac{3}{3x-10}$

$$\therefore\ \frac{1}{x+5}-\frac{3}{3x-4}=\frac{1}{x+3}-\frac{3}{3x-10}$$

$$\frac{3x-4-3x-15}{(x+5)(3x-4)}=\frac{3x-10-3x-9}{(x+3)(3x-10)}$$

$$\therefore\ \frac{-19}{(x+5)(3x-4)}=\frac{-19}{(x+3)(3x-10)}$$

$$\therefore\ (x+5)(3x-4)=(x+3)(3x-10)$$

$$\therefore\ 3x^2+15x-4x-20=3x^2+9x-10x-30$$

$$\therefore\ 12x=-10\ \therefore x=-\frac{5}{6}.\quad \textit{Ans.}$$

(ii) The second equation, when simplified, becomes $ab^2-b^3x-a^2b+a^3y=ab^2x+a^2by$ or $a^3y-a^2by-b^3x-ab^2x=a^2b-ab^2$ or $a^2(a-b)y-b^2(a+b)x=ab(a-b)$.

Multiplying the first equation by b and adding it to the second we have

$$a^2(a-b)y-b^2(a+b)x=ab(a-b)$$

$$-ab(a-b)y+b^2(a+b)x=0$$

$$a(a-b)^2y=ab(a-b)$$

$\therefore y=\dfrac{b}{a-b}$. Substitute the value of y in the first.

$\therefore b(a+b)x=\dfrac{ab(a-b)}{(a-b)}\ \therefore (a+b)x=a\ \therefore x=\dfrac{a}{a+b}$. *Ans.*

13. Let x be the father's age, and y the age of the son.
5 years ago, their respective ages were $x-5$ and $y-5$
$\therefore x-5=3(y-5)$ ....(i)
5 years hence, their respective ages will be $x+5$ and $y+5$ $\therefore x+5=2(y+5)$
$x-3y=-10$ .... (i)
$x-2y=\ 5$ ....(ii)

Subtracting (ii) from (i) we have $y=15\ \therefore x=35$
$\therefore$ the father's age is 35 and the son's age 15. *Ans.*

Euclid.

1. Euclid I. 39.

(Fig. 54). Join DP. Then $\because$ tr. PAQ=tr. BAD (hyp.) take out the common tr. APD $\therefore$ tr. DPQ=tr. DPB. $\therefore$ DP is prll. to QB, *i.e.*, RB (I. 39) and DR is prll. to PB (hyp.) $\therefore$ $DR=PB$ (I. 34) and $\because$ $ABCD$ is a plm. $\therefore$ $DC=AB$ (I. 34) $\therefore RC=AP$.

2. Euclid I. 47.
3. Euclid II. 5.
4. Euclid III. 27.

(Fig. 55.) The arc $AP=2BP$ $\therefore$ ang. ABP = 2 ang. PAB. But ang. APQ = ang. PBA (III. 32) and ang. RPB = ang. BAP (III. 32), $\therefore$ ang. PBA = 2 ang. BPR. But ang. PBA = angs. BPR and BRP (I. 32) $\therefore$ angs. BPR and BRP = $2BPR$ $\therefore$ ang. PRB = ang. RBP = ang. PAB $\therefore$ PR = PA (I. 6.)

Again, $\because$ ang. RAQ = a rt. angle = angs. $AQR+ARQ$, *i.e.*, angs. $RAP+PAQ$. But ang. $RAP=$ ang. ARQ $\therefore$ ang. $AQP=$ ang. QAP $\therefore$ $AP=PQ=PR$, *i.e.*, QR is bisected at P.

(Fig. 56). By III. 21. Cor. All triangles drawn on the same base, and with equal vertical angles, have their vertices on an arc of a circle, of which the given base is the chord.

$\therefore$ if the base and vertical ang. of a tr. are given, then the vertex must move on the segment of a circle described on the base and containing the given angle. Again, the triangle of greatest area is that which has the greatest altitude. Now if BC be the given base the greatest altitude is the straight line AD which bisects BC in D at right angles. If not, take any other point E on the arc of the segment. Join DE and draw EF perp. to BC. Then $\because$ DA passes through the centre (III. 1) $\therefore$ $DA>ED$ (III. 7).

Again, $\because$ ang. EFD is a rt. ang. (constr) $\therefore$ ED, the hypotenuse, is the greatest side in the right angled tr. $\therefore$ $ED>EF$. (I. 19).

The locus of the vertices of triangles drawn on the same base with equal vertical angles is an arc of a circle.

6. Euclid III. 31.

(Fig. 57). Let $ABCD$ be the quadrilateral. Let cirs. be described, having as diameters AB, BC, CD, passing through the common point P. Then the fourth cir. described on AD as diameter will pass through the same point. Join AP, BP, CP, PD.

Now, ang. APB in a semi-circle is a rt. ang. (III. 31). Similarly BPC is a rt. ang. and CPD is a rt. ang. $\therefore$ AC and BD are straight lines (I. 14) $\therefore$ ang. APD is a rt. ang. (I. 15 cor. I.)

$\therefore$ APD must be the angle in the semi-circle of the fourth cir. hence the fourth cir. passes through the same point P.

7. Euclid IV. 12.

8. Euclid IV. 15.

1895.

TUESDAY, 12TH NOVEMBER.

Arithmetic and Algebra.

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1. When a fraction is reduced to its lowest terms, find the form of the denominator so that the fraction may be expressed as a non-recurring decimal. 6

Reduce $\frac{1}{81}$ to decimals. 2

2. A field can be reaped by 10 women in 4 days, or by 6 boys in 10 days, or by 2 men in 12 days. One man, three boys, and three women are employed; what is the total expense, if the wages of a man, a woman, and a boy are 8 annas, 5 annas, and 3 annas respectively? 7

3. The total fare for a journey of 504 miles, partly by main line and partly by branch line, was 17 rupees, 11 annas, and 6 pies. The rates per mile being 6 pies on the main line and 8 pies on the branch line, what distance was travelled on the branch line? 7

4. A certain sum amounts to 186 rupees, 9 annas, and $5\frac{25}{343}$ pies in three years at compound interest, and the amount at the end of the third year is to that at the end of fourth year as $1 : 1\cdot\dot{1}4285\dot{7}$, find the original sum and the rate of interest. 10

5. A person sells £1,600 Russian stock at $75\frac{7}{8}$, and invests the proceeds in Railway stock at 120. The brokerage for selling Russian stocks is $\frac{1}{8}$ per cent. stock, and the expenses of buying Railway stock are one per cent. on the actual value. What amount of stock did he buy? 10

---

6. If a number is equal to the sum of two perfect squares, shew by an algebraical relation that the square 6

of the number is itself the sum of two other perfect squares.

Express $(34)^2$ as the sum of two perfect squares. 3

7. Find an expression containing no higher power of x than the first, which added to 6

$$x^4+6x^3+13x^2+6x+1$$

will make it a complete square.

8. Find the cube root of 7

$$8x^9-12x^8+6x^7-37x^6+36x^5-9x^4+54x^3-27x^2-27.$$

9. (*a*) Find the Least Common Multiple of 8

$$4x^2-6yz-(9y^2+z^2),\ 9y^2+4xz-(4x^2+z^2),\ \text{and}\ z^2-12xy-(4x^2+9y^2).$$

(*b*) Simplify

$$\frac{3x^2-(4a+2b)x+a^2+2ab}{x^3-(2a+b)x^2+(a^2+2ab)x-a^2b}.$$

10. Solve :—

(i) $\dfrac{x^3-4x+4}{x-1}+\dfrac{x^3-3x-1}{x-2}=2\dfrac{x^2-5x+5}{x-3}.$ 7

(ii) $(a+b)x+by=ax+(a+b)y=a^3-b^3.$ 7

11. A certain number of two digits is equal to seven times the sum of the digits. If the digit in the units' place be decreased by 2 and that in the tens' place by 1, and the number thus formed be divided by the sum of the digits, the quotient is 10. Find the number. 8

WEDNESDAY, 13TH NOVEMBER.

Euclid.

1. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are themselves equal and parallel. 4

If two sides of a quadrilateral are parallel and the remaining two sides equal but not parallel, shew that the opposite angles are together equal to two right angles; also that the diagonals are equal. 12

2. Describe a parallelogram which shall be equal 7
to a given triangle and have one of its angles equal to a given rectilineal angle.

3. If a straight line be divided into any two parts 9
the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part together with the square on the other part.

4. In a triangle whose vertical angle is a right 7
angle a straight line is drawn from the vertex perpendicular to the base; shew that the square on this perpendicular is equal to the rectangle contained by the segments of the base.

5. Draw a tangent to a circle from a given point 8
either on or without the circumference.

If two tangents be drawn to a given circle from a 8
fixed point without it and a third line be drawn between the point and the centre of the circle touching the circle, then the perimeter of the triangle formed by the three tangents will be constant.

6. Prove that angles in the same segment of a 6
circle are equal.

Two circles intersect at A and B, and through A 10
any two straight lines PAQ, XAY are drawn terminated by the circumferences; shew that the arcs PX and QY subtend equal angles at B.

7. Describe a circle about a given triangle. 7

8. The diagonals of a quadrilateral $ABCD$ intersect 13
at O; shew that the centres of the circles circumscribed about the four triangles AOB, BOC, COD, DOA are at the angular points of a parallelogram.

9. Inscribe a regular pentagon in a given circle. 9

SOLUTIONS.

1. When a fraction is reduced to its lowest terms the denominator must be solely composed of powers of 2 and 5 either alone or multiplied together so that the fraction can be converted into a terminating or non-recurring decimal :— "For, in reducing a vulgar fraction to a decimal, we affix ciphers to the numerator, *i.e.*, we multiply it by powers of 10, and after dividing the result by the denominator, mark off the proper number of decimal places in the quotient. And as, the fraction being in its lowest terms, the numerator and denominator have no common factor, the division will terminate, and we shall have a terminating or non-recurring decimal, only when the denominator will divide the power of 10 exactly, *i.e.*, when the denominator consists solely of the factors 2 and 5, which are the only factors of which 10 and its powers are composed."

$$\tfrac{1}{81} = \cdot\dot{0}1234567\dot{9}. \quad \textit{Ans.}$$

2. One woman can reap $\frac{1}{40}$ of the field in 1 day ;

„ boy „ „ $\frac{1}{60}$ „ „ „

„ man „ „ $\frac{1}{24}$ „ „ „

∴ 1 man + 3 boys + 3 women reap $\frac{1}{24}+\frac{3}{60}+\frac{3}{40}=\frac{1}{6}$ of the field in one day, *i.e.*, they all will take 6 days to reap the field.

One man gets 8 annas for a day, so the wages for 6 days are equal to 48 annas. One woman gets 5 annas daily, so 3 will get 15 annas and ∴ their wages for 6 days = 90 annas. One boy gets 3 annas daily ∴ the wages of 3 boys = 9 annas ∴ the wages for 6 days = 54 annas. *Ans.*

3. Suppose we travel the whole distance by the main line. The rates per mile being 6 pies on the main line 504 × 6 pies = 3,024 pies would be the fare. But the total fare is Rs. 17 11as. 6ps. actually or 3,402 pies ∴ 3,402 — ,024 = 378 pies more will be required. This increase in the are is due to our supposition that the whole distance is tra-

velled by the main line, whereas part of the distance is travelled by the branch line also. Travelling one mile by the branch line instead of the main line, $(8-6)=2$ pies more will be required.

$\therefore$ 2 pies : 378 pies :: 1 mile $=189$ miles travelled by the branch line. *Ans.*

4. $1\dot{1}4285\dot{7}=1\frac{1}{7}$.

Re. $1\frac{1}{7}$ is the amount of Re. 1 for one year $\therefore$ $\frac{1}{7}$ of a rupee is the interest on Re. 1 for one year, $\therefore$ the interest on Rs. $100=\frac{100}{7}=14\frac{2}{7}$ rate %. *Ans.*

Now the question reduces itself to this:—What sum amounts to Rs. 186 9 as. $5\frac{205}{343}$ ps. in 3 years at $14\frac{2}{7}$ %. The amount of Re. 1 at the end of 1 year $=$ Rs. $1\frac{1}{7}$ $\therefore$ the amount at the end of 3 years $=$ Rs. $(\frac{8}{7})^3$ and Rs. 186-9-$5\frac{205}{343}$ pies $=$ Rs. $\frac{64000}{343}$

$$\therefore (\tfrac{8}{7})^3 : \tfrac{64000}{343} :: \text{Re. } 1 \text{ sum} = \frac{64000}{343} \times \frac{7\times7\times7}{8\times8\times8} = \text{Rs. } 125.$$

Ans.

5. $75\frac{7}{8}-\frac{1}{8}=$ Rs. $75\frac{3}{4}$ obtained by selling £100 Russian stock.

£100 : £1600 :: £$75\frac{3}{4}=$ £1212 sum realized by selling £1600 stock.

The expense of buying Railway stock is one per cent. on the actual price, $\therefore$ £100 : £120 :: £1 $=$ £$\frac{6}{5}$ more is required for the price of £100 Railway stock, *i.e.*, $120+\frac{6}{5}$ $=$ £$\frac{606}{5}$ in all.

£$\frac{606}{5}$: £1212 :: £100 Railway stock $=$ £1000 stock. *Ans.*

---

6. Let x^2+y^2 be the number which is the sum of two perfect squares, the square of the number, *i. e.*, $(x^2+y^2)^2$ $=x^4+2x^2y^2+y^4=x^4-2x^2y^2+y^4+4x^2y^2=(x^2-y^2)^2+(2xy)^2$,

which is a sum of two other perfect squares.

34 is a sum of two perfect squares $25+9$ or 5^2+3^2
$\therefore$ applying the above principle, we have—

$$(34)^2=(5^2-3^2)^2+(2\times5\times3)^2=(16)^2+(30)^2,$$

which is a sum of two perfect squares.

7. Extract the square root of the expression ; thus,

$$\begin{array}{l|l|l}
 & x^4+6x^3+13x^2+6x+1 & x^2+3x+2 \\
 & x^4 & \\
\cline{2-2}
2x^2+3x & \quad 6x^3+13x^2 & \\
 & \quad 6x^3+\ \ 9x^2 & \\
\cline{2-2}
2x^2+6x+2 & \qquad 4x^2+\ \ 6x+1 & \\
 & \qquad 4x^2+12x+4 & \\
\cline{2-2}
 & \qquad\quad -6x-3 & \\
\end{array}$$

The remainder is $-6x-3$, $\therefore$ in order that the expression may be a perfect square, we must add to it $6x+3$. *Ans.*

$$
\begin{array}{lr|l}
8. \quad 3(2x^3)^2 = & 12x^6 & \\
3\times 2x^3 \times -x^2 = & -6x^5 & 8x^9-12x^8+6x^7-37x^6+36x^5-9x^4+54x^3-27x^2-27\ \big(2x^3-x^2-3. \\
(-x^2)^2 = & +x^4 & 8x^9 \\
\hline
 & 12x^6-6x^5+x^4 & -12x^8+6x^7-37x^6 \\
3(2x^3-x^2)^2 = 12x^6-12x^5+3x^4 & & -12x^8+6x^7-\quad x^6 \\
\hline
3(2x^3-x^2)\times -3 = & -18x^3+9x^2 & -36x^6+36x^5-9x^4+54x^3-27x^2-27 \\
(-3)^2 = & +9 & -36x^6+36x^5-9x^4+54x^3-27x^2-27 \\
\hline
 & 12x^6-12x^5+3x^4-18x^3+9x^2+9 & 2x^3-x^2-3. \quad Ans
\end{array}
$$

9. (*a*) $4x^2-6yz-(9y^2+z^2) = 4x^2-(9y^2+6yz+z^2)$

$= 4x^2-(3y+z)^2=(2x+3y+z)(2x-3y-z)$

$9y^2+4xz-(4x^2+z^2)=9y^2-(4x^2-4xz+z^2)$

$= 9y^2-(2x-z)^2=(3y+2x-z)(3y-2x+z)$

$z^2-12xy-(4x^2+9y^2)=z^2-(4x^2+12xy+9y^2)$

$= z^2-(2x+3y)^2=(z+2x+3y)(z-2x-3y)$

$\therefore$ the L. C. M $=(2x+3y+z)(2x-3y-z)(2x+3y-z)$. *Ans.*

(*b*) The numerator

$= 3x^2-4ax-2bx+a^2+2ab$

$= 3x^2-ax-3ax-2bx+a^2+2ab$

$= 3x^2-ax-2bx-3ax+a^2+2ab$

$= x(3x-a-2b)-a(3x-a-2b)$

$= (x-a)(3x-a-2b)$

The denominator

$= x^3-2ax^2-bx^2+a^2x+2abx-a^2b$

$= x^3-bx^2-2ax^2+2abx+a^2x-a^2b$

$= x^2(x-b)-2ax(x-b)+a^2(x-b)$

$= (x^2-2ax+a^2)(x-b)=(x-a)^2(x-b)$

$\therefore$ the fraction $=\dfrac{(x-a)(3x-a-2b)}{(x-a)^2(x-b)}=\dfrac{3x-a-2b}{(x-a)(x-b)}$. *Ans.*

10 (i). The first equation may be written thus:—

$$x-3+\frac{1}{x-1}+x-1-\frac{3}{x-2}=2(x-2)-\frac{2}{x-3}$$

$$\therefore \frac{1}{x-1}-\frac{3}{x-2}=-\frac{2}{x-3}$$

$$\therefore \frac{1}{x-1}+\frac{2}{x-3}=\frac{3}{x-2}$$

$$\therefore \frac{1}{x-1}+\frac{2}{x-3}=\frac{1}{x-2}+\frac{2}{x-2}$$

$$\therefore \frac{1}{x-1}-\frac{1}{x-2}=\frac{2}{x-2}-\frac{2}{x-3}$$

$$\therefore \frac{(x-2)-(x-1)}{(x-1)(x-2)}=\frac{2(x-3)-2(x-2)}{(x-2)(x-3)}$$

$$\therefore \frac{-1}{(x-1)(x-2)}=\frac{-2}{(x-2)(x-3)}$$

$\therefore 2(x-1)=x-3$ $\therefore 2x-x=2-3$ $\therefore x=-1$. *Ans.*

(ii) $ax+bx+by=ax+ay+by$ $\therefore$ $bx=ay$ $\therefore$ $x=\frac{ay}{b}$.

Putting the value of x in the equation—

$(a+b)x+by=a^3-b^3$, we get $\frac{a(a+b)y}{b}+by=a^3-b^3$

$\therefore y\left\{\frac{a(a+b)}{b}+b\right\}=a^3-b^3$ $\therefore y\left\{\frac{a^2+ab+b^2}{b}\right\}$

$=(a-b)(a^2+ab+b^2)$ $\therefore$ $\frac{y}{b}=a-b$ $\therefore$ $y=b(a-b)$. *Ans.*

and $x=\frac{ay}{b}=\frac{ab(a-b)}{b}=a(a-b)$. *Ans.*

11. Let x be the digit in the tens' place, and y the digit in the units' place $\therefore$ the number $=10x+y$

$\therefore$ $10x+y=7(x+y)$ $\therefore$ $3x-6y=0$.

$\therefore$ $x-2y=0$.. ... (i)

If the digit in the units' place be decreased by 2, *i.e.*, if it is $y-2$, and the digit in the tens' place be decreased by 1, *i.e.*, if it is $x-1$, the number formed is $10(x-1)+(y-2)$

$\therefore \frac{10(x-1)+y-2}{x-1+y-2}=10$ $\therefore$ $\frac{10x+y-12}{x+y-3}=10$

$\therefore$ $10x+y-12=10x+10y-30$ $\therefore y=2$. And putting the value of y in $x-2y=0$, we have $x=4$ $\therefore$ the number $=42$ *Ans.*

Euclid.

1. Euclid I. 33.

(Fig. 58). Let $ABCD$ be the quadrilateral, of which the side AB is prll. to CD, and $AD=BC$. Through B draw BE prll. to AD (I. 31). $AD=BE$ (I. 34)$=BC$ (hyp.) $\therefore$ ang. $BCD=$ ang. BEC (I. 5); but ang. BEC is supplementary of ang. BED and ang. $BED=$ ang. BAD (I. 34) $\therefore$ ang. BCD is supplementary of ang. BAD, *i.e.*, angs. BCD and BAD are together$=2$ rt. angs.

Again, since the 4 angs. of the figure $ABCD$ are together equal to 4 rt. angs., the angs. ABC and ADC are supplementary. Join AC, BD. Now, ang. DEB is supplementary of ang. ADE and ang. ABC is also supplementary of ang. ADE $\therefore$ angs. DEB and ABC are equal.

Hence, in trs. DEB, ABC, $DE=AB$ (I. 34) $EB=BC$, and ang. $DEB=$ ang. ABC $\because$ trs. are equal in all respects (I. 4) $\therefore$ $AC=BD$.

2. Euclid I. 42.

3. Euclid II. 7.

4. See Question 4 of 1880, with *Answer.*

5. Euclid III. 17.

Let AB, AC be the tangents touching the circle CFB in B and C, and let ED be drawn between A and the centre G touching the circle in F. Then $EB=EF$, and $DC=DF$ (III. 17 cor.)

$\therefore$ $ED=EB+DC$; add AE, AD to each of these equals; $\therefore$ AD, AE, $ED=AB$, AC and $AB=AC$ (III. 17 cor.)

$\therefore$ $AD+AE+ED=2AB$ or $2AC$, *i.e.*, the perimeter of the tr. AED is a constant quantity.

6. Euclid III. 21.

(Fig. 59). Join PB, XB, YB, QB. Now, ang. PBX $=$ ang. PAX (III. 21) $=$ ang. YAQ (I. 15) $=$ ang. YBQ (III. 21).

7. Euclid IV. 5.

8. (Fig. 60). Let E, F, G, H be the centres of the respective circles. Hence by IV. 5, it can be shewn that HG bisects DO at right angles. Similarly EF bisects OB at right angles. $\therefore$ HG is prll. to EF ($\because$ from I. 28 it can be shewn that straight lines which are perpendicular to the same straight line are parallel to one another).

In the same way it may be shewn that HE is prll. to F $\therefore$ $EFGH$ is a plm.

Euclid IV. 11.

1896.

TUESDAY, 10TH NOVEMBER.

Arithmetic and Algebra.

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1. When a vulgar fraction in its lowest terms is reduced to a decimal, whether recurring or non-recurring, prove that the number of decimal places in the period is never greater than the number representing the denominator diminished by one. 11

Simplify $1\cdot\dot{0}99\dot{0}\times 2\cdot\dot{7}2\dot{9}$; and prove that

$$\frac{5}{9}-3\times\frac{5}{19}+3\times\frac{5}{29}-\frac{5}{39}=\frac{5\times 10\times 20\times 30}{9\times 19\times 29\times 39}.$$

2. Divide £12,540 among A, B, and C, so that A shall receive $\frac{3}{7}$ as much as B and C together, and B shall receive $\frac{20}{33}$ of what A receives. 6

3. Two railway trains on adjacent parallel lines are running in opposite directions, one at the rate of 40 miles and the other of 30 miles per hour. Each has an engine and tender, and the first train has 12 carriages and the second 17. If the length of an engine and tender be 41 feet, the length of a carriage 32 feet, and the coupling spaces be each 5 feet; how much time will elapse from the moment that the engines meet till the last carriages of the trains have passed each other? 8

4. Distinguish clearly between *true* and *false discount*. 9

A banker's discount calculated for one year is 26 times his gain thereby: find the rate per cent. of interest.

5. A person purchases Rs. 10,000 stock partly in the 4 per cents. at 108, and partly in the $3\frac{1}{2}$ per cents. at 104. He sells the former at 106 and the latter at 11

$106\frac{1}{8}$, and loses Rs. 35 by the transaction. How much stock did he buy in the 4 per cents?

---

6. Define *simple* and *compound algebraical expressions*; and give an example of a homogeneous expression of 5 dimensions containing 4 terms. 8

Multiply together the expressions

$$1+ax+\tfrac{1}{2}a(a-1)x^2+\tfrac{1}{6}a(a-1)(a-2)x^3 \text{ and}$$
$$1+bx+\tfrac{1}{2}b(b-1)x^2+\tfrac{1}{6}b(b-1)(b-2)x^3$$

as far as the term involving x^3, and resolve into factors the coefficient of x^3 in the product.

7. (i) Find what quantity not involving higher powers of x beyond the second should be added to $x^9-3x^7-5x^5+2x^4+5x^3+4x^2+1$ to make it exactly divisible by x^3+2x-1. 10

(ii) Resolve into factors—

$$x^4-11x^2y^2+y^4 \text{ and } (a+b+c)^3-a^3-b^3-c^3.$$

8. Find the H. C. F. of—

$$6x^5+35x^4+59x^3+19x^2-17x-6 \text{ and}$$
$$6x^5-5x^4-41x^3+71x^2-37x+6.$$

9. Extract the square root of— 7

$$(a+b)(a+b+c)(a+b+2c)(a+b+3c)+c^4.$$

10. Prove that the value of a fraction is not altered if its numerator and denominator are multiplied by the same quantity. 9

Reduce to a single term—

$$\frac{1}{\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right)}+\frac{1}{\left(1-\frac{c}{b}\right)\left(1-\frac{a}{b}\right)}+\frac{1}{\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)}$$

11. Solve the equations :— 10

(i) $\dfrac{2x+8\frac{1}{2}}{9}-\dfrac{13x-2}{17x-32}+\dfrac{x}{3}=\dfrac{7x}{12}-\dfrac{x+16}{36}$;

(ii) $\begin{cases} 2ab(x-y)=ay(a-b) \\ 2ab(x+y)+ay(a+b+2ab)=0. \end{cases}$

12. A market-woman sells 1,000 oranges, some at a gain of 25 per cent. and the rest at a gain of 15 per cent., and thereby gains 18 per cent. on the whole. How many of each sort does she sell? 6

WEDNESDAY, 11TH NOVEMBER.

Euclid.

1. Divide a given rectilineal angle into two equal parts. 10

Shew how to draw in a triangle ABC a straight line parallel to AC and meeting AB and BC in D and E respectively, so that DE may be equal to AD and CE together.

2. Prove that the interior angles of a quadrilateral are together equal to four right angles. 10

Four points are taken in a plane, such that the distance between any two is equal to the distance between the other two: find the form of the quadrilateral obtained by joining these points.

3. Triangles of equal area standing on the same base and on the same side of it are between the same parallels. 13

If the triangles stand on different sides of the base, prove that the straight line joining their vertices is bisected by the base or the base produced.

4. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side upon which when produced the perpendicular falls, and the straight line intercepted between the perpendicular and the obtuse angle. 16

The sides of a triangle are 3, 5, 7 feet respectively: determine the greatest angle of the triangle.

5. The diameter is the greatest straight line in a circle; and of the rest that which is nearer to the centre is always greater than one more remote; and, conversely, the greater is nearer to the centre than the less. 9

6. The opposite angles of any quadrilateral inscribed in a circle are together equal to two right angles: prove the proposition and state its converse. 20

On the sides of any triangle ABC, equilateral triangles BKC, CLA, AMB are all described externally; prove that the circles described about these equilateral triangles meet in a common point, which is also the point of intersection of the straight lines AK, BL, CM.

7. Describe an isosceles triangle, having each of the angles at the base double of the third angle. 12

8. $ABCDE$ is a regular pentagon; the diagonals BD, CE intersect in F: prove that $ABFE$ is a rhombus. 10

SOLUTIONS.

1. The number of figures in the period can never be greater than the number representing the denominator diminished by one, because the number of different remainders in the process of dividing the numerator by the denominator is less than the denominator of the same fraction.

Thus $\frac{1}{7}$ cannot have more than 6 figures when reduced to a decimal; $\frac{1}{13}$ can have at most 12, and so on.

$1\cdot\dot{0}99\dot{0} = 1\frac{990}{9999} = 1\frac{1}{101} = \frac{102}{101}$; $2\cdot\dot{7}2\dot{9} = 2\frac{729}{999} = 2\frac{27}{37} = \frac{101}{37}$

$$\therefore \quad \frac{102}{101} \times \frac{101}{37} = \frac{102}{37} = 2\frac{28}{37} = 2\cdot\dot{7}5\dot{6}. \quad \textit{Ans.}$$

$$\frac{5}{9} - 3\times\frac{5}{19} + 3\times\frac{5}{29} - \frac{5}{39}$$

$$= \frac{5}{9} - \frac{5}{39} - 3\times\frac{5}{19} + 3\frac{5}{29} = 5\left(\frac{1}{9} - \frac{1}{39}\right) - 3\times 5\left(\frac{1}{19} - \frac{1}{29}\right)$$

$$= 5\left(\frac{30}{9\times 39}\right) - 15\left(\frac{10}{19\times 29}\right) = 5\times 30\left(\frac{1}{9\times 39} - \frac{1}{19\times 29}\right)$$

$$=5\times30\left(\frac{19\times29-9\times39}{9\times19\times29\times39}\right)=5\times30\left(\frac{200}{9\times19\times29\times39}\right)$$

$=\frac{5\times10\times20\times30}{9\times19\times29\times39}$, which was to be proved.

2. Let the portion received by B and C together be represented by 1, then A receives $\frac{3}{7}$, and B receives $\frac{20}{38}$ of $\frac{3}{7}$ or $\frac{20}{77}$, and C receives $1-\frac{20}{77}=\frac{57}{77}$, $\therefore$ we must divide £12,540 in the proportion of $\frac{3}{7}$, $\frac{20}{77}$ and $\frac{57}{77}$. $\frac{3}{7}+\frac{20}{77}+\frac{57}{77}=\frac{10}{7}$.

$\therefore \frac{10}{7} : \frac{3}{7} ::$ £12.540=£ 3,762 received by A

$\frac{10}{7} : \frac{20}{77} ::$ £ 12,540=£2,280 received by B. Hence C gets £12,540 − (3,762+2,280), *i. e.*, £6,498. *Ans.*

3. Each train has one engine and tender. The first train has 12 carriages and the second has 17, $\therefore$ the first train has 12 coupling spaces, and the second has 17 coupling spaces $\therefore$ the length of the first train $=12\times32$ feet (the length of the carriages) $+41$ feet (the length of engine and tender) $+5\times12$ feet (the length of the coupling spaces), *i.e.*, 485 feet.

Similarly the length of the second train—

$=(17\times32+41+5\times17)$ feet $=670$ feet.

Now we are to find the time in which the hindmost surfaces of the two trains running towards each other will meet or come in the same plane, the distance between them being the sum of the lengths of the two trains. The two trains diminish a distance of $(40+30)$ miles, *i.e.*, 70 miles per hour between them; and the length of the two trains $=(485+670)$ feet or 1,155 feet.

Hence, $70\times1760\times3$ feet : 1155 feet :: 60×60 seconds.

$\therefore$ the time required $=11\frac{1}{4}$ seconds. *Ans.*

4. The *true discount* is the interest on the present worth of the sum for the given time; while the *false discount* is the interest on the whole sum, *i.e.*, the sum of money for the period between the date of payment and the date on which it falls due. The excess of the false discount over the true discount, *i e.*, the banker's gain, is equal to the interest on

the excess of the whole sum over the present value, *i.e.*, the interest on the discount. Now let £1 be the banker's gain then the banker's discount =£26; hence true discount, *i.e.*, banker's discount *minus* banker's gain = £26 − £1 = £25 interest−discount, *i. e.*, £26 − £25, *i.e.*, £1 is the interest on £25.

∴ £25 : £160 :: £1 Interest =4%. *Ans.*

5. Finding the quantity of stock which brings the person a loss of £ 35 we get

(£108 − 106) : £35 :: £100 stock=£1,750 stock

∴ £10,000 − £1,750=£8,250 must be divided into two parts such that the gain and loss in the two transactions may be compensated.

When the person sells £100 stock at 106 he has a loss of 2 ; when he sells £100 at 106$\frac{1}{8}$, he gains £2$\frac{1}{8}$.

∴ £8,250 must be divided in the proportion of 2$\frac{1}{8}$ to 2, *i.e.*, of 17 to 16, thus :—

33 : 16 :: £8,250=£4,000 stock is bought in the second.

∴ £10,000 − £4,000=£6,000 stock is bought in 4 per cents. *Ans.*

---

6. A simple algebraical expression is one which consists of one term only; as $3a$. A compound algebraical expression is that which consists of two or more terms, as a^2+ab+b^2. $a^5+a^4b+a^3b^2+a^2b^3$ is a homogeneous expression of 5 dimensions containing 4 terms.

$$\begin{array}{l}
1+ax+\frac{1}{2}a(a-1)x^2+\frac{1}{6}a(a-1)(a-2)x^3 \\
\underline{1+bx+\frac{1}{2}b(b-1)x^2+\frac{1}{6}b(b-1)(b-2)x^3} \\
1+ax+\frac{1}{2}a(a-1)x^2+\frac{1}{6}a(a-1)(a-2)x^3 \\
\quad +bx+abx^2 \qquad\quad +\frac{1}{2}a(a-1)bx^3 \\
\qquad\quad +\frac{1}{2}b(b-1)x^2+\frac{1}{2}a(b-1)bx^3 \\
\qquad\qquad\qquad\qquad +\frac{1}{6}b(b-1)(b-2)x^3 \\
\hline
1+(a+b)x+\{\frac{1}{2}a(a-1)+ab+\frac{1}{2}b(b-1)\}x^2 \\
\quad +[\frac{1}{6}\{a(a-1)(a-2)+b(b-1)(b-2)\}+\frac{1}{2}ab(a-1)+\frac{1}{2}ab \\
(b-1)]x^3.
\end{array}$$

Ans.

The coefficient of $x^2 = \frac{1}{2}\{a(a-1)+b(b-1)\}+ab$.
$= \frac{1}{2}\{a^2-a+b^2-b+2ab\} = \frac{1}{2}(a^2+2ab+b^2-a-b)$
$= \frac{1}{2}\{(a+b)^2-(a+b)\} = \frac{1}{2}(a+b)(a+b-1)$. *Ans.*

7. (i)

$$
\begin{array}{l}
x^3+2x-1 \,\big)\; x^9-3x^7-5x^5+2x^4+5x^3+4x^2+1 \;\big(\, x^6-3x^4-2x^3+2x^2+3x-1. \\
\qquad\qquad\quad x^9+2x^7-x^6 \\
\qquad\qquad\quad\; -3x^7-2x^6-4x^5+2x^4 \\
\qquad\qquad\quad\; -3x^7 \qquad\;\; -6x^5+3x^4 \\
\qquad\qquad\qquad\quad -2x^6+2x^5-x^4+5x^3 \\
\qquad\qquad\qquad\quad -2x^6 \qquad\; -4x^4+2x^3 \\
\qquad\qquad\qquad\qquad\quad 2x^5+3x^4+3x^3+4x^2 \\
\qquad\qquad\qquad\qquad\quad 2x^5 \qquad\quad +4x^3-2x^2 \\
\qquad\qquad\qquad\qquad\qquad 3x^4-x^3+6x^2+1 \\
\qquad\qquad\qquad\qquad\qquad 3x^4 \qquad +6x^2-3x \\
\qquad\qquad\qquad\qquad\qquad\quad -x^3+3x+1 \\
\qquad\qquad\qquad\qquad\qquad\quad -x^3-2x+1 \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 5x
\end{array}
$$

By dividing one expression by the other we have $5x$ as remainder, so in order that the expression may be divided by x^3+2x-1 exactly there should be no remainder, that is, $5x$ must be equal to 0, *i.e.*, we should add $-5x$ to the expression that it could be exactly divided by the other.

(ii) $x^4-11x^2y^2+y^4 = x^4-2x^2y^2+y^4-9x^2y^2 = (x^2-y^2)^2-(3xy)^2$
$=(x^2+3xy-y^2)(x^2-3xy-y^2)$. *Ans.*

$(a+b+c)^3-(a^3+b^3+c^3) = (a+b)^3+3(a+b)^2c+3(a+b)c^2+c^3-(a^3+b^3+c^3)$
$= a^3+3ab(a+b)+b^3+3(a+b)^2c+3(a+b)c^2-(a^3+b^3+c^3)$
$= 3ab(a+b)+3(a+b)^2c+3(a+b)c^2 = 3(a+b)\{ab+(a+b)c+c^2\}$
$= 3(a+b)(ab+ac+bc+c^2)=3(a+b)(b+c)(c+a)$. *Ans.*

8. $6x^5-5x^4-41x^3+71x^2-37x+6\big)\,6x^5+35x^4+59x^3+19x^2-17x-6\,\big(1$

$$6x^5-5x^4-41x^3+71x^2-37x+6$$

$$4\,\big)\,40x^4+100x^3-52x^2+20x-12$$

$$10x^4+25x^3-13x^2+5x-3\,\big)\,6x^5-5x^4-41x^3+71x^2-37x+6\,\big(6x-20$$

$$10$$

$$60x^5-50x^4-410x^3+710x^2-370x+60$$

$$60x^5+150x^4-78x^3+30x^2-18x$$

$$-200x^4-332x^3+680x^2-352x+60$$

$$-200x^4-500x^3+260x^2-100x+60$$

$$84x\,\big)\,168x^3+420x^2-252x$$

$$2x^2+5x-3\,\big)\,10x^4+25x^3-13x^2+5x-3\,\big(5x^2+1$$

$$10x^4+25x^3-15x^2$$

$$2x^2+5x-3$$

$$2x^2+5x-3$$

$\therefore$ the *H. C. F.* $=2x^2+5x-3$. *Ans.*

6. The expression is equal to

$$\{(a+b)(a+b+3c)\}\{(a+b+c)(a+b+2c)\}+c^4$$
$$=\{(a+b)^2+3c(a+b)\}\{(a+b)^2+3c(a+b)+2c^2\}+c^4$$
$$=\{(a+b)^2+3c(a+b)\}^2+2c^2\{(a+b)^2+3c(a+b)\}+c^4$$
$$=[\{(a+b)^2+3c(a+b)\}+c^2]^2$$

$\therefore$ The square root $=(a+b)^2+3c(a+b)+c^2$. *Ans.*

10. Let the fraction be $\frac{a}{b}$, then $\frac{a}{b}=\frac{am}{bm}$.

Because $\frac{a}{b}\times b=a$ $\therefore m\times\frac{a}{b}\times b=ma$, *i.e.*, $\frac{a}{b}\times mb=ma$

Again, $\frac{ma}{mb}\times mb=ma$

$\therefore \frac{a}{b}\times mb=\frac{ma}{mb}\times mb$, *i.e.*, $\frac{a}{b}=\frac{ma}{mb}$

$$\frac{1}{\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right)}=\frac{a^2}{(a-b)(a-c)}$$

$$\frac{1}{\left(1-\frac{c}{b}\right)\left(1-\frac{a}{b}\right)}=\frac{b^2}{(b-c)(b-a)}$$

$$\frac{1}{\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)}=\frac{c^2}{(c-a)(a-b)}$$

$\therefore$ the whole expression

$$=\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}$$
$$=-\frac{a^2}{(a-b)(c-a)}-\frac{b^2}{(b-c)(a-b)}-\frac{c^2}{(c-a)(b-c)}$$
$$=-\left\{\frac{(b-c)a^2+b^2(c-a)+c^2(a-b)}{(a-b)(b-c)(c-a)}\right\}$$

The numerator

$$=a^2(b-c)+b^2c-bc^2-b^2a+ac^2$$
$$=a^2(b-c)+bc(b-c)-a(b^2-c^2)$$

$$= (b-c)\{a^2+bc-a(b+c)\}$$
$$= (b-c)(a^2-ab+bc-ac)=(b-c)(a-b)(a-c)$$
$$= -(a-b)(b-c)(c-a)$$
$$\therefore \frac{-(a-b)(b-c)(c-a)}{-(a-b)(b-c)(c-a)}=1. \quad \textit{Ans.}$$

11. (i) $\frac{7x}{12}-\frac{x+16}{36}-\frac{4x+17}{18}-\frac{x}{3}=-\frac{13x-2}{17x-32}$

$$\therefore \frac{21x-x-16-8x-34-12x}{36}=-\frac{13x-2}{17x-32}$$

$$\therefore -\frac{25}{18}=-\frac{13x-2}{17x-32}$$

$\therefore$ $425x-800=234x-36$ $\therefore$ $x=4$. *Ans.*

(ii) Adding (i) and (ii) we have

$$2abx-2aby-axy+bxy=0$$
$$2abx+2aby+axy+bxy+2abxy=0$$
$$4abx+2bxy+2abxy=0$$

$\therefore$ $2bx(2a+y+ay)=0$ $\therefore$ $y+ay+2a=0$

$\therefore$ $y+ay=-2a$ $\therefore y(1+a)=-2a$

$$\therefore y=-\frac{2a}{1+a}. \quad \textit{Ans.}$$

Again, subtract (ii) from (i)

$\therefore -4aby-2axy-2abxy=0$

$\therefore -2ay(2b+x+bx)=0 \therefore x+bx+2b=0$

$\therefore x(1+b)=-2b$, whence $x=\frac{-2b}{1+b}$. *Ans.*

Or thus :—

Divide both the equations by xy, and add them together

$$\therefore \frac{2ab}{y}-\frac{2ab}{x}=a-b \quad \text{.................................(i)}$$

$$\frac{2ab}{y}+\frac{2ab}{x}=-a-b-2ab$$

$$\frac{4ab}{y}=-(2b+2ab) \therefore \frac{2a}{y}=-\left(1+a\right)$$

$\therefore y=-\frac{2a}{1+a}$, whence $x=-\frac{2b}{1+b}$. *Ans.*

12. Suppose she sold x oranges of the first sort

$\therefore$ she sold $(1000-x)$ oranges of the second sort.

$100 : x :: 25 = \frac{25x}{100}$, gain on x oranges.

$100 : (1000-x) :: 15 = \frac{15(1000-x)}{100}$ gain on $(1000-x)$ oranges.

$100 : 1000 :: 18 = 180$ gain on the whole,

$$\therefore \frac{25x}{100} + \frac{15(1000-x)}{100} = 180$$

whence $x=300$ $\therefore$ $1000-300=700$

$\therefore$ she sold 300 at a gain of 25 % and 700 at a gain of 15 %. *Ans.*

Euclid.

1. Euclid I. 9.

(Fig. 61). Let ABC be a tr. Bisect angs. BAC, ACB by AK, CK, meeting in K; draw DKE prll. to BC.

$\because$ DK is prll. to AC and AK meets them, $\therefore$ ang. DKA $=$ ang. KAC (I. 29); but ang. $DAK=$ ang. KAC (constr.) $\therefore$ ang. $DAK=$ ang. DKA (ax. 1) $\therefore$ $DA = DK$ (1. 6).

Similarly it may be shewn that $KE=EC$ $\therefore$ $DE=AD+CE$.

2. The quadrilateral can be divided into two triangles by joining one pair of opposite angles. Now the three interior angles of one triangle are together equal to two right angles (I. 32). $\therefore$ the interior angles of a quadrilateral are together equal to 4 rt. angs.

Let A, B, C, D be four points in a plane. Join AB, BC, CD, DA. Then $AB=DC$ and $AD=BC$. Join BD.

In trs. ABD, BDC, $AD=DC$ (hyp.), BD is common, AD $=BC$ (hyp.) $\therefore$ trs. are equal in all respects (I. 8) $\therefore$ ang. $ABD=$ ang. BDC; but they are alternate angs. $\therefore$ AB is prll. to DC. Similarly AD is prll. to BC, $\therefore$ $ABCD$ is a plm.

3. Euclid I. 39.

(Fig. 62.) Let ABC and ADB be trs. standing on AB but on opposite sides of it. Let CD meet AB or AB produced in E. Draw DF, CG perp. to AB. Then $DF=CG$ because equal triangles on the same base have equal altitudes.

Hence in trs. CEG, DEF, ang. $DFE=$ ang. CGE $\because$ they are rt. angs., ang. $CEG=$ ang. FED (I. 15), and $CG=DF$; $\therefore$ trs. are equal in all respects (I. 26) $\therefore$ $CE=ED$.

4. Euclid II. 12.

Let ABC be a tr. Let $AB=7$ feet, $BC=5$ ft. $CA=3$ ft. Then, by II. 12, $AB^2=AC^2+BC^2+2\text{rect.}BC.CD$ $\therefore 7^2= 5^2+3^2+2\times5\times CD$ $\therefore$ $CD=\frac{3}{2}$ feet.

Now, in a right-angled triangle the side opposite to the angle of 30° is half the hypotenuse. $\therefore$ $CD=\frac{1}{2}AC$. $\therefore$ in the right-angled triangle ACD, ang. $CAD=30^\circ$ and as ang. $ADC=90^\circ$ $\therefore$ ang. $ACD=60^\circ$ (I. 32). $\therefore$ ang. $ACB=120^\circ$ (I. 13), *i.e.*, the greatest angle of the triangle ABC contains 120°.

5. Euclid III. 15.

6. Euclid III. 22.

The converse of the 22nd :—If a pair of opposite angles of a quadrilateral are together equal to two right angles its vertices are concyclic.

(Fig. 63.) Let CM and BL meet in P.

In trs. MAC, BAL, $AM=AB$, $AC=AL$, ang. $MAC=$ ang. BAL, $\because$ each = the angle of an equilateral tr. with ang. BAC. $\therefore$ $CM=BL$ and ang. $AMP=$ ang. ABP, and ang. $ACP=$ ang. ALP (I. 4) $\because$ ang. $AMP=$ ang. ABP, the quadrilateral $AMBP$ is cyclic (III. 21 Converse).

Similarly quadrilateral $APCL$ is also cyclic. $\therefore$ ang. APB is the supplement of the ang. at M and ang. APC is supplement of the ang. at L, $\therefore$ ang. $APB=$ ang. $APC=$ twice the angle of an equilateral tr. $\therefore$ ang

BPC= twice the angle of an equilateral tr. (I. 15 Cor. 2) and ang. $BKC=\frac{2}{3}$ of a rt. ang. $\therefore$ $BPCE$ is cyclic. Hence the three cirs. meet in a common point P.

Again, ang. $BPK=BCK$ (III. 21) $=\frac{2}{3}$ of a rt. ang. (hyp.) $\therefore$ angs. APB, BPK = 2 rt. angs. $\therefore$ AP is in the same straight line with PK (I. 14). Hence the three straight lines AK, BL, CM meet in a common point P.

7. Euclid IV. 10.

(Fig. 64.) The angle of a regular pentagon is trisected by the straight lines which join it to the opposite vertices: thus:—

In tr. EDC, ang. EDC, which is an angle of the regular pentagon $=\frac{6}{5}$ of a rt. ang. (I. 32 Cor.) $\therefore$ angs. ECD, CED = $\frac{4}{5}$ of a rt. ang. (I. 32); but $DE=DC$ (hyp.) $\therefore$ ang. CED = ang. ECD (I. 5) $=\frac{2}{5}$ths of a rt. ang., *i.e.*, $\frac{1}{3}$ of the angle of a regular pentagon. $\therefore$ ang. FCD = ang. FDC = ang. FED, each being one-third of the angle of a regular pentagon. $\therefore$ ang. $EDF=\frac{2}{3}$ ang. of the pentagon.

Again, ang. EFD = angs. $FCD+FDC$ (I. 32) $=\frac{2}{3}$ ang. of the pentagon. $\therefore$ ang. EDF = ang. EFD $\therefore$ $EF=ED$, a side of the regular pentagon.

Similarly, FB may be proved equal to a side of the regular pentagon. Hence AB, BF, FE, EA are all equal. $\therefore$ $ABFE$ is a rhombus.

1897.

TUESDAY. 9TH NOVEMBER.

Arithmetic and Algebra.

Kavasji Jamshedji Sanjana, M.A.
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N.B.—No algebraical methods are to be employed in solving the first five questions.

1. Define *numerator* and *denominator* of a fraction; and prove that, by multiplying these by the same number, the value of the fraction is not altered. 12

Simplify $1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$; and

shew that $\dfrac{\frac{5}{12}+\frac{1}{99}-\frac{1}{70}+\frac{5}{12}\times\frac{1}{99}\times\frac{1}{70}}{1-\frac{5}{12}\times\frac{1}{99}+\frac{5}{12}\times\frac{1}{70}+\frac{1}{99}\times\frac{1}{70}}=\dfrac{7}{17}.$

2. What is *inverse proportion*? Give two illustrations of it. 8

A contractor undertakes to dig a canal 12 miles long in 350 days, and employs 45 men; he finds that in 200 days he has completed $4\frac{1}{2}$ miles: how many additional men must he employ to get the undertaking finished in time?

3. Guns are fired at intervals of 10 seconds in a town 7
towards which a passenger train is approaching at the rate of 30 miles per hour: if sound travels 1,144 feet per second, find at what intervals the reports will be heard by the passengers.

4. A sum of Rs. 285, put out to compound interest 10
for 3 years, produces Rs. 29 7*a*. $4\frac{32}{75}$p.: find the rate per cent. of interest.

5. I invest a certain sum in the $3\frac{1}{2}$ per cents. at 91, 9
and £4,000 sterling in the 3 per cents. at 75; after paying an income tax of 7*d*. in the £, my net income is £524 5*s*. What sum have I invested in the $3\frac{1}{2}$ per cents.?

---

6. What is a *compound* algebraical expression? 11
State and prove the rule for subtracting one such expression from another.

Simplify

$$\ldots 1\Big)^2+\left(b+\frac{1}{b}\right)^2+\left(ab+\frac{1}{ab}\right)^2-\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)\left(ab+\frac{1}{ab}\right).$$

7. (i) Find the quotient when 11

$$x^3(y^2-z^2)+y^3(z^2-x^2)+z^3(x^2-y^2) \text{ is divided by}$$
$$x^2(y-z)+y^2(z-x)+z^2(x-y).$$

(ii) Resolve into factors

$$(a+b)^3+(b+c)^3+(c+a)^3+a^3+b^3+c^3.$$

8. Simplify 10

$$x^8-\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)\left(x-\frac{1}{x}\right)\Big\};$$

and find the fourth root of

$$\left(x^2+\frac{4}{x^2}\right)^2-8\left(x+\frac{2}{x}\right)^2+48.$$

9. Reduce to a single term 6

$$\frac{1}{x^2-5xy+6y^2}+\frac{a}{x^2-4xy+3y^2}+\frac{1}{x^2-3xy+2y^2};$$

and find the value of a which makes this expression zero.

10. Solve the equations : 10

(i) $\dfrac{a}{x-a}+\dfrac{b}{x-b}=\dfrac{a+b}{x-a-b}$; (ii) $\dfrac{y+x-a}{b+c}=\dfrac{x+a-y}{c+a}=1.$

11. What fraction is that which becomes equal to $\frac{2}{3}$ when its numerator is increased by 2, and equal to $\frac{8}{11}$ when its denominator is diminished by 7 ? 6

1897.

WEDNESDAY, 10TH NOVEMBER.

[2 P.M. TO 5 P.M.]

Euclid.

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1. The greater angle of every triangle has the greater side opposite to it. 10

If D be any point in the side BC of a triangle ABC, then the greater of the sides AB, AC, is greater than AD.

2. If a straight line falling on two other straight lines makes the alternate angles equal, the two straight lines shall be parallel to one another. 10

If the sides BA, CA of a triangle ABC be produced to D, E respectively, so that AD is equal to AB and AE to AC, prove that DE is parallel to BC.

3. To a given straight line apply a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle. 18

On a given straight line as diagonal construct a rhombus, which shall be equal to a given triangle.

4. If a straight line be divided into any two parts, the squares on the whole line and on one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square on the other part. 11

If a straight line be divided at C, so that the rectangle $AB.BC$ is equal to the square on AC, prove that the square on the line made up of AB and BC is five times the square on AC.

5. The angles in the same segment of a circle are equal to one another; prove this proposition, and state its converse. 15

Prove that a circle can be described about an equiangular polygon when its alternate sides are equal.

6. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

A quadrilateral which can be inscribed in a circle, has a circle inscribed within it; prove that the straight lines joining the opposite points of contact are at right angles to each other.

7. Inscribe a circle in a given equiangular pentagon. 9

8. The straight lines OA, OB touch a circle at A, B; a straight line touching the same circle at Q, meets OA, OB in P, R respectively. Prove that PR is less or greater than OA, according as PR cuts OA and OB, or cuts OA produced and BO produced. 10

SOLUTIONS.

1. In any fraction the *number* of parts into which unity is *divided* is called the *denominator* of the fraction; and the number of parts *taken* to make up the quantity is called the *numerator*. Consider the fractions $\frac{2}{3}$ and $\frac{10}{15}$. The first fraction shows that the unit is divided into *three* equal parts and *two* of these parts are taken; the second indicates that the unit is divided into 15 equal parts and 10 of these parts are taken. Now, evidently, one part in the former case is equal to five parts in the latter case, therefore *two* parts taken in the former case are equal to *ten* parts taken in the latter case; hence $\frac{2}{3}=\frac{10}{15}$ and $\frac{10}{15}=\frac{2\times 5}{3\times 5}$.

The expression $=1\div[1+1\div\{1+1\div(1+\frac{1}{2})\}]$

$=1\div[1+1\div\{1+1\div\frac{3}{2}\}]=1\div[1+1\div\{1+\frac{2}{3}\}]$

$=1\div[1+1\div\frac{5}{3}]=1\div[1+\frac{3}{5}]=1\div\frac{8}{5}=\frac{5}{8}$. *Ans.*

$(\frac{5}{12}\times\frac{1}{99}\times\frac{1}{70})+(\frac{5}{12}+\frac{1}{99}-\frac{1}{70})$

$$=\frac{1}{12\times 99\times 70}\left(5+34650+840-1188\right)=\frac{34307}{12\times 99\times 70}$$

$$\frac{5}{12\times 70}+\frac{1}{99\times 70}-\frac{5}{12\times 99}=\frac{495+12-350}{12\times 99\times 70}=\frac{157}{12\times 99\times 70}$$

$$\therefore\ 1+\frac{157}{12\times 99\times 70}=\frac{83317}{12\times 99\times 70}$$

$$\therefore \frac{34307}{12\times 99\times 70}\div\frac{83317}{12\times 99\times 70}=\frac{34307}{83317}=\frac{7\times 4901}{17\times 4901}=\frac{7}{17}.$$

Q. E. D.

2. When any two quantities are so related to each other that the one decreases or increases in proportion as

the other increases or decreases, they are said to vary *inversely*, and the proportion in that case is said to be *inverse*.

(*a*) With a given *work*, the *agency* varies inversely as the *time*; *i.e.*, if a number of men be engaged to do a piece of work, the greater the number of men employed, the smaller the number of days they will take to do the work.

(*b*) With a given *area*, the length varies *inversely* as the breadth.

The contractor has completed $4\frac{1}{2}$ miles in 200 days $\therefore$ 12 miles $-4\frac{1}{2}$ miles, or $7\frac{1}{2}$ miles, are left.

$\therefore 4\frac{1}{2}$ miles : $7\frac{1}{2}$ miles :: 200 days $=\dfrac{1000}{3}$ days required to finish the work.

But according to the contract the work should be finished in $350-200$, or 150 days.

$\therefore$ (*Inverse*) $\dfrac{1000}{3}$ days : 150 days :: 45 men $=$ 100 men.

But 45 men are engaged already $\therefore$ $100-45$, or 55, more are required. *Ans.*

3. 30 miles an hour $=$ 44 ft. per sec. Hence, dividing 10 sec. in the ratio of 1,144 to 44, we find out the parts:—

1188 : 10 :: 1144 $=\frac{260}{27}=9\frac{17}{27}$ secs.

$\therefore$ the second part $=\frac{10}{27}$ sec., *i.e.*, in $9\frac{17}{27}$ secs. the train travels the same distance as sound travels in $\frac{10}{27}$ sec. and the sum of $9\frac{17}{27}$ secs. and $\frac{10}{27}$ is 10 secs. $\therefore$ the interval required $=9\frac{17}{27}$ secs. *Ans.*

4. Rs. 285 + Rs. 29 7as. $4\frac{32}{75}$p. $=$ Rs. $\dfrac{566029}{1800}$ amount on Rs. 285.

$$\text{Rs. } 285 : \text{Rs. } \frac{566029}{1800} :: \text{Re. } 1 = \text{Rs. } \frac{29791}{5^3 \times 3^3 \times 2^3}$$

amount of Re. 1 in 3 years.

$\therefore \dfrac{29791}{5^3 \times 3^3 \times 2^3}$ is the third power of the amount of Re. 1 in 1 year $\therefore$ finding the cube root of $\dfrac{29791}{5^3 \times 3^3 \times 2^3}$ we get Re. $\dfrac{31}{30}$ which is the amount of Re. 1 for 1 year.

$\therefore$ the interest on Re. 1 = Re. $\frac{1}{30}$, *i. e.*, $3\frac{1}{3}$ %. *Ans.*

Or thus :— $\left(1+\dfrac{R}{100}\right)^n = \dfrac{A}{P}$, where R, n, A, P represent, respectively, the rate per cent., number of years, amount and the principal.

$\therefore \left(1+\dfrac{R}{100}\right)^3 = \dfrac{A}{P} = \dfrac{29791}{5^3 \times 3^3 \times 2^3}$ $\therefore$ extracting the cube root of both sides we have

$$1+\frac{R}{100}=\frac{31}{30} \therefore \frac{R}{100}=\frac{1}{30} \therefore R=\frac{10}{3}=3\tfrac{1}{3}\%. \quad \textit{Ans.}$$

5. £$\frac{233}{240}$: £$524\frac{1}{4}$ :: £1 = £540 gross income

£75 : £4,000 :: £3 = £160 income in the second case

$\therefore$ £540 − £160 = £380 income derived by investing in the $3\frac{1}{2}$ per cents.

£$3\frac{1}{2}$: £380 :: £91 = £9880 investment in the $3\frac{1}{2}$ per cents. *Ans.*

---

6. A *compound* algebraical expression consists of two or more terms.

Rule—Change the sign of every term of the *subtrahend* ; then add those terms to the other expression, *i.e.*, the *minuend*, $a+b-(c+d)=a+b-c-d$. Thus: $a+b-(c+d)$ means that from $a+b$ we are to take the sum of $c+d$ and the result is the same whether c and d are subtracted separately or in one sum $\therefore a+b-(c+d)=a+b-c-d$.

Again, $a+b-(c-d)$ means that from $a+b$ we are to subtract $c-d$, *i. e.*, the excess of c over d. If from $a+b$ we take c, we have $a+b-c$, but we thus subtract too much from $a+b$, for we have to subtract not c but a quantity which is less than c, by d. Hence we must add d to the result ; then $a+b-(c-d)=a+b-c+d$.

The expression $=$

$$a^2+2+\frac{1}{a^2}+b^2+2+\frac{1}{b^2}+a^2b^2+2+\frac{1}{a^2b^2}$$

$$-a^2b^2-b^2-a^2-2-\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{a^2b^2}=4. \quad \textit{Ans.}$$

7. (i) $x^2(y^3-z^3)+y^2(z^3-x^3)+z^2(x^3-y^3)$. Arranging the terms according to powers of x, we get

$$x^3(y^2-z^2)-x^2(y^3-z^3)+y^2z^2(y-z)$$
$$=(y-z)\{x^3(y+z)-x^2(y^2+yz+z^2)+y^2z^2\}$$
$$=(y-z)\{-y^2(x^2-z^2)+x^2y(x-z)+x^2z(x-z)\}$$
$$=(y-z)(x-z)\{-y^2(x+z)+x^2y+x^2z\}$$
$$=(y-z)(x-z)\{z(x^2-y^2)+xy(x-y)\}$$
$$=(y-z)(x-z)(x-y)\{z(x+y)+xy\}$$
$$=(y-z)(x-z)(x-y)(xy+yz+zx)$$

$$x^2(y-z)+y^2(z-x)+z^2(x-y)$$
$$=x^2(y-z)-x(y^2-z^2)+yz(y-z)$$
$$=(y-z)\{x^2-x(y+z)+yz\}$$
$$=(y-z)\{-y(x-z)+x(x-z)\}$$

$=(y-z)(x-z)(x-y)$ ∴ the quotient arising from dividing the first expression by the second $=xy+yz+zx$. *Ans.*

(ii) The expression $=$

$$\{(a+b)^3+c^3\}+\{(b+c)^3+a^3\}+\{(c+a)^3+b^3\}$$
$$=(a+b+c)\{(a+b)^2-c(a+b)+c^2\}+(a+b+c)$$
$$\{(b+c)^2-a(b+c)+a^2\}+(a+b+c)\{(c+a)^2-b(c+a)+b^2\}$$
$$=(a+b+c)\{(a+b)^2-c(a+b)+c^2+(b+c)^2-a(b+c)+a^2$$
$$+(c+a)^2-b(c+a)+b^2\}$$

Simplifying the expression within brackets we get

$3(a^2+b^2+c^2)$

∴ the factors are $3(a+b+c)(a^2+b^2+c^2)$. *Ans.*

8. The first expression

$$=\sqrt{\left\{x^3-\left(x^3-\frac{1}{x^6}\right)\right\}}=\sqrt{\frac{1}{x^6}}=\pm\frac{1}{x^3}. \quad \textit{Ans.}$$

The second expression

$$=\left(x^2+\frac{4}{x^2}\right)^2-8\left(x^2+\frac{4}{x^2}+4\right)+48$$

$$=\left(x^2+\frac{4}{x^2}\right)^2-8\left(x^2+\frac{4}{x^2}\right)+16$$

$$=\left(x^2+\frac{4}{x^2}-4\right)^2 \therefore \sqrt{\left(x^2+\frac{4}{x^2}-4\right)^2}=\sqrt{\left(x^2+\frac{4}{x^2}-4\right)}$$

$$=\sqrt{\left(x-\frac{2}{x}\right)^2}=x-\frac{2}{x}. \quad \textit{Ans.}$$

9. The expression

$$=\frac{1}{(x-3y)(x-2y)}+\frac{a}{(x-3y)(x-y)}+\frac{1}{(x-2y)(x-y)}$$

$$=\frac{x-y+a(x-2y)+x-3y}{(x-3y)(x-2y)(x-y)}=\frac{a(x-2y)+2(x-2y)}{(x-3y)(x-2y)(x-y)}$$

$$=\frac{(a+2)(x-2y)}{(x-3y)(x-2y)(x-y)}=\frac{a+2}{(x-3y)(x-y)}. \quad \textit{Ans.}$$

In order that $\frac{a+2}{(x-3y)(x-y)}$ may vanish, $a+2$ must be equal to 0; $\therefore a=-2$. *Ans.*

10. (i) $\frac{a}{x-a}+\frac{b}{x-b}=\frac{a+b}{x-(a+b)}$

$$\therefore \frac{ax-ab+bx-ab}{(x-a)(x-b)}=\frac{a+b}{x-(a+b)}$$

$$\therefore \frac{x(a+b)-2ab}{x^2-x(a+b)+ab}=\frac{(a+b)}{x-(a+b)}$$

$$\therefore x^2(a+b)-2abx-x(a+b)^2+2ab(a+b)$$

$$=x^2(a+b)-x(a+b)^2+ab(a+b)$$

$$\therefore -2abx=-ab(a+b) \therefore x=\frac{a+b}{2}. \quad \textit{Ans.}$$

Or thus—

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{a}{x-a-b}+\frac{b}{x-a-b}$$

$$\therefore \frac{a}{x-a}-\frac{a}{x-a-b}=\frac{b}{x-a-b}-\frac{b}{x-b}$$

$$\therefore\ a\left(\frac{1}{x-a}-\frac{1}{x-a-b}\right)=b\left(\frac{1}{x-a-b}-\frac{1}{x-b}\right)$$

$$\therefore\ a\left\{\frac{x-a-b-x+a}{(x-a)(x-a-b)}\right\}=b\left\{\frac{x-b-x+a+b}{(x-b)(x-a-b)}\right\}$$

$$\therefore\ \frac{-ab}{(x-a)(x-a-b)}=\frac{ab}{(x-b)(x-a-b)}$$

$$\therefore\ -\frac{1}{x-a}=\frac{1}{x-b}\quad\therefore\ x-a=-x+b$$

$$\therefore\ 2x=a+b\ \therefore\ x=\frac{a+b}{2}.\quad \textit{Ans.}$$

(ii) $\dfrac{y+x-a}{b+c}=1$ and $\dfrac{x+a-y}{c+a}=1$

$\therefore\ y+x=a+b+c$ ....(i) and $x-y=c$ ....(ii)

Adding the two, we get $2x=a+b+2c$

$\therefore\ x=\dfrac{a+b+2c}{2}$. Subtracting one equation from the other we get $2y=a+b$ $\therefore\ y=\dfrac{a+b}{2}$. *Ans.*

11. Let x be the numerator and y be the denominator.

$\therefore$ the fraction $=\dfrac{x}{y}$.

$\therefore\ \dfrac{x+2}{y}=\dfrac{2}{3}$ ....(i) and $\dfrac{x}{y-7}=\dfrac{8}{11}$ ....(ii)

$\therefore\ 3x-2y=-6$ ......(i) and $11x-8y=-56$ ....(ii)

Multiplying (i) by 4 and subtracting (ii) from it we get $x=32$; hence $y=51$ $\therefore$ the fraction $=\frac{32}{51}$. *Ans.*

Euclid.

1. Euclid I. 19.

Let AB be greater than AC; then ang. ACB is greater than ang. ABC (I. 18); but ang. ADB is greater than ang. ACD (I. 16) $\therefore$ much more is ang. ADB greater than ang. ABD, $\therefore$ AB is greater than AD (I. 19). Similarly, if AC is greater than AB, we may prove AC also greater than AD.

Fig 1

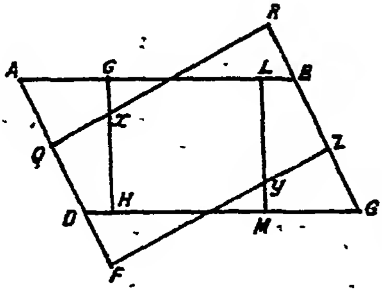


Fig.2

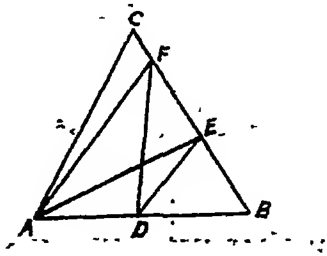


Fig.3

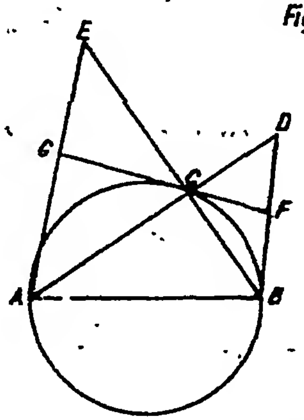


Fig. 4

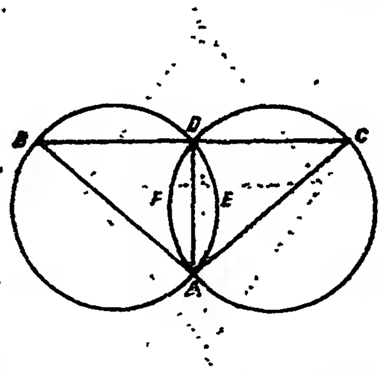


Fig. 5

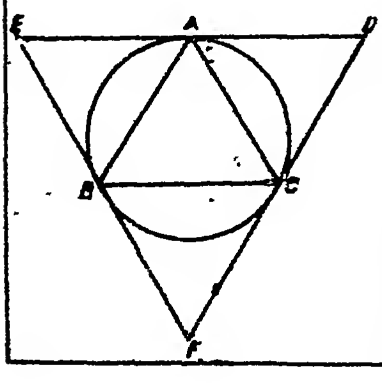
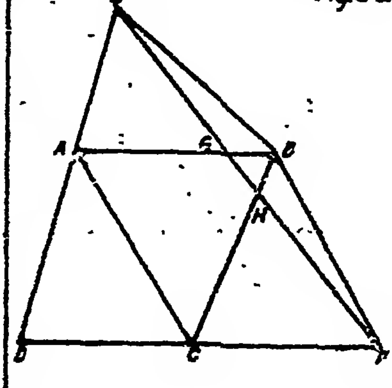


Fig. 5a



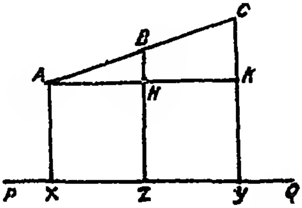


Fig. 6

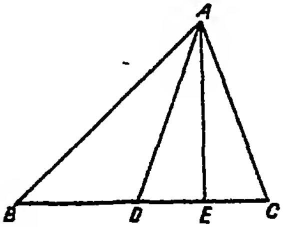


Fig. 7

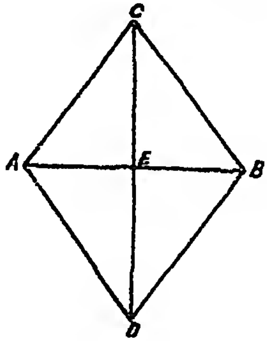


Fig 8

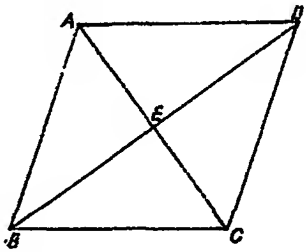


Fig 9

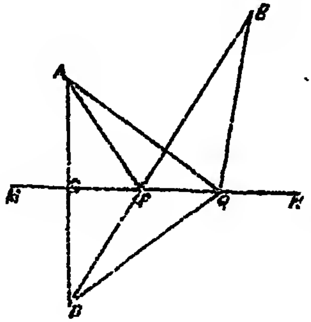


Fig. 10

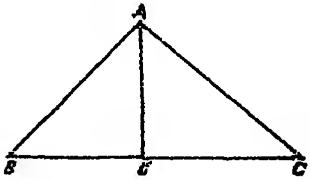


Fig. 11

Fig. 12

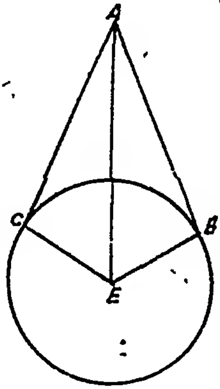


Fig. 13

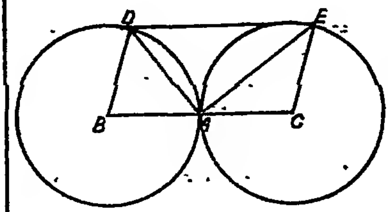


Fig. 14

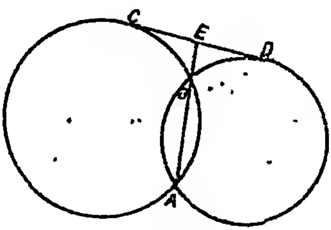


Fig. 15

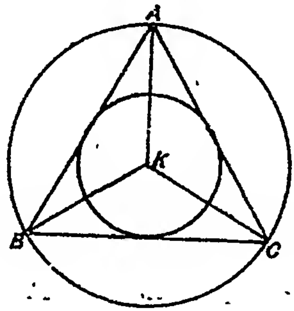


Fig 16

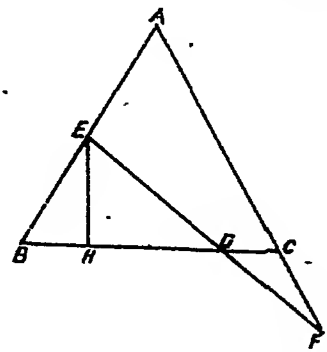


Fig. 17

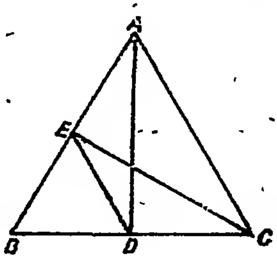


Fig.18

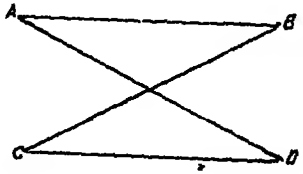


Fig.19

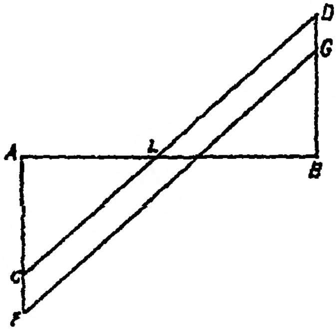


Fig.20

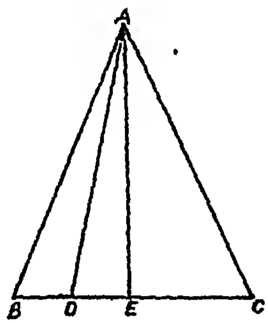


Fig 21

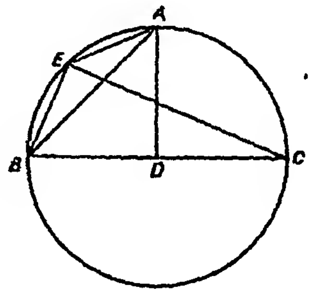


Fig.22

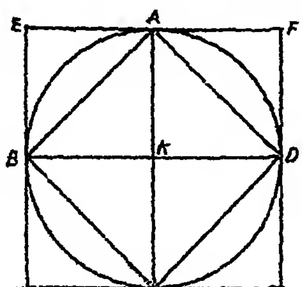
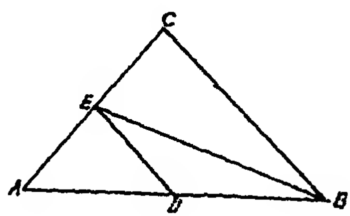


Fig.23



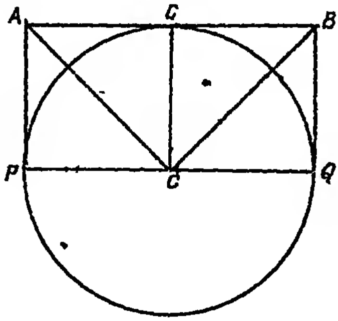


Fig.24

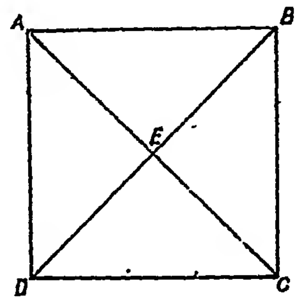


Fig.25

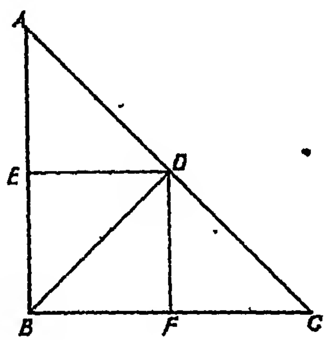


Fig.26

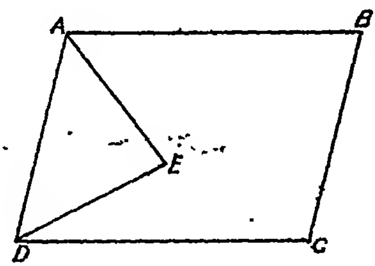


Fig.27

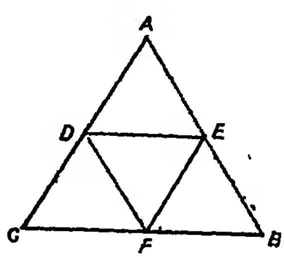


Fig.28

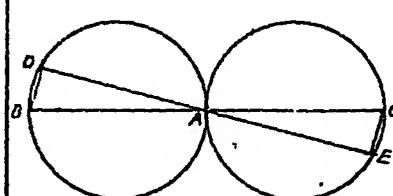


Fig.29

Fig. 30

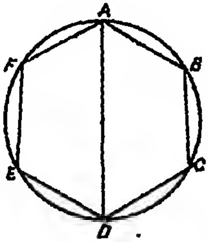


Fig. 31

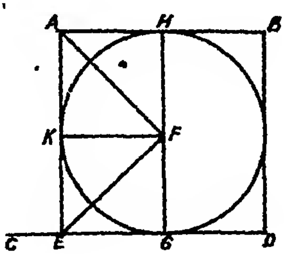


Fig. 32.

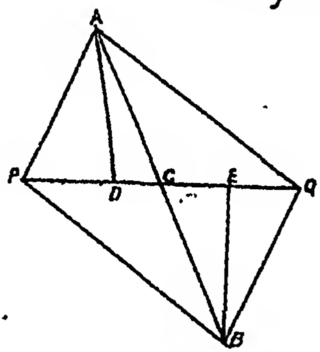


Fig. 33

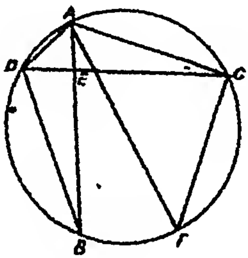


Fig. 34

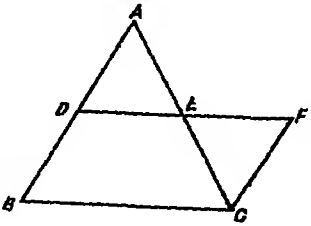


Fig. 35

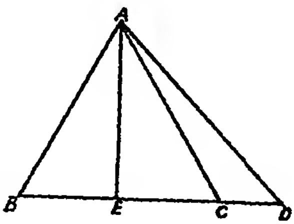


Fig 36

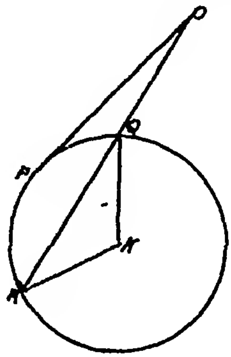


Fig.36a

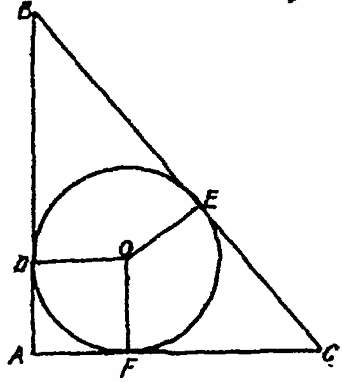


Fig.37

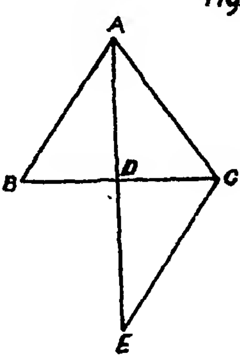


Fig.38

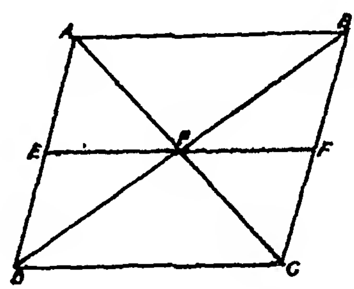


Fig 39

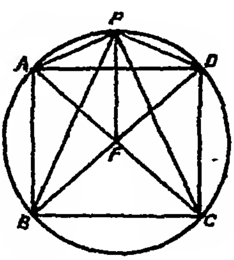
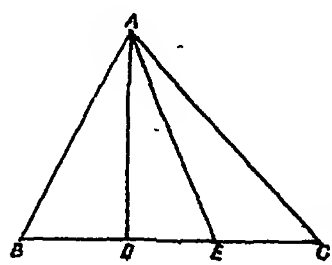


Fig.40



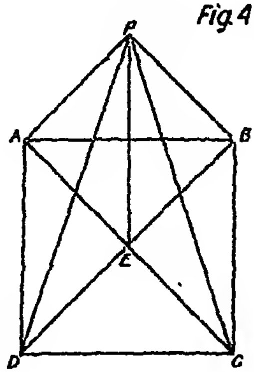


Fig.41

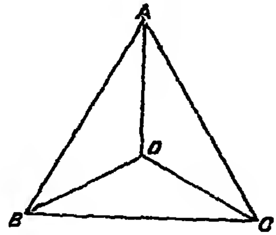


Fig 42

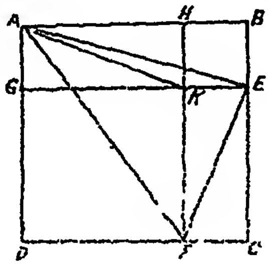


Fig.43

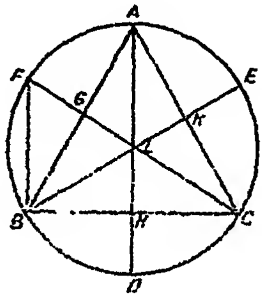


Fig 44

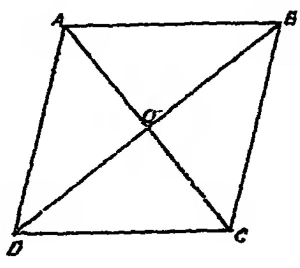


Fig.45

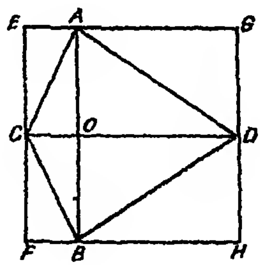


Fig.46

Fig. 47

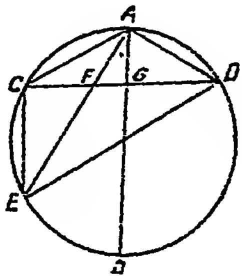


Fig. 48

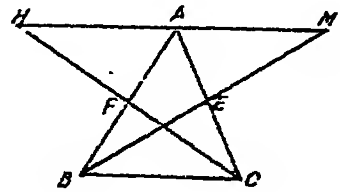


Fig. 49

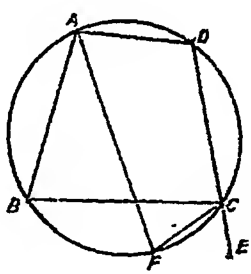


Fig. 50

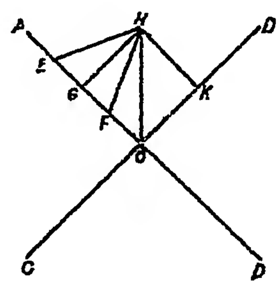


Fig. 51

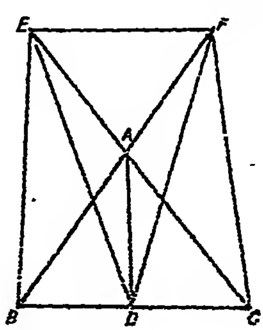
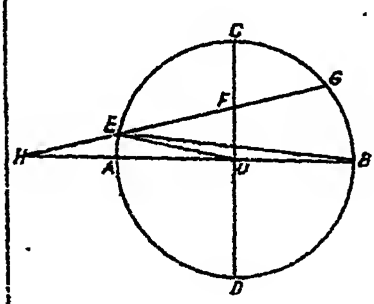


Fig. 52



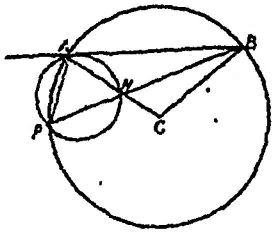


Fig 53

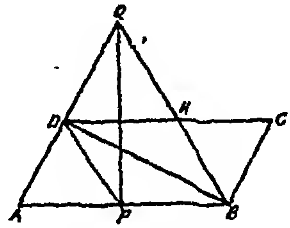


Fig 54

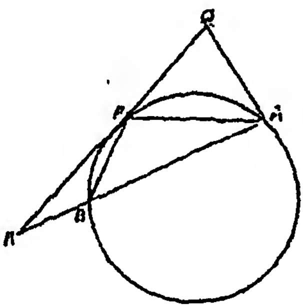


Fig 55

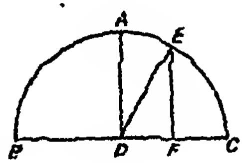


Fig. 56

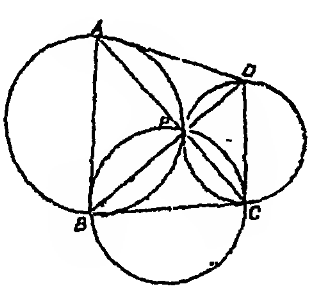


Fig. 57

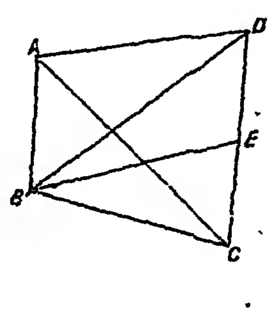


Fig 58

Fig. 59

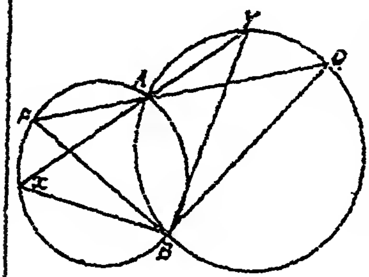


Fig. 60

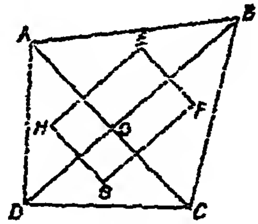


Fig. 61

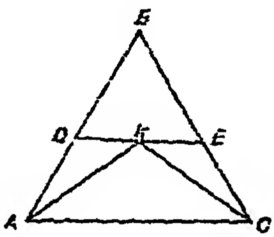


Fig. 62

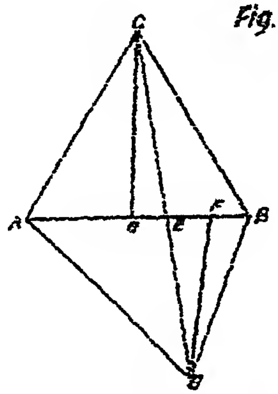


Fig. 63

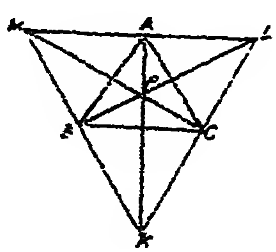
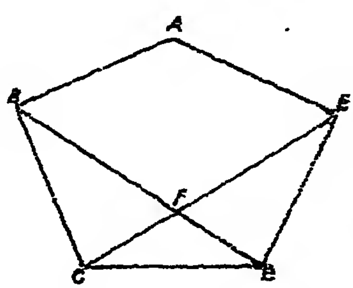


Fig. 64



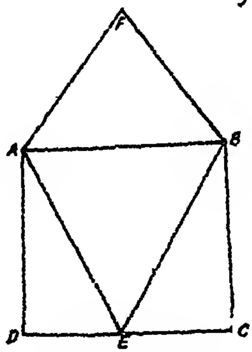


Fig.65

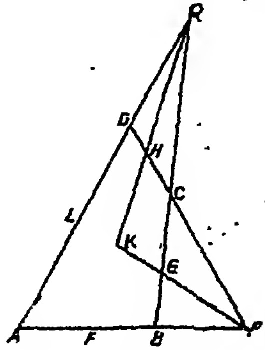


Fig. 66

2. Euclid I. 27.

In the two triangles ABC, AED, $AD=AB$ (hyp.); $AC=AE$ (hyp.); and ang. BAC=ang. EAD (I. 15) ∴ they are equal in all respects (I. 4) ∴ ang. EDA = ang. ABC; but these are alternate angles ∴ ED and BC are parallel (I. 27).

3. Euclid I. 44.

(See Fig 65.) Let AB be the given straight line, and K the given triangle.

On AB describe a parallelogram $ABCD$ equal to the given triangle K. In CD find a point E equidistant from A and B. Through A and B draw AF, BF parallel to BE and AE respectively (I. 31).

Now, the triangle K=the plm. $ABCD$=2 tr. AEB (I. 31).

Again, $AEBF$ is a plm. ∴ $AE=BF$, and $BE=FA$ (I. 34) and tr. AEB=tr. AFB (I. 8). Again, $AE=BE$ (constr.) ∴ $AEBF$ is a rhombus=2 tr. AEB=given tr. K.

4. Euclid II. 7.

Produce AB to D making $BD=BC$ (I. 3). Then four times rect. $AB.BC+AC^2$=square on AB and BC together (II. 8); but rect. $AB{\cdot}BC=AC^2$ (hyp.) ∴ $4AC^2+AC^2$=square on AB and BC together. ∴ the square on the line made up of AB and $BC=5AC^2$.

5. Euclid III. 21.

The converse of this:—Equal angles standing on the same base, and on the same side of it, have their vertices on an arc of a circle, of which the given base is the chord.

Let $ABCDEF$ be an equiangular polygon whose alternate sides are equal. Join AD, AC, BD. In the trs. ABC, BCD, $AB=CD$ (hyp.), BC is common to both and ang. ABC=ang. BCD (hyp.) ∴ the trs. are equal in all respects (I. 4).

∵ ang. BAC=ang. BDC ∴ a circle passes round the points $ABCD$. (converse of III. 21). Similarly it may be

proved that the circle passes round all the points of the equiangular polygon.

6. Euclid III. 32.

(See Fig 66.) Let $ABCD$ be the quadrilateral whose sides AB, BC, CD, DA touch the inscribed circle at F, G, H and L respectively. Let AB and DC be produced to P and AD, BC to Q. Let the bisectors of the angles at P and Q meet at K. Now PK can be proved perp. to the str. line HF and QK to LG; for if two tangents be drawn to a circle from an external point, the chord joining the points of contact is bisected at right angles by the str. lines joining the centre and the external point. Again, $ABCD$ is cyclic $\therefore$ PK, QK are at rt. angles to one another. (*Cf.* *S. F. E.* paper, 1896, Ques. 7.) $\therefore$ HF and LG are perp. to one another.

7. Euclid IV. 13.

The Bombay University

School Final Examination Papers

IN

MATHEMATICS,

WITH FULL SOLUTIONS OF ARITHMETIC

AND OF

Harder Questions in Algebra and Geometry

(From 1889 to 1897)

AND

AN APPENDIX

Comprising some typical examples in Arithmetic
and Algebra from Examination Papers
of various Universities.

University School Final Examination.

1889-90.

TUESDAY, 3RD DECEMBER.

[10 A. M. to 1 P. M.]

GOVIND VITHAL KURKARAY, B A.

FARDUNJI MANCHERJI DASTUR, M. A.

[The figures to the right indicate full marks.]

SECTION I.

1. What are *prime* and *composite* numbers? Resolve 1230, 10584 and 40125 into prime factors and thence find their Greatest Common Measure. 6

2. I bought a parcel of nuts at 49 for two pence. I divided the parcel into two equal parts, one of which I sold at the rate of 24 and the other at the rate of 25 nuts a penny. I spent and received an integral number of pence, but bought the least possible number of nuts. How many nuts did I buy, what did they cost me, and what did I gain on the transaction? 10

3. Two monkeys having stolen a pile of grapes and figs from a garden, are on the point of beginning their feasts when they see the injured owner approaching with a stick. At once they see that he will take $2\frac{1}{2}$ minutes to reach them. There are twice as many grapes as figs and one monkey finishes the latter at the rate of 15 a minute in four-fifths of the time and runs away; the other manages to eat the grapes just in time. If the first monkey had stopped to help the other till all were finished, find when they would have got away (*a*) if they eat grapes at equal rates (*b*) if the first monkey eats grapes at the same rate that he eats figs. 10

4. When fractions are to be added together or subtracted from one another, why is it necessary to convert them into fractions having a common denominator? 8

Simplify—

$$1\frac{5}{6} \text{ of } \frac{\frac{1}{3}+\frac{1}{5}+\frac{1}{4}}{2\frac{1}{2}-3\frac{1}{5}+4\frac{1}{4}} \times \frac{\frac{2\frac{1}{3}}{3\frac{1}{7}}+\frac{\frac{3}{16}}{1\frac{3}{8}}}{\frac{3}{16}\times\frac{4\frac{1}{2}}{3}} \text{ of Rs. } 13 \text{ } 8\text{as.}$$

5. Prove the rule for the division of decimals. 8

Find the value of—

$$\frac{\cdot 2}{37} \text{ of } \frac{8\cdot 3\dot{9} - 1\cdot\dot{1}\dot{8}}{5\cdot\dot{2}9\dot{7} \times 3\cdot 026\dot{6}} \text{ of £264 } 13s.\ 6\tfrac{1}{4}d.$$

Section II.

6. Explain why true discount is the interest on present worth. 13

The discount on a certain sum due 2 years hence is £63 17*s*., and the interest on the same sum for the same time is £71 16*s*. 7½*d*. : find the sum and the rate per cent. per annum.

7. The amount of £4,000 for a certain time at 5 per cent., per annum compound interest is £4,630 5*s*. : find the time. 14

8. The cost of carpeting a room whose length is twice its breadth, at 5*s*. a square yard, was £1 2*s*. 6*d*. and the painting of the walls at 9*d*. a square yard was £2 12*s*. 6*d* : find the height of the room. 11

9. A cubic foot of water weighs 1000 ounces : find the length of the inner edge of a cubical vessel which holds $1398665\frac{71}{512}$ ounces of water 7

10. When is stock said to be at *par*, at a *discount*, and at a *premium*? 13

At what price must a person invest in the 4 per cents, so that after paying 4 pies in the rupee income-tax he may receive 4½ per cent. on his money?

MONDAY, 9TH DECEMBER.

MATHEMATICS (VOLUNTARY)—PAPER I.

Algebra.

1. Define an *algebraical expression* and the *degree* of an expression. What is a *homogeneous* expression? 7

When $x=1$, $y=\frac{1}{2}$, $z=0$, find the value of—

$$\left[x-\left\{y-z-\left(2x-2y-\tfrac{1}{2}\overline{3z-y}\right)\right\}\right] \div \left\{x-y\div\left(z-\frac{1}{x}\right)\right\}. \qquad \underline{1\tfrac{1}{6}}.\ \textit{Ans.}$$

2. Find the factors of— 9

(i) $8x^3+729y^3$. $\underline{(2x+9y)(4x^2-18xy+81y^2)}$. *Ans.*

(ii) $x^5+7x^3-5x^2-35$. $\underline{(x^3-5)(x^2+7)}$. *Ans.*

(iii) $(x^2+4x)^2-2(x^2+4x)-15$.

$\underline{(x+1)(x+3)(x+5)(x-1)}$. *Ans.*

3. Multiply $x^2+y^2+z^2-xy-yz-zx$ by $x+y+z$ and from the result shew that 10

$$(x-y)^3+(y-z)^3+(z-x)^3-3(x-y)(y-z)(z-x)=0$$

$x^3+y^3+z^3-3xyz$). *Ans.* (*See* Solutions.)

4. Find the expression of lowest dimensions which is exactly divisible by 8

$$a^2b-b(b-c)^2,\ ac^2-(a-b)^2 \text{ and } (a+c)^2c-b^2c$$

$abc(a+c-b)(a+b-c)(c-a+b)(a+c-b)$. *Ans.*

5. Extract the square root of 10

$(2x+1)(2x+3)(2x+5)(2x+7)+16$. $4x^2+16x+11$ *Ans.*

6. Explain the meaning of a^{-x}. 11

If $x^2+y^2=a^2$, find the value of

$$\left\{\left(\frac{x}{a}\right)^2+\left(\frac{a}{y}\right)^{-2}\right\}^{\frac{1}{2}}\left\{(x^3+x^2y+xy^2+y^3)(a^2+2xy)^{-\frac{1}{2}}\right\}^{\frac{2}{3}}$$

$a^{\frac{4}{3}}$. *Ans.*

7. Shew that the value of a fraction is unaltered by multiplying its numerator and denominator by the same quantity. 8

Simplify

(i) $$\frac{\frac{a}{b}+\frac{b}{a}+2}{a+b}+\frac{\frac{a}{b}+\frac{b}{a}-2}{a-b}.$$ $\frac{2}{b}$. *Ans.*

(ii) $$\left(\frac{x^6+y^6}{x^6-y^6}\times\frac{x-y}{x+y}\right)\div\frac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^4}.$$ $\frac{x^2+y^2}{(x+y)^2}$. *Ans.*

8. Explain why it is necessary to change the sign of a term when it is transposed from one side of an equation to the other. 15

Solve—

(i) $2x-[3-\{4x+(x-1)\}-5]=8$. $x=1$. *Ans.*

(ii) $$\frac{1}{1-x+y}-\frac{1}{x+y-1}=\frac{2}{3};$$
$$\frac{1}{1-x+y}-\frac{1}{1-x-y}=\frac{4}{3}$$ $x=y=2$. *Ans.*

(iii) $x+\sqrt{(5x+10)}=8$. $x=18$ or 3. *Ans.*

9. A person allowed a pauper 3*s.* 6*d.* per week until the number of shillings left was the same as the number of weeks he had been paying; he then increased the allowance to 4*s.* per week; if the money lasted altogether was 35 weeks, find how many shillings he had given away. 7

126*s.* *Ans.*

10. If α, β are the roots of the equation $x^2+px+q=0$, shew that $\alpha+\beta=-p$, and $\alpha\beta=q$. 15

Prove that the roots of $4x^2+2px+q=0$ are $\frac{\alpha}{2}$ and $\frac{\beta}{2}$.

$x=\frac{p\pm\sqrt{p^2-4q}}{2}$ $\therefore$ If $\alpha=\frac{-p+\sqrt{p^2-4q}}{2}$ and $\beta=\frac{-p-\sqrt{p^2-4q}}{2}$,

Then $\alpha+\beta=\frac{-2p}{2}=-p$ and $\alpha\beta=\frac{p^2-(p^2-4q)}{4}=\frac{4q}{4}=q$.—*Q.E.D.*

Again, $4x^2+2px+q=0$ $\therefore x^2+\frac{p}{2}x+\frac{q}{4}=0$

Let the roots be A and B.

$$-(A+B)=\frac{p}{2}\ AB=\frac{q}{4}=\frac{-(\alpha+\beta)}{2}=\frac{\alpha\beta}{4}=-\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)\frac{\alpha}{2}\times\frac{\beta}{2}$$

$\therefore$ the roots are $\frac{\alpha}{2}$ and $\frac{\beta}{2}$.—*Q.E.D.*

MATHEMATICS (VOLUNTARY)—PAPER II.

Euclid.

1. Explain the terms, axiom, theorom, and corollary. Give Euclid's definition of a straight line. 7

Given the sum and difference of two straight lines, find their lengths. (*See* Solutions.)

2. Euc. I. 18. 10

Prove that the bisectors of adjacent supplementary angles are at right angles to one another. (*See* Solutions.)

3. Euc. I. 32. 10

Shew that isosceles triangles having equal vertical angles have equal base angles. (*See* Solutions.)

4. Euc. II. 12. 13

Prove that the sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the diagonal. (*See* Solutions.)

5. Euc. III. 28. 14

Two equal circles intersect in A and B and any straight line BCD is drawn to cut both circles in C, D. Prove that $AC=AD$. (*See* Solutions.)

6. Euc. III. 33. 16

Given the base, altitude, and vertical angle of a triangle, construct it. (*See* Solutions.)

7. If a quadrilateral figure be described about a circle, shew that the sum of the opposite sides will be equal to each other. (*See* Solutions.) 10

8. Euc. IV. 4. 12

Modify the construction in the above so that the circle may touch one side of the triangle and the other two sides produced. (*See* Solutions.)

9. Euc. IV. 14. 8

1890-91.

MONDAY, 1ST DECEMBER.

[10 A.M. TO 1 P.M.]

Arithmetic.

GOVIND VITHAL KURKARAY, B A.

KHAN BAHADUR BOMANJI SORABJI, L.C E., PH.D., F.C.S., A M I C.E., M.C G.B.

1. Simplify:

$$\frac{(\frac{1}{2}+\frac{1}{3})(\frac{1}{2}+\frac{1}{4})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{2}-\frac{1}{4})}+\frac{(\frac{1}{4}+\frac{1}{2})(\frac{1}{4}+\frac{1}{3})}{(\frac{1}{4}-\frac{1}{2})(\frac{1}{4}-\frac{1}{3})}-\frac{(\frac{1}{3}+\frac{1}{2})(\frac{1}{3}+\frac{1}{4})}{(\frac{1}{3}-\frac{1}{2})(\frac{1}{3}-\frac{1}{4})}.$$

2. How much short of unity is 7

$$\frac{\cdot 21}{\cdot 016} \text{ of } \frac{\cdot 025 \text{ of } 4\cdot 12}{5\cdot 72 - 3\cdot 175}\,?$$

3. A train is exactly 27 minutes in passing through a tunnel, 11,220 metres long: supposing a metre to be 39·39 inches; find the speed of the train in miles per hour. 7

4. The value of nineteen-twentieths of a property exceeds the value of eight-and-ninetieths of the same property by £17 11s. 2d.; find the value of the property. 7

5. Four men working 8 hours a day take 22 days to pave a road 440 yards long and 3 feet broad; how many days will 4 men two of whom work 8 hours and two 10 hours a day, take to pave a road 1575 yards long and 36 feet 6 inches broad? 10

6. If the discount on £678 9s., which is due at the end of a year and a half be £38 8s., what is the rate per cent. of simple interest? 8

7. How may the series of weights 1, 3, 3^2, 3^3, 3^4, &c., be employed to weigh 1,000 lbs? 7

8. Required the square root of 1208·8868379025 and the cube root of 160 103007. 9

9. Define the *prime factors* of a number. Resolve into prime factors 31,752 and 4,450,000. 10

10. Which is greater, $2^{\frac{1}{2}}$ or $3^{\frac{1}{3}}$? Show that $2^{\frac{1}{2}}$ is greater than

11. A, B, and C rent a house together for 2 years, at Rs. 1,500 per annum: A remains in it the whole time, B sixteen months, and C four and a half months during the occupancy of B. How much must each pay of the rent? 11

12 Sound travels at the rate of 1,140 feet per second. If a shot be fired from a ship, moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard at a place $14\frac{1}{4}$ miles off? 8

1890 91.

MATHEMATICS (VOLUNTARY)—PAPER I.

Euclid

MONDAY, 8TH DECEMBER.

10 A.M. TO 1 P.M.

[*The figures to the right indicate full marks.*]

1. Euc. I. 22. 7

2. The straight lines drawn from the angles of a triangle to the points of bisection of the opposite sides, meet at the same point. (*See* Solutions.) 10

3. Euc. I. 47. 8

4. If from the angular points of the squares described upon the sides of a right angled triangle, perpendiculars be let fall upon the hypotenuse produced, they will cut off equal segments, and the perpendiculars will together be equal to the hypotenuse. 9

5. Euc. II. 8. 9

6. Bisect a triangle by a straight line drawn from a given point in one of its sides. (*See* Solutions.) 9

7. Euc III. 17. 7

8. Euc. III. 37. 9

9. Of all triangles on the same base and between the same parallels the isosceles has the greatest vertical angle; and of all triangles on the same base and having the same vertical angle, the isosceles is the greatest. 12

10. If an equilateral triangle be inscribed in a circle and the adjacent arcs cut off by two of its sides, be bisected, the lines joining the points of bisection will be trisected by the sides. (*See* Solutions.) 10

11. Euc. IV. 11. 10

1890-91.

MONDAY, 8TH DECEMBER.

[2 P.M. TO 5 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER II

Algebra.

1. Multiply $x^{(m-1)n} - y^{(n-1)m}$ by $x^n - y^m$ and divide $x^4 + y^4$ by $x^2 - xy\sqrt{2} + y^2$. 7

(a) $x^{mn} - x^n y^{(n-1)m} - x^{(m-1)n} y^m + y^{mn}$. *Ans.*; (b) $x^2 + \sqrt{2}xy + y^2$. *Ans.*

2. If $x + y + z = 0$, prove that $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy} = 3$; also if $ax + by = m$, $bx - ay = n$, and $a^2 = 1 - b^2$, then $x^2 + y^2 = m^2 + n^2$. 13

(Proved, *Vide* Solutions)

3. Simplify— 4

$$\frac{1}{(a-b)(b-c)} + \frac{1}{(b-c)(c-a)} + \frac{1}{(c-a)(a-b)}.$$ 0. *Ans.*

4. Shew that— 10

$$\left(\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{z-x}\right)^2 = \frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2}.$$

(Proved)

5. Find the factors of— 7

$a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$. $(a+b)(b+c)(c+a)$. *Ans.*

6. Find K when $(x+3a)(x-a)(x-3a)(x+a) + K$ is a perfect square. $K = 16a^4$. *Ans.* 5

7. Find the square root of— 11

$$a^2x^2 + 6ac + \frac{12bc}{x} + 4b(ax+b) + \frac{9c^2}{x^2}.$$ $ax + 2b + \frac{3c}{x}$. *Ans.*

And the cube root of—

$$x^3 - \frac{1}{x^3} - 3x^2 - \frac{3}{x^2} + 5.$$ $x - 1 - \frac{1}{x}$. *Ans.*

8. Prove that the sum of any positive quantity and its reciprocal is never less than 2. (Proved). 7

9. Solve the equations— 15

(i) $4x^2 - 6x + 3\sqrt{2x^2 - 3x + 7} = 30$. $x = 3$ or $-1\frac{1}{2}$ or $\frac{3 \pm \sqrt{195}}{4}$. *Ans.*

(ii) $xy = a$, $xz = b$, $xu = c$, $xyzu = d$.

$x = \pm\sqrt{\frac{abc}{d}}$, $y = \pm a\sqrt{\frac{d}{abc}}$; $z = \pm b\sqrt{\frac{d}{abc}}$; $u = \pm c\sqrt{\frac{d}{abc}}$. *Ans.*

10. If $x_1 . x_2$ are the roots of the equation $ax^2+bx+c=0$, shew that $\frac{x_1}{x_2}+\frac{x_2}{x_1}=\frac{b^2-2ac}{ac}$. 11

Since x_1 and x_2 are the roots of the equation $\therefore x_1+x_2 = -\frac{b}{a}$ and $x_1x_2=\frac{c}{a}$; $\frac{x_1}{x_2}+\frac{x_2}{x_1}=\frac{x_1^2+x_2^2}{x_1x_2}=\frac{x_1^2+2x_1x_2+x_2^2-2x_1x_2}{x_1x_2}$

$=\frac{(x_1+x_2)^2-2x_1x_2}{x_1x_2}$; $\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c}{a}}=\frac{\frac{b^2-2ac}{a^2}}{\frac{c}{a}}=\frac{b^2-2ac}{ac}$.—Q.E.D.

11. A boatman rows to a place 120 miles distant and back in 35 hours; it is found that he can row 4 miles with the current in the same time as 3 miles against the current. Find the current's rate. 10

Rate of current 1 mile per hour
Rate of rowing 7 miles per hour } . Ans.

1891-92.

MONDAY, 7TH DECEMBER.

[10 A. M. TO 1 P. M.]

Arithmetic.

GOVIND VITHAL KURKARAY, B.A.
RAMKRISHNA SAKHARAM ATHAVALE, M.A.

1. Subtract $\cdot\dot{2}8571\dot{4}$ of 13s. 6d. from the sum of ·75 guineas and $\cdot\dot{2}14285\dot{7}$ of 13s. 4d. 7

2. Two friends during a walk take steps of $2\frac{2}{11}$ and $2\frac{2}{21}$ feet: if they start in step, how far will they have walked before they are in step again and how many steps will each have taken? 9

3. A journey of 560 miles was made by rail, steamer, and coach. The distance by coach was one-fourth, and the distance by sea three-fourths of that by rail. The fare per mile by coach was double, and by sea four-fifths of that by rail. What was the expense of the whole journey, railway fare being $1\cdot\dot{5}7142\dot{8}d.$ per mile? 10

4. Two persons agreed to pay £81 for the use of a certain tract of pasture meadows for 10 months; the first put in 27 oxen for three months, the second 270 sheep for seven months: supposing the feed equally good throughout and that 3 oxen eat as much as 11 sheep, how much of the rent ought each to pay? 10

5. What sum would in three years amount to £2,811 18s. at compound interest, if the rate were 3 per cent. per annum for the first year, 4 per cent. per annum for the second year, and 5 per cent. per annum for the third year? 10

6. Allowing interest at $3\frac{1}{2}$ per cent. per annum, what sum of money will now discharge a debt of £[illegible]3 11*s.* 3*d.* which becomes due 1[illegible] months hence? 9

7. A person having to pay £102 3*s.* 9*d.* two years hence, invests a certain sum in the three per cent. consols, and also an equal sum next year together with the interest already received. Supposing the price of consols to remain throughout at 96, what must be the sum invested on each occasion so that there may be just sufficient to pay the debt at the proper time? 11

8. A cubic foot of water weighs $62\frac{1}{2}$ lbs and a room 18 feet 9 inches by 13 feet 4 inches is flooded to a depth of 2 inches; what is the weight of water in the room? 7

9. A merchant sells 49 quarters of corn at a profit of 7 per cent and 8[illegible] quarters at a profit of 1[illegible] per cent; if he had sold it all at a profit of 9 per cent., he would have received £2 10*s.* 9*d.* less than he actually did: what was the price he paid for the corn? 11

10 A certain number of persons agree to subscribe as many guineas each as there are subscribers in all; the whole subscription being £[illegible],047,901 1*s.*: how many subscribers were there? 8

11. A passenger train going 41 miles an hour and 431 feet long, overtakes a goods train on a parallel line of rails; the goods train is going 28 miles an hour and is 71[illegible] feet long; how long does the passenger train take in passing the other? 8

1891-92.

FRIDAY, 11TH DECEMBER.

[2 P. M. TO 5 P. M.]

MATHEMATICS (VOLUNTARY)—PAPER I.

Algebra.

1. Resolve the following expressions into their simplest factors:— 7

(i) $x^4-3a^2x^2+a^4$. $\underline{(x^2+a^2-ax)(x^2+a^2+ax)}$. *Ans.*

(ii) $x^3+z^3-3xz+1$. $\underline{(x+z+1)(x^2+z^2+1-xz-x-z)}$. *Ans.*

(iii) $x^2+5xy-24y^2+x-3y$. $\underline{(x-3y)(x+8y+1)}$. *Ans.*

2. Divide $2x^3-6x+5$ by $\sqrt[3]{2}\,x+\sqrt[3]{4}+1$. 9

3. If $ax+by=1$, and $ax^2+by^2=\dfrac{1}{a+b}$, prove that $ax^n+by^n=(a+b)^{1-n}$. (*Vide* Solutions.) 10

4. Find the value of $\dfrac{x+2a}{2b-x}+\dfrac{x-2a}{2b+x}+\dfrac{4ab}{x^2-4b^2}$ when $x=\dfrac{ab}{a+b}$ 10
(*Vide* Solutions).

5. Find the Greatest Common Measure of the numerator and denominator of the fraction— 10

$$\frac{3x^2-(4a+2b)x+a^2+2ab}{x^3-(2a+b)x^2+(a^2+2ab)x-a^2b}$$

and reduce it to its lowest term.

G. C. M. $=(x-a)$; the fraction $=\frac{3x-a-2b}{(x-a)(x-b)}$. *Ans.*

6. Find the square root of— 7

$a^6+\frac{1}{a^6}-6\left(a^4+\frac{1}{a^4}\right)+15\left(a^2+\frac{1}{a^2}\right)-20$. $a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)$. *Ans.*

7. Find the cube root of— 7

$1-6x+21x^2-44x^3+63x^4-54x^5+27x^6$. $1-2x+3x^2$. *Ans.*

8. If p be the difference between any fraction and its reciprocal, q the difference between the square of the same fraction and the square of its reciprocal, shew that $p^2(p^2+4)=q^2$. (*Vide* Solutions). 9

9. The trinomial ax^2+bx+c becomes 8, 22, 42, respectively, when x becomes 2, 3, 4, what does it become when $x=-\frac{1}{8}$. (*Vide* Solutions) 12

10. Solve the following equations:— 9

(i) $x^3+\frac{1}{x^3}=\frac{65}{8}$. $x=2$ or $\frac{1}{2}$. *Ans.*

(ii) $2^x=8^{y+1}$, $9^y=3^{x-9}$. $x=21$; $y=6$. *Ans.*

11. A person selling a horse for £72 finds his loss per cent. is one-eighth of the number of pounds that he paid for the horse: what was the cost price? £80 or £720. *Ans.* 10

1891-92.

SATURDAY, 12TH DECEMBER.

[10 A.M. TO 1 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER II.

Euclid.

1. Euc. I. 28. 7

2. If a quadrilateral figure have two sides parallel, and the parallel sides be bisected the line joining the point of bisection shall pass through the point in which the diagonals cut one another. 13

3. Euc. II. 4. 7

4. Euc. III 11.

5. Euc. III. 31. 10

6. If two circles touch each other internally, any chord of the greater circle which touches the less shall be divided at the point of its contact into segments which subtend equal angles at the point of contact of the two circles. 12

7. Prove that the perpendiculars drawn from the angles of an acute angled triangle on the opposite sides meet at the same point. (*See* Solutions.) 13

8. Describe a circle about a given triangle. 8

9. If O be the centre of the circle inscribed in the triangle ABC, and AO be produced to meet the circumscribed circle at F, shew that FB, FO, FC, are all equal. (*See* Solutions.) 12

10. Euc. IV. 15. 11

1892-93.

MONDAY, 5TH DECEMBER.

[10 A. M. to 1 P. M.]

Arithmetic.

GOVIND VITHAL KURKARAY, B.A.
RAMKRISHNA SAKHARAM ATHAVALE, M.A.

1. Simplify— 7

(i) $\dfrac{1484915}{4946403}$.

(ii) $\left\{\frac{1}{7}-\frac{1}{9}-\frac{1}{11}+\frac{1}{13}\right\}\left\{\frac{117}{64}+\frac{9}{26}\right\}$

2. Find the difference between $\cdot 027$ of a guinea and $\cdot 0291\dot{6}$ of 15*s.*; and express the difference as a decimal of 14*s.* 8*d.* 8

3. The length of $\frac{1}{360}$ of the earth's circumference is about $69\frac{1}{12}$ miles. The circumference of a circle is equal to $3\frac{1}{7}$ times the diameter. Find the earth's diameter. 7

4. The premium on my life insurance is 15 per cent. of my income; after deducting this I pay income-tax at 5 pies in the rupee and my net income is then Rs. 910 10a. 5p.: what is my gross income? 7

5. A bankrupt's estate amounts to £455 1s. $6\frac{3}{4}$*d.* and his debts to £937 10*s.* What can he pay in the pound and what will a creditor lose on a debt of £114? 8

6. If the discount on a sum due at the end of 3 years be $\frac{220}{221}$ of the simple interest, find the rate per cent. 9

7. If 15 men, 12 women and 9 boys can complete a piece of work in 50 days, what time would 9 men, 15 women and 18 boys take to do four times as much, the parts done by each in the same time being as the numbers 3, 2, 1? 10

8. I have £932 5s. 8*d*. to invest. I want to produce a total income of £36 12*s*. in three equal portions. For this purpose I invest some in 3 per cent. stock at 95 and some in 4 per cent. stock at 119. What percentage must I obtain on the remainder? 9

9. Ten loads of gravel are laid evenly on a path. 80 yards long and $4\frac{1}{2}$ feet wide. Each load contains a cubic yard. Find in inches the thickness of the layer. 10

10. Prove that the difference between the numbers $\cdot\dot{3}0864197\dot{5}$ and $\cdot\dot{1}9753086\dot{4}$ is equal to the difference between their square roots. 10

11. Express in inches the cube root of 41·421736 solid feet. 6

12. A train 88 yards long overtook a man walking along the line at the rate of 4 miles an hour and passed him completely in 10 seconds; it afterwards overtook another man and passed him in 9 seconds; at what rate per hour was the second man walking? 9

FRIDAY, 9TH DECEMBER.

[2 P.M. to 5 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER I.

Algebra.

1. Prove without actual division that $5x^4+6x^3-7x^2-8x-480$ is divisible by $x-3$. (*See* Solutions.) 7

2. Divide $x^2(4n^3+16n^2+21n+9)+2xy(6n^3+23n^2+29n+12)+y^2(8n^3+28n^2+32n+12)$ by $(n+1)(2n+3)(x+2y)$. (Solved). 10

3. Simplify— 7

$$\frac{x^2}{ab}+\frac{(x-a)^2}{a(a-b)}-\frac{(x-b)^2}{b(a-b)}.$$ $\underline{1.}$ *Ans.*

4. Find the G. C. M. of— 7

$x^4+2x^3+3x^2+2x+1$ and $x^5+5x^4+2x^3-4x-1$. $\underline{x^2+x+1}$. *Ans.*

5. Extract the square root of—

$(x+x^{-1})-2(x^{\frac{1}{2}}+x^{-\frac{1}{2}})+3$. $\underline{x^{\frac{1}{2}}+x^{-\frac{1}{2}}-1}$. *Ans.*

6. Find the cube root of 9

$27x^6-108x^5+171x^4-136x^3+57x^2-12x+1$. $\underline{x^2-4x+1}$. *Ans.*

7. Shew that 10

$$(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)$$

is an exact square. (*See* Solutions.)

8. If $a^b = b^a$, shew that $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$. (*Vide* Solutions.)

9. Solve the equations— 11

(i) $\frac{x-4a}{x-3a} + \frac{x-5a}{x-4a} = \frac{x+6a}{x-4a} + \frac{x+8a}{x-7a}$. $\underline{\frac{69}{20}a}$. *Ans.*

(ii) $x^2 + \frac{1}{x^2} + x + \frac{1}{x} = 4$. $\underline{x=1 \text{ or } x = \frac{-3 \pm \sqrt{5}}{2}}$. *Ans.*

10. If α and β are the roots of the equation $x^2 - px + q = 0$, prove that the expression $x^2 - px + q$ is identical with $(x-\alpha)(x-\beta)$. 7

11. If of two numbers of two digits the sum is $\frac{11}{7}$ of the difference, and the numbers consist of the same digits reversed, find the numbers. <u>81, 18.</u> *Ans.* 7

12. A man bought a number of pieces of cloth for £32 and sold them at £4 10*s* each, thus gaining as much as one piece cost him. How many pieces were there? <u>8 pieces.</u> *Ans.* 9

SATURDAY, 10TH DECEMBER.

[10 A.M. to 1 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER II.

Euclid.

1. Euc. I. 32. 8

2. In a right-angled triangle, if one of the acute angles be double the other, prove that the square on the greater of the sides containing the right angle is three times the square on the lesser side. 9

3. In a given straight line, find a point such that the perpendiculars drawn from it to two given straight lines which intersect shall be equal. (*See* Solutions). 9

4. Euc. II. 9. 10

5. Euc. III. 18. 7

6. Euc. III. 32.

7. Two circles touch each other externally at A; shew that the square on the common tangent (not at A) is equal to the rectangle contained by the diameters of the circles. 10

8. Two tangents are drawn to a circle at the opposite extremities of a diameter which cut off from a third tangent a portion AB; if C be the centre of the circle, shew that ACB is a right angle. (*See* Solutions.) 10

Euc. IV. 2. 8

10. Circumscribe a regular pentagon about a given circle. 9

11. If from any point in the circumference of a given circle, straight lines be drawn to the four angular points of an inscribed square, the sum of the squares on the four straight lines is double the square on the diameter. 12

1893 94.

MONDAY, 4TH DECEMBER.

[10 A.M. TO 1 P. M.]

Arithmetic.

GOVIND VITHAL KURKURAY, B.A.[1]

RAMKRISHNA SAKHARAM ATHAVALE, M.A.

1. Simplify :—

$$\text{(i)}\quad \frac{\frac{1}{7}-\frac{1}{10}+\frac{4}{11}-\frac{1}{3}}{9-(\frac{1}{7}-\frac{1}{10})(\frac{4}{11}-\frac{1}{3})}$$

$$\text{(ii)}\quad (2\tfrac{3}{8}\text{ of }\tfrac{7}{13})\div(2{\cdot}59\times{\cdot}3\dot{1}4\dot{8}).$$

2. If the third satellite of Jupiter be ·0000835 of the mass of the planet and ·02347 of the mass of the Earth, find the mass of the Earth in terms of the mass of Jupiter. 7

3. A steam-engine of $4\frac{1}{2}$ horse-power, working 51 days of 6 hours, consumes 25 tons of coal; how much coal will be consumed by a 17 horse-power engine working at the same work for 3 days of $8\frac{1}{2}$ hours? 8

4. At compound interest, the interest on a certain sum for the first year is Rs. 145 13a. 4p., and for the second year Rs. 153 2a.; find the interest for the third year. 8

5. A man built a house and sold it for £800, losing 4 per cent. by the transaction; what did it cost him to build? If he had not sold it, at what rent would he have had to let it, so as to make 6 per cent. on his outlay? 8

6. A hare is 500 leaps of himself before a greyhound and takes in the same time 4 leaps to his 3, but 2 of the greyhound's leaps are as much as 3 of the hare's; in how many leaps will the greyhound catch the hare? 10

7. Two rods, each a foot long, are divided,—the one into 10 equal parts, the other into 11 equal parts. They are placed side by side, so that the seventh points of division in each coincide. What is the distance between their extremities? 10

8. Two trains, 92 and 82 feet long, respectively, move with uniform velocities on parallel rails in opposite directions and pass each other in $1\frac{1}{2}$ seconds; when moving in the same direc- 10

tion, with the same velocities as before, the faster train passes the other in 6 seconds; find the rate at which each moves.

9. Express $\sqrt[3]{(\cdot 0448)}$ as a decimal of $\sqrt[3]{(\cdot 0875)}$. 10

Prove that $\sqrt[3]{(\cdot 0\dot{3}\dot{7})} = \cdot\dot{3}$.

10. A snail climbs over a slippery wall 20 feet high; he crawls as fast as he can and ascends 3 feet in a day and sleeps at night, during which he slips 2 feet: the instant he arrives at the top, he commences to descend the other side, crawling by day as fast as he can and resting by night as before. How long will he take to arrive at the bottom on the other side? 10

11. The difference between the incomes derived from investing a certain sum in 5 per cent. stock at 127 and $5\frac{1}{2}$ per cent. stock at 135 is £4 14s. Find the amount invested and the income resulting from each investment. 10

1893-94.

FRIDAY, 8TH DECEMBER.

[2 P. M. to 5 P. M.]

MATHEMATICS (VOLUNTARY)—PAPER I.

Algebra.

1. Find the factors of:—

(i) $a^4+4b^4+2a^2b^2$.

(ii) $a^3(b-c)+b^3(c-a)+c^3(a-b)$. (See Solutions).

2. Shew that the sum of the co-efficients of x^3 and x^9 in the cube of $1-x+x^3+x^4$ is zero. 7

3. Determine the values of c which make $3x^2+x-4$ and $12x^2-5x+c$ have a common measure. ($c=-7$ or -28). Ans. 7

4. Simplify— 7

$$\frac{2x^3+29x^2+63x+24}{2x^3+25x^2+11x-12}. \qquad \frac{2x^2+5x+2}{2x^2+x-1}. \text{ Ans.}$$

5. If $x+y+z-1 = \sqrt{2(1-x)(1-y)(1-z)}$, shew that $x^2+y^2+z^2-1+2xyz=0$. (Vide Solutions). 8

6. Prove that— 8

$$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=(a+b)(b+c)(c+a).$$

(Vide Solutions).

7. Shew that $x^{2n}+a^{2n}+ax(x^{2n-2}+a^{2n-2})-a^{n-1}x^{n-1}(x+a)^2$ is divisible by x^2-a^2 when n is an even integer. 10

8. Extract the square root of— 8

$x^{\frac{3}{2}}-8x^{\frac{5}{4}}+20x-22x^{\frac{3}{4}}+28x^{\frac{1}{2}}-12x^{\frac{1}{4}}+9$. $\underline{x^{\frac{3}{4}}-4x^{\frac{1}{2}}+2x^{\frac{1}{4}}-3}$. *Ans.*

9. Find the cube root of $8x^6+12x^5+18x^4+13x^3+9x^2+3x+1$. 9

Hence solve the equation $8x^6+12x^5+18x^4+13x^3+9x^2+3x=7$. (*Vide* Solutions.)

10. If $x+y+z=0$, prove that 7

$x(y-z)^2+y(z-x)^2+z(x-y)^2+9xyz=0$. (*Vide* Solutions.)

11. Find a number of two digits, such that, if it be divided by the product of the digits, the quotient is 2; and if 27 be added to the number, the order of the digits is reversed. 36. *Ans.* 10

12. Shew that the equation $ax^2+bx+c=0$ cannot have more than two roots. 12

Make the quadratic whose roots are $\dfrac{\sqrt{17}+\sqrt{5}}{3}$ and $\dfrac{4}{\sqrt{17}+\sqrt{5}}$ and reduce it to its simplest form. $\underline{x^2-2\sqrt{17}x+4=0}$. *Ans.*

1893-94

SATURDAY, 9TH DECEMBER.

[10 A.M. to 1 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER II.

Euclid.

1. Euc. I. 44. 8

2. Find a point in the diagonal of a square produced, from which if a straight line be drawn parallel to any side of the square and meeting another side produced, it will form, together with the produced diagonal and the produced side, a triangle equal to the square. 9

3. If a straight line be drawn from one of the acute angles of a right-angled triangle bisecting the opposite side, the square upon that line is less than the square upon the hypotenuse by three times the square upon half the line bisected. 9

4. Euc. II. 11. 9

5. The squares on the diagonals of a trapezium are together equal to the sum of the squares on its two oblique sides with twice the rectangle contained by its parallel sides. (*See* Solutions.) 12

6. Euc. III. 22. 7

7. $ABCD$ is a quadrilateral inscribed in a circle, and the opposite sides AB, DC are produced to meet at P, and CB, DA to meet at Q; if the circles circumscribed about the triangles PBC and QAB intersect at R, show that P, R, Q are collinear. (*See* Solutions.) 14

8. If from any point without a circle a tangent and a secant be drawn, the rectangle contained by the whole secant and the part of it without the circle shall be equal to the square on the tangent. 10

9. Euc. IV. 10. 12

10. Euc. IV. 10. 10

1894.

MONDAY, 3RD DECEMBER.

Arithmetic.

RAMKRISHNA SAKHARAM ATHVALE, M.A.

K. G. DESHPANDE, B.A.

1. Simplify— 12

$$\frac{1\cdot\dot{1}8\dot{5} - \cdot\dot{6}\dot{1}}{19\cdot5 - 12\cdot8} \times \frac{211}{212} \div \frac{1\cdot55 \times \cdot\dot{0}3\dot{7}}{4\cdot\dot{3}\dot{0}}.$$

In an examination paper a question was printed thus:— "Add together $\frac{1}{14\cdot\dot{6}}$, $\frac{1}{19\cdot25}$, $\frac{3\cdot4}{\quad}$, $\frac{1}{13\cdot75}$." The answer required was $1\frac{1}{3}$. Required the missing denominator.

2. What is meant by the G. C. M. of two or more fractions? Find the G. C. M. of $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$. 6

3. Express the square root of $4\cdot3\dot{5}$ qrs., as the decimal of a ton. 8

4. The minute hand of a clock overtakes the hour hand at intervals of 64 minutes of true time. How much a day does the clock gain? 14

5. The outer and inner boundaries of a gravel path are squares, and the path is 4 ft. wide. The side of the square enclosed by the path is 50 yards. How much would it cost to gravel the path at 1*s*. 6*d*. a square yard? 10

6. A railway train 120ft. long starting from a station passes completely through a tunnel in 5 seconds. Another train 90 ft. long starting an hour later from the same station passes completely through the tunnel in 3 seconds and overtakes the first train in two hours. Find the length of the tunnel and the velocities of trains per hour. 12

7. A certain sum is invested at compound interest. The interest for the first year is £60: for the second year £63. At the end of the second year a portion of the money is with- 12

drawn, and the interest for the third year is thus reduced to £56 3s. Find the amount withdrawn.

8. *A* starts in trade with a capital of £624; after 4 months *B* joins him with £1,728; after two months more *C* joins them with £312. At the end of the year from the commencement the profits are £316 6s. 8*d*. How should the profits be divided? 10

9. The real cost of an article is 65 per cent. of the price at which it is marked for sale. It is, however, sold at a trade discount of 25 per cent. How much does the seller gain per cent? 8

10. The investment of a certain sum at 3 per cent. produces an income of £501 15s. A portion of the capital is withdrawn and invested at 5 per cent. If this produces the same income as the whole formerly did, find the amount of income drived from the whole when the new investment has been effected. 10

FRIDAY, 7TH DECEMBER.

Algebra (Voluntary).

1. Write down the co-efficient of x^3 in the product of— 8

$$ax^7+bx^5-cx^4+cx^2+fx-b \text{ by } ha^4+la^3+m,$$

without actual multiplication, and state how the result is obtained. $hf+cl+mb$. *Ans.*

2. Simplify —

$$\frac{a+b}{ax+by}+\frac{a-b}{ax-by}+\frac{2(a^2x+b^2y)}{a^2x^2+b^2y^2}-\frac{4(a^4x^3-b^4y^3)}{a^4x^4-b^4y^4}. \quad 0. \quad \textit{Ans.}$$

3. If an expression involving x vanish where a is put for x wherever x occurs, then the expression is exactly divisible by $x-a$. Shew that $x^{2n}-a^{2n}$ is divisible by $x\pm a$ when n is a positive integer. 10

4. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$, prove that $\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{1}{a^3+b^3+c^3}$. 8
(*Vide* Solutions.)

5. The *H. C. F.* of two expressions is $x-7$ and their *L.C.M.* is $x^3-10x^2+11x+70$; one of the expressions is $x^2-5x-14$; find the other. (*See* Solutions.) 8

6. Solve— 10

(i) $\frac{x-a}{bc}+\frac{x-b}{ca}+\frac{x-c}{ab}=\frac{2}{a}+\frac{2}{b}+\frac{2}{c}$. $x=a+b+c$. *Ans.*

(ii) $x(x-1)(x-2)(x-3)=9.8.7.6$, $9, -6, = \frac{3\pm\sqrt{-217}}{2}$. *Ans.*

7. Find the cube root of 8

$1-9x^2+33x^4-63x^6+66x^8-36x^{10}+8x^{12}$. $1-3x^2+2x^4$. *Ans.*

8. Solve the equation—

$(x-a)^2+(y-b)^2=0$ (See Solutions.)

The sum of the squares of the ages of 2 persons is 32 times the sum of the ages eight years ago. What are their ages? (Their ages are equal, *i.e.*, they are each 16 years old. *Ans.*

9. *A* and *B* walk from *P* to *Q* and back. *A* starts one hour after *B*, overtakes him 2 miles from *Q*, meets him 32 minutes afterwards, and arrives at *P* when *B* is four miles off. Find the distance from *P* to *Q*. 12

10. (i) Shew that if the roots of the equation— 14

$(b^2+b'^2)x^2+2(ab+a'b')x+a^2+a'^2=0,$

be real, they will be equal.

(ii) If the roots of the equation $px^2+qx+1=0$ be equal to $\frac{1}{p}$ and $\frac{1}{q}$, what are their numerical values? $q=1,\ p=-2$. *Ans.*

SATURDAY, 8TH DECEMBER.

Euclid.

1. Euc. I. 41. 7

P is a point within a parallelogram *ABCD*. Show that the sum of the triangles *APD* and *BPC* is half the parallelogram *ABCD*. (*See* Solutions.) 7

2. Define a median. 10

BY and *CZ*, two medians of a triangle *ABC*, intersect at *O*, Prove that $BO=2OY$. (*See* Solutions.)

3. Euc. II. 13. 8

The base *BC* of a triangle *ABC* is bisected in *P*. Shew that square on $AB+$ square on AC, $=2$ square on $BP+$ square on AC. (*See* Solutions.)

4. Euc. III. 3.

In a circle of radius 5 feet, *P* is a point 3 feet away from the centre. Draw a chord that shall be bisected at P, and find its length. (*See* Solutions.)

5. Shew that it is impossible to solder together two unequal circular rings at more than two points. 8

6. Euc. III. 31. 10

AB is a diameter of a semicircle. *D* and *E* are any two points in the circumference. If the chords joining *A* and *B* with *D* and *E* each way intersect at *F* and *G*, prove that *FG* produced is at right angles to *AB*. 10

7. The base BC of a triangle ABC. touches the inscribed circle at D and the corresponding escribed circle at D'. Shew that DD' is equal to the difference of AB and AC. 12

8. Inscribe a regular pentagon in a given circle. 12

1895

MONDAY, 2ND DECEMBER.

Arithmetic.

K. G. DESHPANDE, B.A.
LIEUT. A. J. PRILE, R A.

1. Simplify— 6

(a) $$\frac{5\frac{1}{2} \text{ of } \cdot\dot{2} \times 2\cdot\dot{5}7142\dot{8} - 1 \div (\frac{1}{5} + \frac{1}{2})}{1 - 1\frac{3}{4} \text{ of } \left\{ \frac{1}{2} + \frac{1}{2} \text{ of } \dfrac{\frac{1}{20}}{\frac{1}{7} \text{ of } 1\cdot05} \right\}} \div \frac{1\cdot\dot{2}8571\dot{4}}{\cdot2\dot{1}4285\dot{7}}.$$

(b) $$\frac{(\cdot005185)^2}{(18\cdot5)^2}.$$

2. Two punkhas swing side by side. One makes 144 complete swings in 5 minutes, the other makes 208 complete swings in 7 minutes. They start swinging together: how long will it be before they start on their swing together again, and how many swings will each punkha have made? 10

3. A cubical tank holds 597160402·461 cubic inches of water: find to 5 places of decimal what decimal of a mile is the side of the tank? 10

4. Two kinds of sherry are mixed in one cask in the proportion of 4 : 1 and in another cask in the proportion of 6 : 5 respectively. Find how many gallons should be taken from each cask to get a mixture of 10 gallons containing 7 gallons of the first sherry and 3 of the other. 12

5. If 56 Indian workmen each earning 6 annas a day can do the same piece of work in 25 days that takes 20 English workmen each earning 3s. 6d. a day, 15 days to complete. Taking the value of the rupee at 1s. 1d. determine which class of workmen it is more profitable to employ. 10

If a piece of work done by Indian workmen cost Rs. 3,000 what would be the cost in £ of the same work done by English workmen.

6. A cistern has 3 pipes. The first two can fill it in 4 and 5 hours respectively, the third can empty it in 2 hours; if the pipes be open in order at 1, 2, 3 A. M., when will the cistern be empty. 10

7. A person borrows £1,261 at 5 per cent. compound interest; he wishes to pay out the loan with 3 equal yearly instalments, commencing with the end of the first year. What ought he to pay yearly to effect this? 12

8. A person invests £10,000 in the 3 per cents. at 75 and when 8
they rise to 78 he sells and invests the proceeds in shares at £208 which pay £8 a share. Find the difference in the income.

9. A and B rent a pasture for £132 a year. A puts in 200 12
sheep and B puts in 160 sheep. At the end of 6 months they dispose of $\frac{1}{2}$ their stock and allow another man C to put in 120 sheep. What are their shares of the rent.

10. A tradesman bought goods and sold $\frac{2}{5}$ at a profit of 6 per 9
cent. On the remainder he got 10 per cent.; his total profit was Rs. 114. What did he lay out?

FRIDAY, 6TH DECEMBER.

[2 P.M. TO 5 P.M.]

Algebra.

R. G. DESHPANDE, B.A.

LIEUT. A. J. PEILE, R.A.

1. Simplify—

$$\left(\frac{y^2-yz+z^2}{x}+\frac{x^2}{y+z}-\frac{3}{\frac{1}{y}+\frac{1}{z}}\right)\frac{\frac{9}{y}+\frac{2}{z}}{\frac{1}{yz}+\frac{1}{zx}+\frac{1}{xy}}+(x+y+z)^3.$$

$3(x^2+y^2+z^2)$. *Ans.*

2. If $a+b=x$, $a-b=y$, express a^3-b^3 in the terms of x and y. Hence find the value of $(5002)^3-(4998)^3$. (*See* Solutions.)

3. If $a+\frac{1}{b}=b+\frac{1}{c}=1$, prove that $c+\frac{1}{a}=1$ and $abc+1=0$ (*See* Solutions.)

4. Find by resolution into factors the H. C. F. of—

x^6-y^6, x^8-y^8, $x^{12}-y^{12}$. x^2-y^2. *Ans.*

And the L.C.M. of—

$x^3+8ax^2+20a^2x+16a^3$, $x^3-12a^2x-16a^3$, $x^3+2ax^2-4a^2x-8a^3$.

$(x^2-16a^2)(x^2-4a^2)(x+2a)$. *Ans.*

5. If m and n be any positive integers, prove that $(x^m)^n=x^{mn}$.

Find the 6th root of—

$1+6a+9a^2-10a^3-30a^4+6a^5+41a^6-6a^7-30a^8+10a^9+9a^{10}-6a^{11}+a^{12}$. $1+a-a^2$. *Ans.*

6. Solve the equations :—

(i) $\frac{a}{x+a}+\frac{b}{x+b}=\frac{a-c}{x+a-c}+\frac{b+c}{x+b+c}$. $x=0$ or $-\frac{(a+b)}{2}$. *Ans.*

(ii) $(y-z)(x+y)=22$; $(x+y)(x-y)=33$; $(x-y)(y-z)=6$.

$x=7$; $y=4$; $z=2$. *Ans.*

7. If α and β be the roots of the equation $ax^2+bx+c=0$, find the value of $\alpha+\beta$ and $\alpha\beta$ in terms of the co-efficients.

If α and β be the roots of the equation $3x^2-6x+2=0$, form the equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. $3x^2-18x+2=0$. *Ans.*

8. Find when $x^4+bx^3+cx^2+dx+e$ is a perfect square. Hence, or otherwise, find the value of a which will make $x^4+2x^3+6x^2+4x+a$ a perfect square. $a=1$ *Ans.*

9. Two travellers, *A* and *B*, set out from two places, *P* and *Q*, respectively and travel so as to meet. When they meet, it is found that *A* has travelled 30 miles more than *B* and that *A* will reach *Q* in 4 days and *B* will reach *P* in 9 days after their meeting. Find the distance between *P* and *Q*. Distance = 150 miles; rate of walking: *A* = 15 miles, *B* = 10 miles, per day. *Ans.*

10. A number consists of two digits whose sum is 8. A second number is obtained by reversing the digits. If the product of the two numbers is 1855, find the original numbers. 53 and 35 *Ans.*

Euclid.

1. Any two sides of a triangle are together greater than the third side. 14

In any triangle *ABC*, the two sides *AB*, *AC*, are together greater than twice the line joining the apex *A* to the middle point of the base *BC*.

2. If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a right angle. 8

3. The straight line joining the middle points of any two sides of a triangle is parallel to the third side and equal to half of it. 10

4. Prove any theorem of Euclid which corresponds to the algebraical formula $(a+b)(a-b)=a^2-b^2$. 10

5. The angle at the centre of a circle is double of the angle at the circumference on the same side, that is, on the same arc. 8

6. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of circle. 9

7. *ABC* is a triangle having the angle *A* acute. Prove that BC^2 is less than AB^2+AC^2 by twice the square on the tangent drawn from *A* to the circle of which *BC* is a diameter. 11

8. Describe a circle touching one side of a triangle and the other two sides produced. 8

9. Let ABC be a triangle in a circle. P the centre of the inscribed circle, Q the centre of the escribed circle; PQ shall be bisected by the circumference of the circle in which ABC is inscribed. 12

10. Inscribe an equilateral and equiangular hexagon in a given circle. 10

1896.

MONDAY, 7TH DECEMBER.

[10 A. M. TO 1 P. M.]

Arithmetic.

K. G. DESHPANDE, B.A.

LIEUT. A. J. PEILE, B.A.

1. Simplify:—

$$\frac{\frac{1}{3} \text{ of } 2\cdot179 - \frac{5}{6} \text{ of } \cdot8684}{\frac{2\frac{1}{3}}{5} - \frac{2}{5\frac{5}{8}} + \frac{1\frac{5}{6}}{4\frac{6}{7}}}.$$ 4

A property worth £1,237 10*s.* is divided among four brothers; the eldest obtains $\cdot 2\dot{7}$ of the property, the second $\cdot\dot{2}\dot{7}$ and the third $\cdot 27$. What sum does the fourth receive. 4

2. Three persons start to walk round a circular tract in the same direction, at the rates of 60, 80, and 100 yds. per minute respectively. If at the end of 2 hours they again find themselves together what is the lenght of the track? 8

3. Shew that $\sqrt{2}$ lies between $\frac{17}{12}$ and $\frac{4}{3}$. 8

Shew whether it is possible to put a cubical case whose content is 4019·679 cubic feet through a square hatchway whose area is 37791·3 square inches.

4. The cost of tiling a verandah, 3 ft. broad all round a square room is Rs. 304 at the rate of Rs. 1-5-4 per sqr. ft. What is the size of the room? 9

5. A person goes out between 9 and 10 o'clock and returns between 10 and 12 o'clock when he finds that the hands of his clock have exactly changed places. What was the time when he went out and how long had he been away? 12

6. A merchant bought goods at 16 guineas a cwt., and by retailing at 3*s.* 8*d.* per lb. made 10 per cent. more than if he had sold the whole for £151 10*s.* What weight did he buy? 10

7. A person arranged with his broker in London to pay a sum of 7990 roubles through Paris: he pays the broker when the exchange between London and Paris is £1 = 23·50 francs and between Paris and St. Petersburg 2·40 francs = 1 rouble. The broker delays the remittance till the exchange is £1 = 23·80 francs and 7 francs = 3 roubles. Find the broker's profit or loss. 9

8 Two sums of money amounting to £1,946 5s. were invested, the smaller at 4 per cent and the larger at 4½ per cent. per annum. At the end of 18 months the simple interest on the two sums amounted together to £125 4s. 10½d. What were the two sums? 10

9. A sum of money was lent at compound interest. The first year's interest was £91 5s. and the third year's is £100 12s. ½d. What is the sum and the rate of interest? 12

10. How much in the 3½ per cents. at 105⅞ (brokerage ⅛ per cent.) must be sold out to pay a bill of Rs. 1,751 nine months before it is due, rate of interest being 4 per cent.? 10

Algebra (Voluntary).

FRIDAY, 11TH DECEMBER.

1. Simplify— 8

$$\frac{(a-b+c)(a+b-c)}{(a-b)(a-c)}+\frac{(a+b-c)(-a+b+c)}{(b-c)(b-a)}+\frac{(-a+b+c)(a-b+c)}{(c-a)(c-b)}.$$

2. Find the algebraical expression which when divided by x^2+x-1 gives x^3-3x^2+4x-7 for the quotient and $11x-7$ for the remainder. 3

x^5-2x^4. *Ans.*

Find the factors of $(x^2+4x+8)^2+3x(x^2+4x+8)+2x^2$ and of $4(ab+cd)^2-(a^2+b^2-c^2-d^2)^2$. 3

$(x+2)(x+4)(x^2+5x+8)$. *Ans.*

$(a+b-c+d)(a+b+c-d)(c+d-a+b)(c+d+a-b)$. *Ans.*

3. If $xy=ab(a+b)$ and $x^2-xy+y^2=a^3+b^3$ prove that— 10

$$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{b}-\frac{y}{a}\right)=0.$$

(See Solutions.)

4. If m and n are any positive integers prove that $a^m\times a^n=a^{m+n}$. 8

Find the value of $(x^{2n}+x^{2n-1}+1)(x^{2n}-x^{2n-1}+1)$

5. Find the square root of 8

$$a^6+\frac{1}{a^6}-6\left(a^4+\frac{1}{a^4}\right)+15\left(a^2+\frac{1}{a^2}\right)-20.$$

$a^3-3a+\frac{3}{a}-\frac{1}{a^3}$ *Ans.*

6. Solve the following equations:— 6

(i). $\frac{M(x+a)}{x+b}+\frac{N(x+b)}{x+a}=M+N.$

$x=\frac{Nb+Ma}{M-N}$. *Ans.*

(ii). $x+y+z=a+b+c$; $ax+by+cz=bc+ca+ab$; $(b-c)x+(c-a)y+(a-b)z=0$. 8

$x=\frac{b+c}{2}$; $y=\frac{a+c}{2}$; $z=\frac{a+b}{2}$. *Ans.*

7. Find the condition that one root of the equation— $ax^2+bx+c=0$ may be n times the other. $na^2=ac(n+1)^2$. *Ans.* 8

8. If Z be real prove that $\frac{x^2+34x-71}{x^2+2x-7}$ *can* have no value between 5 and 9. 12

9. If there were no stoppages, it would take half as long to travel the distance from A to B by rail-road as by coach; but three hours being allowed for occasional stoppages by the former, the coach will travel the distance all but 15 miles in the same time; if the distance were two-thirds as great as it is and the same time allowed for Railway stoppages, the coach would exactly take same time as the train; what is the distance between A and B. 10

10. What is the price of eggs per dozen when 2 more in a shilling's worth lowers the price a penny a dozen. 10

9*d*. per dozen. *Ans.*

Euclid.

SATURDAY, 12TH DECEMBER.

1. $ABCD$ is a quadrilateral frame loosely jointed at the corners. A diagonal bar AC will make the frame rigid. Prove by Euclid that this must be so. 9

2. Euc. I. 32. 6

The straight line joining the middle point of the hypotenuse of a right-angled triangle to the right angle is equal to half the hypotenuse. 9

3. Bisect a triangle by a straight line drawn through a given point in one of its sides. 10

4. Euc. II. 11. 10

5. Euc. III 15.

6. Euc. III. 33. 8

7. The opposite sides of a quadrilateral in a circle are produced to meet; prove that the bisectors of the angles thus formed are perpendicular to each other. 10

8. Euc. IV. 5. 9

9. ABC is a triangle and O is the point of intersection of the perpendiculars from A, B, C on the opposite sides of the triangle: the circle which passes through the middle points of OA, OB, OC will pass through the feet of the perpendiculars and through the middle points of the sides of the triangle. 12

10. Give the construction, without the proof, of a twelve-sided regular figure in a circle. 8

1897.

MONDAY, 6TH DECEMBER.

[10 A.M. TO 1 P.M.]

Arithmetic

LIEUT. A. J. PEILE, R.A.
NARAYAN BALVANT PENDSE, M.A. LL.B.

1. Define *prime number, fraction, percentage.* 10

What are the factors of 6083?

Divide $3\cdot\dot{7}1428\dot{5}$ by $5\cdot0\dot{5}71428\dot{}$, shewing answer correct to four decimal places.

2. A certain number of men, twice as many women, and thrice as many boys earn in a week Rs. 43 5 as. Each man gets 3 as. 6p., each woman 2 as., and each child 1a. 6p. per day. The week consists of 7 days. What were the numbers of men, women, and children employed? 8

3. It is required to build a terrace 100 yards long, 14 ft. wide, and 5 ft. 6 in. high, with earth obtained by making a cubical tank. What must be the depth of the tank if the earth increases in bulk 10 per cent. when excavated? (Answer to be in feet and inches, correct within one inch.) 10

4. A milkman buys milk at $2\frac{1}{2}$ annas a seer, dilutes it with water, and sells it at 3 annas a seer, making a profit of 60 per cent. How much water is in each seer sold? 8

5. It is between 3 and 4 o'clock. It is observed that the number of minute spaces between the hands is to the number of spaces between them 10 minutes before as 26 : 15. What is the time? 12

6. One vessel contains 24 gallons of water, another contains 24 gallons of wine. One gallon is taken from each and poured into the other. This operation is done three times. How much wine and how much water will each vessel then contain? 10

7. A man near the seashore sees the flash of a gun fired on a ship moving directly towards him and hears the report in 15 seconds. He then walks towards the ship at the rate of 3 miles an hour and sees a second flash 5 minutes after the first and immediately stops. The report follows in $10\frac{1}{2}$ seconds. Find the speed of the ship in feet per second. (Velocity of sound 1,200 ft. per second) 11

8. A leaky cistern can be filled in 5 hours by 30 jugs of 3 gallons each and in 3 hours by 40 jugs of 2 gallons each. How much does the cistern hold, and after being filled how long will it take to empty itself? 10

9. £900 are due 5 years hence. The debt is to be paid by five equal instalments payable at the end of each year. What should be the value of each instalment, interest being 4 per cent.? 10

10. A man sells $2\frac{3}{4}$ per cent. consols at $96\frac{3}{8}$, and by investing the proceeds in shares which pay an annual dividend of £4 per share, raises his income by 5 per cent. What was the price of each share? Brokerage in each case $\frac{1}{8}$ per cent. 11

FRIDAY, 10TH DECEMBER.

[2 P.M. TO 5 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER I.

Algebra.

1. Simplify $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}-\frac{1}{a+b+c}$. $\frac{(a+b)(b+c)(c+a)}{(a+b+c)abc}$. *Ans.* 6

2. What quantity when multiplied by 5

$x-\frac{1}{x}$ gives $x^3-\frac{1}{x^3}-\left(x-\frac{1}{x}\right)^2$? $\left(x^2-x+1+\frac{1}{x}+\frac{1}{x^2}\right)$. *Ans.*

Find the factors of $(a-b)^3+(b-c)^3+(c-a)^3$. 6

$3(a-b)(a-c)(c-b)$. *Ans.*

3. Find the G. C. M. of 6

$x^4-5x^3-6x^2-5x+1$ and $x^4+3x^3-2x^2+3x+1$.

(x^2+4x+1). *Ans.*

Find the L. C. M. of $x^2-4x+3-y^2+2y$ and $x^2-5x+4-y^2+3y$. $(x-y-1)(x+y-3)(x+y-4)$. *Ans.* 6

4. Find for what values of p and q, $m^6+6m^5-40m^3+pm-q$ is a perfect cube. $p=q=64$. *Ans.* 8

5. Give the meanings of $a^{\frac{1}{n}}$, a^{-n}, a^0. 6

Simplify— 8

$$\left\{\left(p+\frac{1}{q}\right)^p\left(p-\frac{1}{q}\right)^q\right\}\div\left\{\left(q+\frac{1}{p}\right)^p\left(q-\frac{1}{p}\right)^p\right\}.\quad \left(\frac{p}{q}\right)^{p+q}.\ \textit{Ans.}$$

6. Solve the equations—

(i) $\frac{x+a}{x+b}=\left(\frac{2x+a+c}{2x+b+c}\right)^2$. $x=\frac{ab-c^2}{2c-a-b}$. *Ans.* 6

(ii) 9

$$\left.\begin{aligned}\frac{a}{x}+\frac{b}{y}&=\frac{3}{2};\\ \frac{b}{x}+\frac{c}{z}&=\frac{4b+a}{4a};\\ \frac{c}{y}+\frac{a}{z}&=\frac{2c^2+ab}{4bc}.\end{aligned}\right\}\qquad \left.\begin{aligned}x&=a\\ y&=b\\ z&=c\end{aligned}\right\}.\ \textit{Ans.}$$

7. Show that the equation $ax^2+bx+c=0$ cannot have more than two roots. 6

8. Divide 12 into two parts so that the sum of their squares may be the least. 6 and 6. *Ans.* 8

9. *A* person walks from *A* to *B*, a distance of $7\frac{1}{2}$ miles, in 2 hours $17\frac{1}{2}$ minutes, and returns in 2 hours 20 minutes. His rates of walking, in miles per hour, are, up hill, down hill and on the level, 3, $3\frac{1}{2}$ and $3\frac{1}{4}$ respectively. Find the length of level road between *A* and *B*. $4\frac{7}{8}$ miles. *Ans.* 12

10. *A* and *B* can do a piece of work in $14\frac{2}{5}$ days. *A* alone can do it in 12 days less than *B* alone; find the time in which *A* alone can do it. 24 days. *Ans.* 8

SATURDAY, 11TH DECEMBER.

[10 A.M. TO 1 P.M.]

MATHEMATICS (VOLUNTARY)—PAPER II.

Euclid.

1. Draw a straight line at right angles to a given straight line through a given point without it. 6

From two given points on the same side of a given straight line draw two straight lines which will meet at a point in the given line and make equal angles with it. 8

2. The opposite sides and angles of a parallelogram are equal. 6

3. Divide a straight line into two parts so that the square on one part may be double the square on the other. 10

4. Describe a square that shall be equal to a given rectilineal figure. 8

The largest rectangle, the sum of whose sides is given, is a square. 8

5. In a parallelogram, the sum of the squares on the sides is equal to the sum of the squares on the diagonals. 10

6. (*a*) The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles. 8

(*b*) State the converse.

7. In equal circles equal angles stand on equal arcs whether they be at the centres or circumferences. 6

Describe a circle cutting the sides of a given square in eight points, such that they shall be the angular points of a regular octagon. 10

8. Inscribe a regular pentagon in a given circle. 8

9. If from any point in the circumference of a circumscribing circle, perpendiculars be drawn to the sides of the inscribed triangle (produced if necessary), prove that their feet will lie in the same straight line. 12

SOLUTIONS.

Arithmetic.

1889.

1.
$$\begin{array}{r|l} 2 & 1260 \\ 2 & 630 \\ 3 & 315 \\ 3 & 105 \\ 5 & 35 \\ & 7 \end{array}$$

$\therefore$ the factors are $2^2 \times 3^2 \times 5 \times 7$.

$$\begin{array}{r|l} 2 & 10584 \\ 2 & 5292 \\ 2 & 2646 \\ 3 & 1323 \\ 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ & 7 \end{array}$$

$\therefore$ the factors are $2^3 \times 3^3 \times 7^2$.

$$\begin{array}{r|l} 5 & 40425 \\ 5 & 8085 \\ 3 & 1617 \\ 7 & 539 \\ 7 & 77 \\ & 11 \end{array}$$

$\therefore$ the factors are $3 \times 5^2 \times 7^2 \times 11$

$\therefore$ the *G. C. M.* $=3\times7=21$. *Ans.*

2. Suppose I buy 49 nuts, then by selling half of these at 24 a penny, *i.e.*, 48 for 2 pence and the remaining half at 25 a penny, *i.e.*, 50 for 2 pence, I get Rs. $\left(\frac{49}{48}+\frac{49}{50}\right)=$ Rs. $\frac{4802}{2400}=$ Rs. $\frac{2401}{1200}$. The fraction $\frac{2401}{1200}$ is in its lowest terms $\therefore$ the least number by which it must be multiplied so as to become an integer $=1200$ $\therefore$ the least sum I must receive $=$ Rs. $1200\times\frac{2401}{1200}$ $\therefore$ least number of nuts I buy $=1200\times49=$ 58,800. *Ans.*

49 nuts : 58,800 :: 2*d.* $=$ £10 *C.P.* of nuts. *Ans.*

$\frac{58,800}{2}=29400$. 24 nuts : 29400 nuts :: 1*d.* : 1225 *d.*, *S.P.* of half the number of nuts

nuts 25 : 29400 nuts :: 1*d.* $=$ 1176*d.* *S.P.* of the remaining half

$\therefore$ total *S.P.* $=1225d.+1176d.=2401d.$; but *C. P.* $=2400d.$

$\therefore$ the gain $=1d.$ *Ans.*

3. The first monkey finishes the figs in $\frac{4}{5}\times2\frac{1}{2}$ min. $=2$ min. and he eats 15 figs. in 1 min. $\therefore$ the total number of figs robbed $=15\times2=30$. As the grapes are twice as many, their number is 60. The first monkey assists the second in eating grapes for $\frac{1}{2}$ min.

$2\frac{1}{2}$ min. : 2 min :: 60 grapes : 48 grapes, eaten by the second monkey when the first was engaged in eating the figs.

∴ $60-48=12$ grapes were divided between the two. The second eats 48 grapes in 2 min., *i.e.*, 24 in 1 min.; ∴ if they eat grapes at equal rates, 48 would be consumed in 1 min.

Hence $48 : 12 :: 1$ min. $=\frac{1}{4}$ min.

∴ they would have got away in $2\frac{1}{4}$ min. *Ans.*

Now, had their respective rates been 15 and 24 per min., they would have finished 39 grapes in 1 min.; hence 12 would have been consumed in $\frac{4}{13}$ min. ∴ they would have got away in $2\frac{4}{13}$ min. *Ans.*

4. Rs. $\frac{21}{8}\times\frac{13}{12}\times\frac{12}{21}\times\frac{41}{42}\times\frac{32}{5}\times\frac{27}{2}$ = Rs. 26. *Ans.*

5. $£\frac{1}{5}\times\frac{1}{37}\times\frac{122}{55}\times\frac{37\times75}{23099}\times\frac{254089}{4\times240}=£\frac{61}{160}=$ 7*s.* 7½*d.* *Ans.*

6. Int. *minus* Dis. = Int. on Dis.

£71 16*s.* 7½*d.* Int. − £63 17*s.* 0*d.* Dis. = £7 19*s.* 7½*d.* Int. on £63 17*s.*

£7 19*s.* 7½*d* = £$\frac{1277}{160}$; £63 17*s* = £$\frac{1277}{20}$ £71 16*s.* 7½*d.* = £$\frac{11493}{160}$.

£$\frac{1277}{160}$ int. : £$\frac{11493}{160}$ int. :: $\frac{1277}{20}$ sum = £574 13*s.* *Ans.*

Now, £71 16*s.* 7½*d.* is the interest on £574 13*s.* for two years at the required rate. ∴ the rate $=6\frac{1}{4}\%$ *Ans.*

7. $\left(1+\frac{R}{100}\right)^n=\frac{A}{P}$, where R, n, A, P represent respectively the rate, number of years, amount and principal.

$$\therefore\ \left(1+\tfrac{5}{100}\right)^n=\frac{4520\frac{1}{4}}{4000}\ \therefore\ \left(\tfrac{21}{20}\right)^n=\tfrac{18081}{16000}.$$

Now, we are to find out what power of $\frac{21}{20}$ is $\frac{18081}{16000}$. $\frac{18081}{16000}\times\frac{20}{21}\times\frac{20}{21}=\frac{41}{40}$. The division here is performed two times ∴ 2 represents the number of the units of period, $\frac{41}{40}$ which remains is less than $\frac{21}{20}$ ∴ $£\frac{41}{40}-£1=£\frac{1}{40}$ is the interest for the remaining period; but $£\frac{5}{100}$ is the interest on £1 for 1 year. Hence— $£\frac{5}{100} : £\frac{1}{40}$ :: 1 year $=\frac{1}{2}$ year ∴ the total period $=2\frac{1}{2}$ years. *Ans.*

8. Let $h=$ height, $b=$ breadth and $l=$ length.

∴ $2h(b+l)=$ area. 9*d.* : £$\frac{21}{8}$ :: 1 sq. yd. : 70 sq. yd, area of room.

5*s.* : £$\frac{49}{8}$ :: 1 sq. yd. : $\frac{49}{2}$ sq. yds., area of floor.

∴ $l\times b=\frac{49}{2}$ ∴ $2b\times b=\frac{49}{2}$ ∴ $2b^2=\frac{49}{2}$ ∴ $b^2=\frac{49}{4}$

∴ $b=\frac{7}{2}$ ∴ breadth $=\frac{7}{2}$ yds.

Now, $2h(b+2b)=70$ sq. yds. ∴ $6b\times h=70$ sq. yds. ∴ $6\times\frac{7}{2}\times h=70$ sq. yds. ∴ $3b=10$ ∴ $h=\frac{10}{3}$ yds. ∴ height = 10ft. *Ans.*

9. $1398665\frac{71}{216}=\frac{302111711}{216}$

oz. $1000 : \frac{302111711}{216}$ oz. :: 1 c. ft. $=\frac{302111711}{216000}$ c. ft.

		302111711	671
187	10800	216	
	1309	86111	
	12109	84763	
	49	1348711	
2011	1346700	1348711	
	2011		
	1348711		

$\sqrt[3]{216000}=60$

∴ the length of the edge $=\frac{671}{60}$ ft. $=10\frac{11}{60}$ ft. *Ans.*

10. Let Re. 1 be the gross income

∴ Re. 1−4 pies = 192−4 = 188 pies = Re. $\frac{47}{48}$ the net income.

Rs. $\frac{47}{48}$: Rs. $4\frac{1}{2}$ :: Re. 1 = Rs. $\frac{216}{47}$ gross income.

Rs. $\frac{216}{47}$: Rs. 4 :: Rs. 100 sum invested = Rs. $\frac{2350}{27} = 87\frac{1}{27}$ price of 4 per cents. *Ans.*

1890.

1. $(\frac{[illegible]}{10}\times\frac{[illegible]}{3}\times\frac{[illegible]}{[illegible]}\times\frac{[illegible]}{1}) + (\frac{[illegible]}{20}\times\frac{[illegible]}{12}\times -\frac{[illegible]}{1}\times -\frac{[illegible]}{1}) - (\frac{[illegible]}{12}\times\frac{1}{2}\times -\frac{[illegible]}{1}\times\frac{1[illegible]}{1})$.

$= \frac{60}{5} + 9 - (-\frac{92}{5}) = \frac{60}{5} + 9 + \frac{92}{5} = \frac{197}{5} = 39\frac{2}{5}$. *Ans.*

2. $\frac{[illegible]}{100}\times\frac{1000}{10}\times\frac{\frac{[illegible]}{1000}\times 4\frac{[illegible]}{100}}{2\frac{[illegible]}{1000}} = \frac{[illegible]}{100}\times\frac{1000}{10}\times\frac{[illegible]}{1000}\times\frac{[illegible]}{100}\times\frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]}$

∴ $1 - \frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]}$. *Ans.*

3. 1 metre : 11220 metres :: 39·39 inches $= \frac{3939 \times 561}{5}$ inches.

27min : 60min :: $\frac{3939 \times 561}{5 \times 1760 \times 3 \times 12}$ miles $= \frac{22321}{1440}$ miles.

$= 15\frac{721}{1440}$ miles per hr. *Ans.*

4. $(\frac{19}{20}-\frac{18}{19})$ of property = £17 11*s.* 2*d.* = £$\frac{2107}{120}$

∴ $\frac{1}{380}$ of the property = £$\frac{2107}{120}$

∴ the whole property = $\frac{2107}{120} \times \frac{380}{1}$ = £$\frac{40033}{6}$ = £6672 3*s.* 4*d.* *Ans.*

5. $(440 \times \frac{35}{3})$yds : $\left(\frac{1575 \times 73}{2 \times 3}\right)$yds. :: 22×32 hrs. $= 73 \times 36$ hrs.

Of 4 men, 2 work 8 hrs. a day and 2 work 10 hrs. a day.

∴ altogether they work for $(2\times8)+(2\times10)$ or 36 hrs.

Hence they take $\frac{73 \times 36}{36}$ or 73 days. *Ans.*

6. £678 8*s.*−£38 8*s.* = £640.

$\left.\begin{array}{l} £640 : 100 \\ \text{yrs. } 1\frac{1}{2} : 1 \end{array}\right\}$ :: £$38\frac{2}{5}$ = 4%. *Ans.*

7.
```
3) 1000
3)  333−1
3)  111−0
3)   37−0
3)   12−1
3)    4−0
      1−1
```
∴ the required number =

$3^6\ 3^5\ 3^4\ 3^3\ 3^2\ 3^1\ 3^0$

1 1 0 1 0 0 1, the small figures above showing the local value of each digit

∴ the weights to be selected are—

$1\times3^6+1\times3^5+1\times3^3+1\times3^0 = 3^6+3^5+3^3+1$. *Ans.*

8. $\sqrt{1208\cdot8838379025} = 34\cdot76905$. *Ans.*

	7500	160·103,007	5 43
154	616	125	
	8116	35103	
	16	32464	
1623	874800	26390·7	
	4869	2639007	
	879669		

5·43. *Ans.*

9. The factors are $2^3 \times 3^4 \times 7^2$ and $2^4 \times 3^4 \times 5^3$. *Ans.*

10. $2^{\frac{1}{2}}$ and $3^{\frac{1}{3}} = 2^{\frac{3}{6}}$ and $3^{\frac{2}{6}} = \sqrt[6]{2^3}$ and $\sqrt[6]{3^2} = \sqrt[6]{8}$ and $\sqrt[6]{9}$

$\therefore$ $3^{\frac{1}{3}}$ is greater than $2^{\frac{1}{2}}$; $2^{\frac{1}{5}}$ and $3^{\frac{1}{8}} = 2^{\frac{8}{40}}$ nad $3^{\frac{5}{40}} = \sqrt[40]{2^8}$ and $\sqrt[40]{3^5} = \sqrt[40]{256}$ and $\sqrt[40]{243}$ $\therefore$ $2^{\frac{1}{5}}$ is greater than $3^{\frac{1}{8}}$.

11. Each has to pay $\frac{1}{3}$ of the rent for $4\frac{1}{2}$ months. During $11\frac{1}{2}$ months each of *A* and *B* has to pay $\frac{1}{2}$ of the rent due for the period. During the remaining 8 months *A* has to pay the whole of the rent due for the period.

12 months : $4\frac{1}{2}$ :: Rs $\frac{1500}{8}$ = Rs. 187 8 as. each pays.

12 months : $11\frac{1}{2}$ :: Rs. $\frac{1500}{2}$ = Rs. 718 12 as. each of *A* and *B* pays.

12 months : 8 :: Rs. 1500 = Rs. 1,000 *A* pays.

Hence *A* pays Rs. 1000 + Rs. 718 12 as. + Rs. 187 8 as. = Rs. 1906 4 as. *B* pays Rs. 718 12 as. + Rs. 187 8 as. = Rs. 906 4 as. and *C* pays Rs. 187 8 as. *Ans.*

12. 1140ft. : $(14\frac{1}{4} \times 1760 \times 3)$ ft. :: 1 sec. = $\frac{11}{600}$ hr.
1 hr. : $\frac{11}{600}$ hr. :: 10 miles = $\frac{11}{60}$ miles or 968 ft. *Ans.*

1891.

1. $\frac{285714}{999999} = \frac{2}{7}$; $\cdot2\dot{1}42857\dot{7} = \frac{2\frac{1}{7}}{10} = \frac{3}{14}$; 13*s.* 6*d.* $= \frac{27}{2}$ *s.*

·75 guineas $= \frac{63}{4}$ *s.*; 13*s.* 4*d.* $= \frac{40}{3}$ *s.*

$\therefore$ $\frac{2}{7} \times \frac{27}{2} = \frac{27}{7}$ *s.*; $\frac{3}{14} \times \frac{40}{3} = \frac{20}{7}$; $\frac{63}{4} + \frac{20}{7} = \frac{521}{28}$ *s.*

$= \frac{521}{28} - \frac{27}{7} = \frac{413}{28}$ *s.* = 14*s.* 9*d.* *Ans.*

2. $2\frac{22}{49}$ ft. and $2\frac{23}{56}$ ft. $=\frac{120}{49}$ ft. and $\frac{135}{56}$ ft. $\frac{\text{The } L.C.M. \text{ of Num.}}{\text{The } G.C.M. \text{ of Den.}}$ $=\frac{1080}{7}=154\frac{2}{7}$. *Ans.*

$$\left.\begin{array}{l}\frac{1080}{7}\times\frac{49}{120}=63 \text{ steps of the first} \\ \frac{1080}{7}\times\frac{56}{135}=64 \text{ steps of the second}\end{array}\right\} \text{ Ans.}$$

3. Let the distance travelled by rail be 1 mile; then the distance by coach $=\frac{1}{4}$ mile and the distance by sea $=\frac{3}{4}$ mile. Dividing 560 in the proportion of 1, $\frac{1}{4}$, $\frac{3}{4}$ we get the distances: 280 miles by rail, 70 miles by coach, and 210 miles by sea. The railway fare per mile $=1\cdot\dot{5}71428\dot{}$ *d.* $=1\frac{4}{7}d.$ ∴ the fare per mile by coach $=\frac{11}{7}\times 2=\frac{22}{7}d.$ and the fare per mile by sea $=\frac{11}{7}\times\frac{4}{5}=\frac{44}{35}d.$

∴ the expense of the whole journey $=440d.$ by rail, $220d.$ by coach, and $264d.$ by sea; altogether $924d.=£3$ 17*s.* *Ans.*

4. 3 oxen = 11 sheep ∴ 27 oxen = 99 sheep.

Hence, $99\times 3+270\times 7 : 99\times 3 :: £81=£11$ the first ought to pay.

∴ the second pays $£81-£11=£70$. *Ans.*

5. $£1\cdot03\times£1\cdot04\times£1\cdot05=£1\cdot12476$, amount of £1 for 3 years
∴ $£1\cdot12476 : £2811\frac{6}{10} :: £1=£2504$ sum. *Ans.*

6. 12 mos. : 11 mos. :: $£\frac{25}{3} : £\frac{275}{36}$. £903 11*s.* 3*d.* $=£\frac{14457}{16}$
$£(100+\frac{275}{36}) : £\frac{14457}{16} :: £100=£878$ 8*s.* *Ans.*

7. Let £96 be invested.

$£96 : £96 :: £3=£3$ interest for the first year.

During the second year $96+96+3=£195$ are invested.

∴ $96 : 195 :: 3=£\frac{195}{32}$ interest.

∴ $(195+\frac{195}{32})$ or $\frac{6435}{32}$ is the amount of £96 at the end of two years.
Hence $£\frac{6435}{32} : £402\frac{3}{10} :: £96=£192$. *Ans.*

8. $18\frac{3}{4}$ft. $\times$ $13\frac{1}{3}$ ft. $\times$ $\frac{2}{12}$ ft. $=\frac{125}{3}$ c. ft.

1 c. ft. : $\frac{125}{3}$ c. ft. :: $62\frac{1}{2}$lbs. $=\frac{15625}{6}$lbs. $=2604\frac{1}{6}$lbs. *Ans.*

9. 100qrs. : 49 :: $7=\frac{343}{100}$. $\frac{343}{100}+\frac{231}{25}=\frac{1267}{100}$

100qrs. : 84 :: $11=\frac{231}{25}$

100qrs. : 133 :: $9=\frac{1197}{100}$. $\frac{1267}{100}-\frac{1197}{100}=\frac{7}{10}$

∴ the price of $\frac{7}{10}$ quarter $=£2$ 10*s.* 9*d.* $=£\frac{203}{80}$.

$\frac{7}{10}$ qrs. : $(49+84)$ :: $£\frac{203}{80}=\frac{3857}{8}=£482$ 2*s.* 6*d.* *Ans.*

10. £1047901 1*s.* $=20958021s.=\frac{20958021}{21}$ or 998001 guineas.

Hence $\sqrt{998001}=999$ subscribers. *Ans.*

11. 431ft. + 713ft. = 1144ft. 1144ft. are passed over at the rate of $(41-28)$ or 13 miles an hour in the required time. Hence,—

13 miles : $\frac{1144}{1760\times 3}$ :: 1 hr. $=\frac{1}{60}$ hr. $=$ 1 min. *Ans.*

1892.

1. (i) [illegible] $=\frac{143}{[illegible]}$

$\frac{1}{7}-\frac{1}{9}-\frac{1}{11}+\frac{1}{13}=\frac{1287-1001-819+693}{7\times9\times11\times13}=\frac{160}{7\times9\times11\times13}$

[illegible] $=\frac{1001}{64\times25}$ $\therefore\ \frac{160}{7\times9\times11\times13}\times\frac{1001}{64\times25}=\frac{1}{10}$. *Ans.*

2. $\frac{27}{1000}\times21=\frac{567}{1000}$s. [illegible] $\times 15s.=\frac{7}{16}$; $\frac{567}{1000}-\frac{7}{16}=\frac{259}{2000}$.

14s 8d. $=\frac{44}{3}$s. $\frac{259}{2000}\times\frac{3}{44}=\frac{777}{88000}=\cdot0088295\dot{4}$. *Ans.*

3. $\frac{1}{360}\times$ circum. $=69\frac{1}{22}=\frac{1519}{22}$ miles $\therefore$ circum. $=\frac{1519}{22}\times360$

$=\frac{1519\times180}{11}$ miles. diameter $\times\frac{22}{7}=$ circum. $=\frac{1519\times180}{11}$

$\therefore$ diameter $=\frac{1519\times180\times7}{11\times22}=\frac{956970}{121}=7908\frac{102}{121}$ miles. *Ans.*

4. Rs. 910 10as. 5ps. = Rs. $\frac{174845}{192}$. If the gross income is Re. 1, the net income = Re. 1 − 5 pies = Re. $\frac{187}{192}$.

Re. $\frac{187}{192}$: Re. $\frac{174845}{192}$ :: Re. 1 = Rs. 935 gross income.

Rs. 85 : Rs. 935 :: Rs. 100 = Rs. 1,100. *Ans.*

5. £937½ : £1 :: £455 1s. 6¾d. = 9s. 8[illegible]d. *Ans.*

£1 : £114 :: £[illegible] = £55 6s. 9d.

$\therefore$ the creditor gets £114 − £55 6s. 9d. = £58 13s. 3d. *Ans.*

6. If £100 be the interest, then the dis. $=\frac{220}{221}\times$ £100 = £$\frac{22000}{221}$.

£100 − £$\frac{22000}{221}$ = £$\frac{100}{221}$ interest on £$\frac{22000}{221}$.

£$\frac{100}{221}$ int. : £100 int. :: £$\frac{22000}{221}$ = £$\frac{22000}{21}$ sum.

$\left.\begin{array}{l}£\frac{22000}{21}\text{ sum : 100 sum.}\\ \text{3 yrs. : 1 yr.}\end{array}\right\}$ :: 100 int = $3\frac{1}{2}$ %. *Ans.*

7. 3 boys = 1 man ; 2 boys = 1 woman.

Inv. $\left.\begin{array}{l}(27+30+18)\text{ boys : }(45+24+9)\text{ boys}\\ \quad 1 \quad : \quad 4\end{array}\right\}$ :: 50 days = 208 days.

Ans.

8. £36 12s. ÷ 3 = £12 4s. = £$\frac{61}{5}$

£3 : £$\frac{61}{5}$ :: £95 = £$\frac{1159}{3}$ = £386 6s. 8d.

£4 : £$\frac{61}{5}$ :: £119 = £$\frac{7259}{20}$ = £362 19s.

£749 5s. 8d.

£	s.	d.
932	5	8
749	5	8
183	0	0

£183 : £100 :: £[illegible] $=\frac{20}{3}=6\frac{2}{3}$ %. *Ans.*

20

9. The length of 1 load = 8yd

$\therefore$ 1 cub. yd. = 8 yds. $\times 4\frac{1}{2} \times \frac{1}{3}$ yd. $\times$ thickness.

$\therefore$ thickness $= \frac{1}{12}$yd. or 3 inches. *Ans.*

10. $\cdot\dot{3}0864197\dot{5} = \frac{25}{81}$ and $\cdot\dot{1}9753086\dot{4} = \frac{16}{81}$. Hence $\frac{25}{81} - \frac{16}{81} = \frac{5^2}{9^2} - \frac{4^2}{9^2}$

$= (\frac{5}{9} + \frac{4}{9})(\frac{5}{9} - \frac{4}{9}) = 1 \times (\frac{5}{9} - \frac{4}{9}) = \frac{5}{9} - \frac{4}{9}$ the diff. of the sq. rts. *Ans.*

11. $\sqrt[3]{41\cdot421736} = 3\cdot46 = 3\cdot46 \times 12$ inches $= 41\cdot52$ inches. *Ans.*

12. In 1 hr. or 360 secs. the first man walks (4×1760) yds.

$\therefore$ in 10 secs. " " $\frac{10 \times 4 \times 1760}{60 \times 60} = \frac{176}{9}$ yds.

$\therefore$ in 10 secs. the train travels $88 + \frac{176}{9} = \frac{968}{9}$ yds.

$\therefore$ in 9 secs. " " $\frac{968}{9} \times \frac{9}{10} = 96\frac{4}{5}$ yds.

$\therefore$ in 9 secs. the second man walks $96\frac{4}{5} - 88$ or $\frac{44}{5}$ yds.

$\therefore$ in 1 hr. " " " $\frac{44}{5} \times \frac{60 \times 60}{9 \times 1760} = 2$ miles. *Ans.*

1893

1. (i) $\frac{1}{7} - \frac{1}{10} + \frac{4}{11} - \frac{1}{3} = \frac{169}{7 \times 10 \times 11 \times 3}$; $\frac{1}{7} - \frac{1}{10} = \frac{3}{70}$; $\frac{4}{11} - \frac{1}{3} = \frac{1}{33}$

$\therefore \frac{3}{70} \times \frac{1}{33} = \frac{1}{770} \therefore 9 - \frac{1}{770} = \frac{6929}{770} \therefore \frac{169}{7 \times 10 \times 11 \times 3} \times \frac{770}{6929} = \frac{1}{123}$. *Ans.*

(ii) $(\frac{17}{5} \times \frac{7}{10}) \div \left(2\frac{53}{100} \div \frac{3148-3}{9990}\right) = \frac{17}{5} \times \frac{7}{10} \times \frac{100}{253} \times \frac{9990}{3145}$

$= \frac{80}{37} = 2\frac{6}{37}$. *Ans.*

2. $\cdot02947$ mass of Earth $= \cdot0000885$ mass of Jupiter.

$\therefore$ mass of the Earth $= \frac{\cdot0000885}{\cdot02947}$ mass of Jupiter $= \cdot003003$. *Ans.*

3 (Direct) $4\frac{1}{2} : 17$ \
(Inverse) $306 : \frac{51}{2}$ $\Big\}$ :: 25 tons coal $= 8\frac{1}{3}$ tons coal. *Ans.*

4. Rs. 153 2as. 0p. − Rs. 145 13a. 4p. = Rs. 7 4as. 8p. int. on Rs. 145 13as. 4p. for one year at the given rate.

Rs. 7 4as. 8p. $= \frac{175}{24}$ and Rs. 145 13as. 4p. = Rs. $\frac{875}{6}$

Rs. $\frac{875}{6}$ sum : Rs. 100 sum :: Rs. $\frac{175}{24}$ int. = 5 %

ow, the interest for the third year = Rs. 153 2as + 5 % on s. 153 2as. = Rs. $153\frac{1}{8} + \frac{5}{100} \times$ Rs. $153\frac{1}{8}$ = Rs. 160 12as. 6p. *Ans.*

5. £96 s.p. : £800 s.p. :: £100 c.p. = £833 6s. 8d. *Ans.*

£100 : £833$\frac{1}{3}$ :: £6 = £50. *Ans.*

6. 3 leaps of greyhound = 4 leaps of hare

∴ 6 ,, ,, = 8 ,, ,,

and ∵ 2 of the greyhound's leaps = 3 of the hare's,

∴ 6 ,, ,, ,, = 9 ,, ,,

∴ for every 8 leaps of hare's the greyhound gains (9 − 8) = 1 leap

∴ 500 × 6 = 3000 leaps of greyhound. *Ans.*

(1 : 500 :: 8 = 4,000 number of leaps hare takes before it is overtaken.)

7. One rod is 10 inches long, the other 11 inches, and the seventh points coincide. ∴ the difference between one pair of their extremeties = $\frac{4}{11}$ ft. − $\frac{3}{10}$ ft. = $\frac{7}{110}$ ft. And the diff. between the other pair of extremeties = $\frac{7}{10}$ ft. − $\frac{7}{11}$ ft = $\frac{7}{110}$ ft. ∴ $\frac{7}{110}$ ft. *Ans.*

8. Dis. travelled by faster + dis. travelled by slower in 1$\frac{1}{2}$ secs. = 92 + 82 = 174 ft.

∴ dis. travelled by faster + dis. travelled by slower in 1 sec. = 116 ft.

Dis. travelled by faster − dis. travelled by slower in 6 secs. = 174 ft

,, ,, ,, ,, ,, ,, in 1 sec. = 29 ft.

∴ twice dis. travelled by faster in 1 sec. = 145 ft.

∴ distance ,, ,, ,, ,, ,, ,, = $\frac{145}{2}$ ft.

∴ distance travelled by slower in 1 sec. = 116 − $\frac{145}{2}$ = $\frac{87}{2}$ ft.

∴ dis. travelled by faster in 1 hr. = $\frac{145 \times 60 \times 60}{2 \times 1760 \times 3}$ = 49$\frac{19}{44}$ miles. *Ans.*

and dis. travelled by slower in 1 hr. = $\frac{87}{2} \times \frac{60 \times 60}{1760 \times 3}$ = 29$\frac{29}{44}$ miles. *Ans.*

9. $\sqrt[3]{\frac{\cdot 448}{\cdot 875}} = \sqrt[3]{\frac{64}{125}} = \frac{4}{5} = \cdot 8$. *Ans.*

$\sqrt[3]{\cdot 0\dot{3}\dot{7}} = \sqrt[3]{\frac{37}{999}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3} = \cdot \dot{3}$. *Ans.*

10. The snail ascends 3ft. in 12 hrs. and slips down 2ft. in 12 hrs. ∴ he gains 1ft. in 24hrs. ∴ he will gain (30 − 3) or 17ft. in 17 days. Now, when the snail crawls the remaining 3ft., he does not slip down as he is at the top of the wall, ∴ he takes altogether 12 hrs. for crawling, and 12 hrs. for sleeping at the top. While descending he slips 3 + 2 = 5ft. in 24 hrs. or 1 day.

∴ 5ft. : 20 :: 1 day = 4 days he takes in descending.

∴ the total time taken by the snail = 17 + 1 + 4 = 22 days. *Ans.*

11. Let £127 be invested. ∴ £5 is the income in the first case.

£135 : £127 :: £5$\frac{1}{2}$ = $\frac{1397}{270}$. $\frac{1397}{270}$ − 5 = $\frac{47}{270}$.

£$\frac{47}{270}$: £4$\frac{7}{10}$ :: £127 = £3429. *Ans.*

£127 : £3429 :: £5 = £135 total income in the first case.

£135 : £3429 :: £5$\frac{1}{2}$ = £139 14s. *Ans.*

1894.

1. $\dfrac{1\frac{186}{999}-\frac{55}{90}}{19\frac{5}{9}-17\frac{8}{9}}\times\frac{241}{215}\div\dfrac{1\frac{53}{100}\times\frac{34}{999}}{4\frac{30}{99}}$

$$=\frac{1147}{1998}\times\frac{3}{20}\times\frac{241}{213}\times\frac{20\times27\times142}{31\times33}=\frac{241}{33}=7\tfrac{10}{33}.\quad \textit{Ans.}$$

$$\frac{3}{44}+\frac{4}{77}+\frac{4}{55}=\frac{297}{11\times4\times7\times5}\therefore\frac{3\cdot4}{\text{Denom.}}=\frac{11}{28}-\frac{297}{11\times4\times7\times5}=\tfrac{1}{5}$$

$\therefore$ Denominator $=5\times3\cdot4=17.$ *Ans.*

2. $\dfrac{G.C.M.}{L.C.M.}=\dfrac{1}{144}$ *Ans.*

3. $4\cdot3\dot{5}$ qrs. $=4\frac{32}{90}=\frac{196}{45}$ qrs. $\therefore\ \frac{196}{45}\times\frac{1}{80}=\frac{49}{900}$

$\therefore\ \sqrt{\frac{49}{900}}=\frac{7}{30}=\cdot2\dot{3}.$ *Ans.*

4. When the hands of a clock are once together, they again coincide when the minute hand gains 60 divisions on the hour hand; hence, 55 : 60::60 $=\frac{720}{11}$ min. taken by the minute hand to overtake the hour hand. Thus the hands of a clock are together after every $\frac{720}{11}$ min.

Now, 64 mins. of true time $=\frac{720}{11}$ or $65\frac{5}{11}$ min. of the clock.

$\therefore$ the clock gains $65\frac{5}{11}-64=1\frac{5}{11}$ min. in every 64 mins.

$\therefore$ 64 mins : 24×60 mins. ::$1\frac{5}{11}$ min. gain $=32\frac{8}{11}$ min. gained by the clock during the day. *Ans.*

5. One side of the outer square $=(50\times3)$ ft. $+(2\times4)$ ft. $=158$ ft.

" " " inner " $=(50\times3)$ft. $=150$ ft.

$\therefore$ area of the path $=(158)^2$ sq. ft $-(150)^2$ sq. ft.

$=(158+150)(158-150)=308\times8=2464$ sq. ft.

9 sq. ft : 2464 sq. ft ::$1\frac{1}{2}$s. $=$ £20 10*s.* 8*d.* *Ans.*

6. The time taken by the trains are as 3 : 2.

3 : 2 :: 5 : $\frac{10}{3}$ secs. taken by the second train to travel the same distance as is travelled by the first in 5 secs. But by the question, the second train takes 3 secs. to pass through the tunnel $\therefore$ in $(3\frac{1}{3}-3)$ secs. the second train travels a distance of $(120-90)$ or 30 ft. $\therefore$ in 3 secs. it passes 270 ft.

Hence, the length of the tunnel $=270-90$ or 180 ft. *Ans.*

$\therefore$ $\frac{1}{3}$ sec. : 1 hr. :: 30 ft. $=\frac{675}{11}$ miles per hour or $61\frac{4}{11}$ miles rate of the second train. *Ans.*

Proceeding as above, we find that the first train travels a distance of 30 ft. in $\frac{1}{2}$ sec. ($\therefore$ in 5 secs. it travels a distance of 300 ft. $\therefore$ the length of tunnel $=300-120$ or 180).

Hence,—$\frac{1}{3}$ sec. : 1 hr. :: 30ft. $=\frac{450}{11}$ miles per hour

i.e., $40\frac{10}{11}$ miles rate of the 1st train. *Ans.*

$$\left.\begin{array}{l} 5 \text{ mins.} : \frac{35}{16} \text{ min.} :: 144 = 63 \text{ swings} \\ 7 \text{ mins.} : \frac{35}{16} \text{ min.} :: 208 = 65 \quad ,, \end{array}\right\}. \quad \textit{Ans.}$$

3. $\sqrt[3]{597160402{\cdot}461} = 842{\cdot}1$. The edge of the tank $= 842{\cdot}1$ inches.

$$\therefore \frac{842{\cdot}1}{5280 \times 12} = \frac{2807}{211200} = {\cdot}01329. \quad \textit{Ans.}$$

4. In the first cask, out of 5 gallons, there are 4 gallons of the first kind of sherry and 1 of the second kind $\therefore$ 1 gallon from the first cask contains $\frac{4}{5}$ gall. of the first kind of sherry. Similarly 1 gallon from the second cask contains $\frac{6}{11}$ gall. of the first kind of sherry; and one gallon of the mixture contains $\frac{7}{10}$ gallon of the first kind of sherry. $\therefore$ the required proportion will be in the inverse ratio of $(\frac{4}{5}-\frac{7}{10}) : (\frac{7}{10}-\frac{6}{11})$, *i.e.*, of $\frac{1}{10} : \frac{17}{110}$, *i.e.*, of $11 : 17$

$\therefore$ the proportion $= 17 : 11$. *Ans.*

5. $\left.\begin{array}{r} \text{Workmen } 1 : 56 \\ \text{day } 1 : 25 \end{array}\right\} :: 6 \text{ as.} = 8400 \text{ as.} = \text{Rs. } 525$

$\left.\begin{array}{r} \text{Workmen } 1 : 20 \\ \text{days } 1 : 15 \end{array}\right\} :: 3\frac{1}{2} \text{ s.} = 1050 s. = £\frac{105}{2} = £52\frac{1}{2}$

Re. 1 : Rs. 525 :: $1\frac{1}{12}$ s. $= £\frac{455}{16} = £28\frac{7}{16}$.

$\therefore$ It is more profitable to employ Indian workmen.

6. *A* is open for 2 hrs. and *B* for 1hr. $\therefore \frac{1}{4} \times 2 + \frac{1}{5}$ or $\frac{1}{2}+\frac{1}{5}$ or $\frac{7}{10}$ of the cistern is filled at 3 o'clock, *i.e.*, before *C* is opened.

In one hour $\frac{1}{2}-\frac{1}{4}-\frac{1}{5}$ or $\frac{1}{20}$ of the cistern is emptied.

$\therefore \frac{1}{20} : \frac{7}{10} :: 1$ hr. $= 14$ hr.

$\therefore$ at 5 p. m. the cistern will be empty. *Ans.*

7. The first instalment runs for 2 yrs. and
the second ,, ,, 1 yr. at 5%.

(1) The amount of £ 1 for 1 year $= 1{\cdot}05$
(2) ,, ,, 2 years $= (1{\cdot}05)^2$
(3) ,, ,, 3 ,, $= (1{\cdot}05)^3$

$\therefore$ yearly instalment $\times (1{\cdot}05)^2 +$ yearly instalment $\times 1{\cdot}05$ $+$ yearly instalment $= £1261 \times (1{\cdot}05)^3$

$\therefore$ (yearly instalment) $\{(1{\cdot}05)^2 + 1{\cdot}05 + 1\} = 1261 \times (1{\cdot}05)^3$

$\therefore$ yearly instalment $\times 3{\cdot}1525 = £1261 \times 1{\cdot}157625$

$$\therefore \text{ yearly instalment} = £\frac{1261 \times 1{\cdot}157625}{3{\cdot}1525} = £463 \quad 1s. \quad \textit{Ans.}$$

8. $£75 : £10000 :: £100 = £\frac{40000}{3}$ stock.
$£75 : £10000 :: £3 = £400$ income.
$£100 : £\frac{40000}{3} :: £78 = £10400$ cash.
$£208 : £10400 :: £8 = £400$ income.

$\therefore$ there is no difference in income. *Ans.*

9. A puts 200 sheep for 6 months and 100 sheep for 6 months, *i. e.*, 1200 + 600 or 1800 sheep for one month.

B puts 160 sheep for 6 months, and 80 sheep for 6 months, *i. e.*, 960 + 480 or 1440 sheep for 1 month.

C puts 120 sheep for 6 months, *i.e.*, 720 sheep for 1 month.

$\therefore$ the shares to be paid by A, B, C, are proportional to 1800, 1440 and 720, *i. e.*, to 5, 4, and 2.

$$\left.\begin{aligned} \text{Hence } A\text{'s share} &= \tfrac{5}{11} \text{ of } £132 = £60 \\ B\text{'s share} &= \tfrac{4}{11} \text{ of } £132 = £48 \\ C\text{'s share} &= \tfrac{2}{11} \text{ of } £132 = £24 \end{aligned}\right\} \quad \textit{Ans.}$$

10. Let the $C. P.$ of goods be £1

£100 : £$\frac{3}{5}$ :: £6 = £$\frac{9}{250}$ profit. £100 : £$\frac{2}{5}$ :: £10 = £$\frac{1}{25}$ profit.

£$\frac{9}{250}$ + £$\frac{1}{25}$ = £$\frac{19}{250}$, total profit.

£$\frac{19}{250}$: £1 :: £114 = £1500 $C. P.$ of goods. *Ans.*

1896.

1. $$\frac{\frac{1}{3}\times 2\frac{173}{1000}-\frac{3}{8}\times\frac{5884}{10000}}{\frac{7}{12}-\frac{10}{15}+\frac{7}{18}}=\frac{\frac{2173}{3000}-\frac{2171}{3000}}{\frac{7}{12}-\frac{10}{15}+\frac{7}{18}}=\frac{1}{575}\div\frac{1}{2}=\frac{2}{375}. \quad \textit{Ans.}$$

$\frac{25}{90}=\frac{5}{18}$; $\frac{27}{99}=\frac{3}{11}$; $\frac{27}{100}$; $\frac{5}{18}+\frac{3}{11}+\frac{27}{100}=\frac{8133}{9900}$

$1 : 1-\frac{8133}{9900}$ :: £$1237\frac{1}{2}$ = £222. $\frac{1}{8}$. *Ans.*

2. 1 min. : 120 mins. :: 60 yds. = 7,200 yds.

: 120 mins. :: 80 = 9,600 yds. 1 : 120 :: 100 = 12,000 yds.

The G.C.M. of 7200, 9600, 12000 = 2400.

$\therefore$ $\frac{2400}{1760}$ miles or $\frac{15}{11} = 1\frac{4}{11}$ miles. *Ans.*

3. $\frac{17}{12}=\sqrt{(\frac{17}{12}\times\frac{17}{12})}=\sqrt{\frac{289}{144}}=\sqrt{(2+\frac{1}{144})}$

$\frac{41}{29}=\sqrt{(\frac{41}{29}\times\frac{41}{29})}=\sqrt{\frac{1681}{841}}=\sqrt{(2-\frac{1}{841})}$

i.e., $\frac{17}{12}$ is greater than $\sqrt{2}$ and $\frac{41}{29}$ is less than $\sqrt{2}$.—*Q.E.D.*

$\sqrt[3]{4019\cdot679}=15\cdot9$ $\therefore$ the edge of the cubical box = 15·9 ft.

$\sqrt{37971\cdot36}$ = 194·4 $\therefore$ the side of the square = 194·4 inches = $\frac{194\cdot4}{12}$ ft. = 16·2 ft.

$\therefore$ the cubical box can be put into the square hatchway.

4. Rs. $1\frac{1}{3}$: Rs. 304 :: 1 sq. ft. = 228 sq. ft. area of verandah. Area = length × breadth = $l \times b$. The length of verandah = length of 4 sides + 4 times 3ft. at the end of the corners. The breadth = 3ft.

$\therefore$ $4(l\times 3)+4\times 3^2=228$ sq. ft. $\therefore$ $12l+36=228$

$\therefore$ length = 16 ft. $\therefore$ breadth also = 16 ft. *Ans.*

5. The space passed over by the minute hand = 12 times the space passed over by the hour-hand ∴ the two hands together go over 13 times the space passed over by the hour hand. But the man leaves home between 9 and 10 and returns between 11 and 12 when he finds that the hands have changed places, ∴ the two hands have gone over 2×60 minute spaces

∴ 13 times the space gone over by the hr. hand = 120 minute spaces ∴ space passed over by the hour hand = $\frac{120}{13}$ min. In order to find the time when he returned we are to find the time after 11 when the minute hand is $\frac{120}{13}$ minute spaces behind the hour-hand,

$$\therefore 11 : 55 - \tfrac{120}{13} :: 12 = \tfrac{7140}{143} = 49\tfrac{133}{143} \text{ min.}$$

∴ the man returned after $49\frac{133}{143}$ min. past 11 o'clock.
Now, to find the time when he went out: $11 : 45 + \frac{120}{13} :: 12 = \frac{8460}{143} = 59\frac{23}{143}$ the man left home at $59\frac{23}{143}$ min past 9 o'clock. ∴ the man was away for (11 hrs. $49\frac{133}{143}$ min.) − (9 hrs $59\frac{23}{143}$ min.) = 1 hr. $50\frac{10}{13}$ min. *Ans.*

6. £1 : £112 :: 3*s.* 8*d.* = £$\frac{308}{15}$ *S.P.* of 1 cwt.
C.P. of 1 cwt. = 16 guineas = £$\frac{84}{5}$ ∴ £$\frac{308}{15}$ − £$\frac{84}{5}$ = £$\frac{56}{15}$ gain
£$\frac{84}{5}$: £100 :: £$\frac{56}{15}$ = $\frac{200}{9}$ = $22\frac{2}{9}$ gain %. $122\frac{2}{9} - 101 = 12\frac{2}{9}$
Had the man sold the whole for £$151\frac{1}{4}$ he would have gained only $1\frac{2}{9}$ %. ∴ £$112\frac{2}{9}$: £$151\frac{1}{4}$ :: £100 : £135 total cost. 1 cwt. costs £$\frac{84}{5}$.
∴ the total cost being £135 the man buys 8 cwts. 0 qr. 4 lbs. *Ans.*

7. 1 rouble = 2·4 francs. 23·5 francs = £1 ∴ £1 = $\frac{235}{24}$ roubles.
$\frac{235}{24}$ roubles : 7990 roubles :: £1 = £816.
Again, 3 roubles = 7 francs. 23·8 francs = £1 ∴ £1 = $\frac{51}{5}$ roubles.
$\frac{51}{5}$ roubles : 7990 roubles :: £1 = £$\frac{2350}{3}$ = £$783\frac{1}{3}$.
∴ £816 − $783\frac{1}{3}$ = £$32\frac{2}{3}$ gain of broker. *Ans.*

8. Let the whole sum be invested in 4%; then the income for 18 mos would be £$1946\frac{1}{4} \times \frac{4}{100} \times \frac{18}{12}$ = £116 15*s.* 6*d.* ∴ diff. in income = (£125 4*s.* $10\frac{1}{2}$*d.*) − (£116 15*s.* 6*d.*) = £8 9*s.* $4\frac{1}{2}$*d* = £$\frac{271}{32}$, and the difference of income of £1 for 18 months at 4% and $4\frac{1}{2}$% = $\frac{27}{400} - \frac{3}{50}$ = £$\frac{3}{400}$
∴ sum invested in $4\frac{1}{2}$% = £$\frac{271}{32} \times \frac{400}{3}$ = £1129 3*s.* 4*d.*
∴ " " " 4 % = £817 1*s.* 8*d.* *Ans.*

9. The amt. of £91 5*s.* or £$\frac{365}{4}$ in 2 yrs = £100 12*s.* $0\frac{3}{4}$*d.* = £$\frac{32193}{320}$
∴ the amount of £1 in 2 years = $\frac{32193}{320} \times \frac{4}{365}$ = £$\frac{441}{400}$ = £1·1025; hence the amount of £1 for 1 year = $\sqrt{1 \cdot 1025}$ = £1·05. ∴ rate per cent = 5.
£5 : £$91\frac{1}{4}$ :: £100 sum = £1825. *Ans.*

10. The P. W. of £1751 due 9 mos. hence at 4% = £ $\dfrac{1751 \times 100}{103}$ = £1700.
£($106\frac{3}{8} - \frac{1}{8}$) : £1700 :: £100 stock = £1600 stock. *Ans.*

Algebra.

1889.

3. $(x^2+y^2+z^2-yz-zx-xy)(x+y+z)=x^3+y^3+z^3-3xyz$
Now $(y-z)^3+(z-x)^3+(x-y)^3-3(y-z)(z-x)(x-y)$ has a factor $(y-z)+(z-x)+(x-y)$ which is equal to 0
$\therefore \quad (y-z)^3+(z-x)^3+(x-y)^3-3(y-z)(z-x)(x-y)=0$

5. $(2x+1)(2x+7)(2x+3)(2x+5)+16.$
$= \quad (4x^2+16x+7)(4x^2+16x+15)+16$
$=\{(4x^2+16x)^2+22(4x^2+16x)+105\}+16$
$= \quad (4x^2+16x)^2+22(4x^2+16x)+121=(4x^2+16x+11)^2$
$\therefore$ the sq. rt. $=4x^2+16x+11.$ *Ans.*

1890.

2. $$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{x^3}{xyz}+\frac{y^3}{xyz}+\frac{z^3}{xyz}=\frac{x^3+y^3+z^3}{xyz}$$
$=\dfrac{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz}{xyz}$; but $x+y+z=0$
$\therefore =\dfrac{3xyz}{xyz}=3.$ *Q.E.D.* Or $x+y+z=0$, cube both sides
$\therefore \; x^3+y^3+z^3+3(x+y)(y+z)(z+x)=0$
$\because \; x+y=-z,\; y+z=-x,\; z+x=-y$
$\therefore \; x^3+y^3+z^3+3\times-z\times-x\times-y=0 \;\; \therefore \; x^3+y^3+z^3=3xyz$, &c. Or
$z=-(x+y) \;\; \therefore \; x^3+y^3+z^3=x^3+y^3-(x+y)^3$
$=x^3+y^3-x^3-y^3-3xy(x+y)=-3xy(x+y)=-3xy\times-z=3xyz$, &c.

2. $m^2=a^2x^2+b^2y^2+2abxy.\; n^2=b^2x^2+a^2y^2-2abxy$
$\therefore \quad m^2+n^2=a^2x^2+b^2x^2+a^2y^2+b^2y^2=(a^2+b^2)(x^2+y^2)$;
ut $a^2+b^2=1 \;\; \therefore \; m^2+n^2=x^2+y^2.$ *Q E.D.*

4. The left hand side
$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}+\frac{2}{(y-z)(z-x)}+\frac{2}{(z-x)(x-y)}+\frac{2}{(x-y)(y-z)}$$
$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}+\frac{2}{z-x}\left(\frac{1}{y-z}+\frac{1}{x-y}\right)+\frac{2}{(x-y)(y-z)}$$
$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}+\frac{2}{z-x}\left\{\frac{x-z}{(y-z)(x-y)}\right\}+\frac{2}{(x-y)(y-z)}$$
$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}-\frac{2}{(y-z)(x-y)}+\frac{2}{(y-z)(x-y)}$$
$$=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}.\quad Q.E.D.$$

5. The expression.

$= a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)+2abc.$

$= a^2(b+c)+ac(b+c)+ab(b+c)+bc(b+c)$

$= (b+c)(a^2+ac+ab+bc)=(b+c)(c+a)(a+b).$ *Ans.*

6. The expression $=(x+3a)(x+a)(x-a)(x-3a)+K$

Now, the product of any four numbers which increase or decrease by the same quantity *together with the fourth power of that quantity* is a square. The numbers here decrease by $2a$

$\therefore K=(-2a)^4 =16a^4$. *Ans.*

8. Let x be the positive quantity, then the reciprocal of $x=\frac{1}{x}$.

$$\therefore x+\frac{1}{x}=\frac{x^2+1}{x}=\frac{x^2-2x+1+2x}{x}=\frac{(x^2-2x+1)+2x}{x}=\frac{(x-1)^2+2x}{x}$$

$=2+\frac{(x-1)^2}{x}$; and this is positive. $\because$ $(x-1)^2$ is positive. Q.E.D.

1891.

3. $ax^2+by^2=\frac{1}{a+b}\therefore (a+b)(ax^2+by^2)=1$

$\therefore a^2x^2+b^2y^2+abx^2+aby^2=1.$

And $ax+by=1$ $\therefore a^2x^2+b^2y^2+2abxy=1.$

$\therefore a^2x^2+b^2y^2 = 1-2abxy.$

$\therefore 1-2abxy+abx^2+aby^2=1 \therefore x^2-2xy+y^2=0.$

$\therefore (x-y)^2=0 \therefore x-y=0 \therefore x=y$

Putting the value of y in $ax+by=1$, we get

$(a+b)x=1 \therefore x=\frac{1}{a+b} \therefore x=y=\frac{1}{a+b}.$

Hence $ax^n+bx^n=x^n(a+b)=\left(\frac{1}{a+b}\right)^n(a+b)=(a+b)1-n.$

4. The expression $=\frac{(2b+x)(x+2a)+(2b-x)(x-2a)-4ab}{4b^2-x^2}$

$=\frac{x^2+2x(a+b)+4ab-4ab+2x(a+b)-x^2-4ab}{4b^2-x^2}$

$=\frac{4x(a+b)-4ab}{4b^2-x^2}$; but $x(a+b)=ab$. $\therefore =\frac{4ab-4ab}{4b^2-x^2}=0.$ *Ans.*

8. $x-\frac{1}{x}=p \therefore x^2-\frac{1}{x^2}=q. \therefore p^2(p^2+4)=\left(x-\frac{1}{x}\right)^2\left\{\left(x-\frac{1}{x}\right)^2+4\right\}$

$=\left(x-\frac{1}{x}\right)^2\left(x^2-2+\frac{1}{x^2}+4\right)=\left(x-\frac{1}{x}\right)^2\left(x+\frac{1}{x}\right)^2$

$=\left(x^2-\frac{1}{x^2}\right)^2=q^2.$ Q. E. D.

9. We get three equations :—

$4a+2b+c=8$ (i) ; $9a+3b+c=22$ (ii) ; $16a+6b+c=42$ (iii).

Solving these equations we get $a=3$, $b=-1$, $c=-2$.

When $x=-\frac{1}{3}$, $ax^2+bx+c=\frac{1}{3}+\frac{1}{3}-2=-\frac{4}{3}$. *Ans.*

10. (i) $x^3+\frac{1}{x^3}=\frac{65}{8}$. $\therefore 8x^6+8=65x^3 \therefore 8x^6-65x^3+8=0$. $\therefore (x^3-8)$

$(8x^3-1)=0 \therefore x^3=8$ or $\frac{1}{8} \therefore x=2$ or $\frac{1}{2}$. *Ans.*

(ii) $2^x=2^{3(y+1)} \therefore x=3y+3$. $3^{2y}=3^{x-9} \therefore 2y=x-9$

Solving the two equations, we get $x=21$, $y=6$.

1892.

1. Putting $x=3$, we get $5\times81+6\times27-7\times9-8\times3-480$ which is 0.

$\therefore$ one of the factors of the expression $=x-3$

$\therefore$ it is exactly divisible by $x-3$.

2. $4n^3+16n^2+21n+9=4n^3+6n^2+10n^2+15n+6n+9$.

$=2n^2(2n+3)+5n(2n+3)+3(2n+3)=(2n^2+5n+3)(2n+3)$

$=(2n+3)(2n+3)(n+1)$.

$6n^3+23n^2+29n+12=6n^3+9n^2+14n^2+21n+8n+12$.

$=3n^2(2n+3)+7n(2n+3)+4(2n+3)=(3n^2+7n+4)(2n+3)$.

$=(2n+3)(3n+4)(n+1)$

$8n^3+28n^2+32n+12=8n^3+12n^2+16n^2+24n+8n+12$

$=4n^2(2n+3)+8n(2n+3)+4(2n+3)$

$=(4n^2+8n+4)(2n+3)=4(2n+3)(n^2+2n+1)=4(2n+3)(n+1)^2$

The dividend

$\therefore =(2n+3)(n+1)\{x^2(2n+3)+2xy(3n+4)+4y^2(n+1)\}$

The divisor $=(n+1)(2n+3)(x+2y)$

$\therefore$ dividing $x^2(2n+3)+2xy(3n+4)+4y^2(n+1)$ by $x+2y$, we get

$(2n+3)x+2(n+1)y$. *Ans.*

7. The expression $=(x^2-yz+y^2-zx+z^2-xy)\{(x^2-yz)^2$

$+(y^2-zx)^2+(z^2-xy)^2-(x^2-yz)(y^2-zx)-(y^2-zx)(z^2-xy)$

$-(z^2-xy)(x^2-yz)\}$

Simplifying the second factor, and arranging the terms, we get

$x^4+x^3y+x^3z)+(y^3x+y^4+y^3z)+(z^3x+z^3y+z^4)-3x^2yz-3y^2zx-3z^2xy$

$=x^3(x+y+z)+y^3(x+y+z)+z^3(x+y+z)-3xyz(x+y+z)$

$(x+y+z)(x^3+y^3+z^3-3xyz)$

$(x^2-yz+y^2-zx+z^2-xy)(x+y+z)(x^3+y^3+z^3-3xyz)$

$(x^3+y^3+z^3-3xyz)^2$, a perfect square.

Or, let $a=x^2-yz$, $b=y^2-zx$, $c=z^2-xy$

$\therefore$ the expression $=a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$

$=(a+b+c)\{(a^2-bc)+(b^2-ca)+(c^2-ab)\}$

Now, $a^2-bc=(x^2-yz)^2-(y^2-zx)(z^2-xy)$

$=\{x^4-2x^2yz+y^2z^2\}-\{y^2z^2-x(y^3+z^3)+x^2yz\}$

$=x^4+x(y^3+z^3)-3x^2yz=x(x^3+y^3+z^3-3xyz)$.

Similarly

$b^2-ca = y(y^3+z^3+x^3-3xyz)$ and $c^2-ab=z(z^3+x^3+y^3+3xyz)$

$\therefore \quad (a^2-bc)+(b^2-ca)+(c^2-ab)$

$=x(x^3+y^3+z^3-3xyz)+y(x^3+y^3+z^3-3xyz)+z(x^3+y^3+z^3-3xyz)$ and

$a+b+c=(x^2-yz)+(y^2-zx)+(z^2-xy)=x^2+y^2+z^2-xy-yz-zx$.

$\therefore$ the whole expression $=(x^3+y^3+z^3-3xyz)^2$

8. $a^b=b^a \therefore a=b^{\frac{a}{b}}$. Now, $\left(\frac{a}{b}\right)^{\frac{a}{b}}=\dfrac{a^{\frac{a}{b}}}{b^{\frac{a}{b}}}=\dfrac{a^{\frac{a}{b}}}{a}=a^{\frac{a}{b}-1}$. Q. E. D.

1893.

1. $a^4+4b^4+2a^2b^2 = a^4+4a^2b^2+4b^4-2a^2b^2$

$= (a^2+b^2)^2-(\sqrt{2}ab)^2 = (a^2+\sqrt{2}ab+b^2)(a^2-\sqrt{2}ab+b^2)$ Ans

$a^3(b-c)+b^3(c-a)+c^3(a-b)$

$=a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2)$

$=(b-c)\{a^3-a(b^2+bc+c^2)+bc(b+c)\}$

$=(b-c)\{a^3-ab^2-abc-ac^2+b^2c+bc^2\}$

$=(b-c)\{a(a^2-b^2)-bc(a-b)-c^2(a-b)\}$

$=(b-c)(a-b)\{a(a+b)-bc-c^2\}=(b-c)(a-b)(a^2-c^2+ab-bc)$

$=(b-c)(a-b)(a-c)(a+b+c)=-(a-b)(b-c)(c-a)(a+b+c)$. Ans.

5. Squaring both sides, we get

$x^2+y^2+z^2+1+2xy+2xz-2x+2yz-2y-2z$

$=2(1-x-y+xy-z+xz+yz-xyz)$.

Hence, simplifying, we get $x^2+y^2+z^2-1+2xyz=0$

6. The numerator

$=b^6-c^6-3b^2c^2(b^2-c^2)+c^6-a^6-3c^2a^2(c^2-a^2)+a^6-b^6-3a^2b^2(a^2-b^2)$

$=-3\{a^2b^2(a^2-b^2)+b^2c^2(b^2-c^2)+c^2a^2(c^2-a^2)\}$

$=-3\{a^4(b^2-c^2)-a^2(b^4-c^4)+b^2c^2(b^2-c^2)\}$

$=-3(b^2-c^2)\{a^4-a^2(b^2+c^2)+b^2c^2\}$

$=-3(b^2-c^2)(a^4-a^2b^2-a^2c^2+b^2c^2)$

$=-3(b^2-c^2)(a^2-b^2)(a^2-c^2)=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$=3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

Or thus, $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

$$=(a^2-b^2)+(b^2-c^2)\{(a^2-b^2)^2-(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2\}+(c^2$$
$$=(a^2-c^2)\{(a^2-b^2)^2-(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2\}-(a^2-c^2)^3$$
$$=(a^2-c^2)\{(a^2-b^2)^2-(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2-(a^2-c^2)^2\}$$
$$=(a^2-c^2)[\{(a^2-b^2)^2-(a^2-c^2)^2\}-(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2]$$
$$=(a^2-c^2)\{(a^2-b^2+a^2-c^2)(c^2-b^2)-(a^2-b^2)(b^2-c^2)+(b^2-c^2)^2\}$$
$$=(a^2-c^2)(c^2-b^2)\{(2a^2-b^2-c^2)+(a^2-b^2)+(c^2-b^2)\}$$
$$=(a^2-c^2)(c^2-b^2)(2a^2-b^2-c^2+a^2-b^2+c^2-b^2)$$
$$=3(a^2-c^2)(c^2-b^2)(a^2-b^2)=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$$

Similarly, the denominator $=3(b-c)(c-a)(a-b)$

$\therefore$ the given expression $=(a+b)(b+c)(c+a)$

7. $x^{2n}+a^{2n}+ax^{2n-1}+a^{2n-1}x-a^{n-1}x^{n+1}-2a^nx^n-a^{n+1}x^{n-1}$

$$=x^{2n}-a^nx^n-a^nx^n+a^{2n}+ax^{2n-1}-a^{n+1}x^{n-1}+a^{2n-1}x-a^{n-1}x^{n+1}$$
$$=x^n(x^n-a^n)-a^n(x^n-a^n)+ax^{n-1}(x^n-a^n)-a^{n-1}x(x^n-a^n)$$
$$=(x^n-a^n)(x^n-a^n+ax^{n-1}-a^{n-1}x)$$

$\therefore$ when n is an even integer, the expression is divisible by x^2-a^2. *Q.E.D.*

9. The cube root of the expression $=2x^2+x+1$

Add 1 to both sides

$$\therefore 8x^6+12x^5+18x^4+13x^3+9x^2+3x+1=7+1=8$$

Extract the cube root of both sides $\therefore 2x^2+x+1=2$

$\therefore 2x^2+x-1=0 \therefore (x2-1)(x+1)=0 \therefore x=-1$, or $\frac{1}{2}$. *Ans.*

10. $x(y-z)^2+y(z-x)^2+z(x-y)^2+9xyz$

$$=\{x(y-z)^2+y(z-x)^2+z(x-y)^2+8xyz\}+xyz$$

$=(x+y)(y+z)(z+x)+xyz$ (See Q. 5, 1890).

Again, $x+y=-z,\ y+z=-x,\ z+x=-y$

$\therefore -z\times-x\times-y+xyz=0$.—Q E.D.

1894.

4. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c} \quad \therefore \frac{bc+ac+ab}{abc}=\frac{1}{a+b+c}$

$$\therefore \frac{(ab+ac+bc)(a+b+c)}{abc}=1 \therefore (ab+ac+bc)(a+b+c)-abc=0$$

$\therefore (a+b)(b+c)(c+a)=0 \therefore$ any of these factors $=0 \therefore a=-b$

$$\therefore \frac{1}{a}=-\frac{1}{b} \therefore \frac{1}{a^3}=-\frac{1}{b^3} \therefore \frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=-\frac{1}{b^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{1}{c^3}$$

$$=\frac{1}{-b^3+b^3+c^3}=\frac{1}{a^3+b^3+c^3}. \quad Q.E.D.$$

5. $\therefore$ Product $= H.C.F. \times L.C.M.$

$$\therefore \frac{(x-7)(x^3-10x^2+11x+70)}{(x^2-5x-14)} = x^2-12x+35. \quad \textit{Ans.}$$

8. If the sum of two or more squares be zero, then each is zero.

$$\therefore (x-a)^2=0,\ (y-b)^2=0 \ \therefore\ x=a,\ y=b. \quad \textit{Ans.}$$

1895.

2. $a+b=x,\ a-b=y,\ a^3-b^3=(a-b)^3+3ab(a-b)$ and ab

$$=\frac{(a+b)^2-(a-b)^2}{4} \therefore\ a^3-b^3=(a-b)^3+3(a-b)\left\{\frac{(a+b)^2-(a-b)^2}{4}\right\}$$

$$=y^3+3y\left(\frac{x^2-y^2}{4}\right)=\frac{4y^3+3x^2y-3y^3}{4}=\frac{y^3+3x^2y}{4}=\frac{y}{4}(y^2+3x^2)$$

Putting the values given in the result above obtained, we have

$$\frac{y}{4}(y^2+3x^2)=\left(\frac{5002-4998}{4}\right)\{(5002-4998)^2+3(5002+4998)^2\}$$

$$= \frac{4}{4}\left\{4^2+3(10000)^2\right\} = 300000016. \quad \textit{Ans.}$$

3. See Matric. Paper, 1887-88, Ques. 6.

8. $\left(c-\frac{b^2}{4}\right)^2=4c$ and $d^2=b^2c$. Hence, $c=\frac{d^2}{b^2}\ \therefore\ a=\frac{16}{4}=4.$ *Ans.*

1896.

3. $$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{b}-\frac{y}{a}\right)=\frac{1}{ab}(x^2+y^2)-xy\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

Now, $x^2-xy+y^2 = a^3+b^3$; $xy = ab(a+b)$.

Adding the two together we get—

$$x^2+y^2=a^3+b^3+ab(a+b)=(a^2+b^2)(a+b)$$

$$\therefore \text{ the expression} = \frac{1}{ab}(a+b)(a^2+b^2)-ab(a+b)\left(\frac{a^2+b^2}{a^2b^2}\right)$$

$$=\frac{1}{ab}(a+b)(a^2+b^2)-\frac{1}{ab}(a+b)(a^2+b^2)=0. \quad Q.\ E.\ D.$$

Euclid.

1889.

1. Add half the difference to half the sum, and the result is the greater of the sought magnitudes; and subtract half the difference from half the sum, and the remainder is the less.

2. Let the adjacent supplementary angles be ABC and CBD and let the bisectors of these angles be FB and BE respectively,

The angles FBC and CBE together = half the angles ABC, CBD; but angles ABC, CBD together = 2 right angles (I. 13) $\therefore$ angles FBC, CBE, *i. e.*, angle FBE = a right angle.

3. Let ABC and DEF be two isosceles triangles which have their vertices A and D equal to each other. Then the base angles shall be equal. The three angles of a triangle are together = 2 rt. angles (I. 32), and as the vertical angles A, D are equal, the base angles at B and C are equal to the base angles at E and F.

4. Let $ABCD$ be a plm. Let the diagonals meet at E. Now the diagonals of a plm. bisect each other $\therefore$ E is the middle point of AC BD.

Again, in any tr. the sum of the sqrs. on the two sides is equal to twice the sq. on half the third side, together with twice the sq. on the median which bisects this side $\therefore$ in tr. ABC, $AB^2+BC^2=2AE^2+2BE^2$. Again in tr. ADC, $AD^2+DC^2=2EC^2+2ED^2$; but $AE = EC$ and $EB = ED$ $\therefore$ $AB^2+BC^2+CD^2+DA^2=4AE^2+4EB^2$.

Again, sq. on a str. line = 4 times the sq. on half the line $\therefore AB^2+BC^2+CD^2+DA^2=AC^2+BD^2$.—Q.E.D.

5. Construct the figure as directed. Join AC, AD. The chord AB is common to both the equal circles $\therefore$ arc AB, of one circle = arc AB of another (III 28) $\therefore$ ang. ADB = ang. ACB (III. 27) $\therefore$ $AC=AD$.

6. See Potts' Euclid, Proposition V. appended to Book III.

7. Let the quadrilateral be described about the circle $EFGH$ Let AB, BC, CD, DE, touch the circle in the points E, F, G, H, respectively. $AH=AE$ (III 17 cor). Similarly, $EB=BF$, $DG=DH$, $GC=CF$; $\therefore$ $AB+DC=AD+BC$.

8. To draw an escribed circle of a given triangle. (For proof, see Hall and Stevens' Euclid, p. 255.)

1890.

2. The medians of a triangle are concurrent. (For proof, see Hall and Stevens' Euclid, p. 15)

6. (See Matric Paper, 1878, Ques. 4.)

10. Let ABC be the equilateral tr. inscribed in the circle and let the arcs BDC and BEA, cut off by BC, BA of the equilateral triangle ABC be bisected in D and E respectively. Join ED, and let it cut BA and BC in F and G. Join FB. The arcs BD, AE are equal. $\therefore$ ang. BED = ang. EBA (III. 27). $\therefore$ $FE=FB$ (I.). Similarly, $BG=GD$. Again, arc DC = arc AE $\therefore$ ang. ADE = ang. DAC and these are alternate, $\therefore$ ED is prll. to AC (I. 27). $\because$ FG is prll. to AC, $\therefore$ ang. BFG = ang. FAC and ang. BGF = ang. GCA; $\therefore$ the tr. BFG is equilateral $\therefore$ $EF=EB=FG=GB=GD$. *i.e.*, the str. line ED is trisected.

1891.

7. (For proof, see Hall and Stevens' Euclid, pp 106 and 227.)

9. Construct as directed. Join AO, BO, CO. AO, BO, CO, bisect angles at A, B, C. respectively; ang. FOB = angs. $OAB+OBA$ $=\frac{1}{2}$ ang. $A+\frac{1}{2}$ang. B, ang. FBO = angs. $FBC+CBO$ = ang. FAC (III. 21) and ang. $CBO=\frac{1}{2}$ ang. $A+\frac{1}{2}$ ang. B $\therefore$ ang. FOB = ang. FBO. $\therefore$ $FB=FO$ (I. 6), and $FC=FB$ (III. 26, 29) $\therefore$ $FC=FB=FO$.

1892.

3. Let LM, BAB' and CAC' be three given str. lines and let BAB' and CAC' intersect each other at A. Draw AD bisecting ang. BAC or $B'AC$. Let AD meet LM in P. Draw PE, PF perps. to AB, AC, respectively. Now, ang. AEP=ang AFP, ∵ they are rt. angs., and ang. EAP=ang. PAF and AP is common, ∴ $PE=PF$ (I. 26) ∴ P is the required point.

8. (See Matric. Paper, 1883. Ques. 6.)

1893.

5. Let $ABCD$ be the trapezium, having the sides AB, DC prll. Draw CE, DF perps. to AB.

Then $AC^2 = AB^2+BC^2-2AB.BE$ (II. 13). Again, $BD^2 = AB^2 + AD^2-2AB.AF$ (II. 13). ∴ $AC^2+BD^2 = BC^2+AD^2+2AB^2-2AB.BE-2AB.AF$. But $AB^2=AB.BE+AB.EF+AB.AF$ (II. 1). ∴ $2AB^2-2AB.BE-2AB.AF=2AB.EF=2AB.CD$. ∴ $AC^2+BD^2 = BC^2+AD^2+2AB.CD$.

7. Construct as directed. Angs. $QAB+BAD$=2rt. angs. (I. 13): angs. $BCP+BCD$=2rt. angs. (I. 13). ∴ angs. $QAB+BAD+BCP+BCD$=4 rt. angs. Now, the angs. $BAD+BCD$=2rt. angs. (III. 22) ∴ angs. $QAB+BCP$=2rt. angs. Again, angs. $QAB+BRQ$=2rt. angs. (III. 22). And angs. $BRP+BCP$=2rt. angs (III. 22) ∴ angs. $QAB+BRQ+BRP+BCP$=4 rt. angs. But angs. $QAB+BCP$=2rt. angs. (proved) ∴ angs $BRP+BRQ$=2rt. angs. ∴ the points P, R, Q, are in the same str. line (I. 14)

1894.

1. (See Matric. Paper, 1888, Ques. 2)

2. Join AO. ∵ $BZ=AZ$ (hyp.) ∴ tr. BZC=tr. AZC and tr. BZO =tr. AZO ∴ tr. BOC=tr. AOC=2 tr. COY ∴ $BO=2OY$.

3. (For proof, see Hall and Stevens' Euclid, p. 147.)

4. Let P be the point within the circle. Find O the centre of the circle, and join OP and draw APB perp to OP meeting the circumference in A and B. Then OP bisects AB (III. 3) To find the length of AB. Join OA. OA=5 feet, OP=3ft. ∴ AP=4ft. (I. 47) ∴ AB=8ft.

1895.

1. (See Matric. Paper, 1888, Ques. 1)

3. (See Matric. Paper, 1887, Ques. 1)

8. (For proof, see Hall and Stevens' Euclid, p. 255.)

1896.

3. (For proof. see Matric. Paper, 1884, Ques. 2.)

7. Let $ABCD$ be the quadrilateral inscribed in the circle. Let AB, DC be produced to meet in P, and AD, BC to meet in Q and let the bisectors of the angles so formed meet in K.

Ang. KPQ=½(angs. $CPQ+APQ$). ang. KQP=½(angs $CQP+AQP$): ang. PKQ=½(angs. $PCQ+PAQ$) (I. 32) =½(angs. $BCD+BAD$) 15) = one right angle.

APPENDIX.

The singular properties of the decimal $\dot{1}4285\dot{7}$, which is equal to $\frac{1}{7}$, should be carefully noted; for from it we can at once read off the decimals corresponding to multiples of $\frac{1}{7}$ less than unity. Thus by taking the order of largeness of these figures which is 1, 2, 4, 5, 7, 8, we get the following relations :— $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$; $\frac{4}{7} = \cdot\dot{5}7142\dot{8}$; $\frac{5}{7} = \cdot\dot{7}1428\dot{5}$; $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$.

So, also the properties of the decimals $\dot{0}7692\dot{3}$, which is equal to $\frac{1}{13}$, and $\dot{1}5384\dot{6}$ which is equal to $\frac{2}{13}$, may, with advantage, be remembered: because from these two periods we can read off the decimals corresponding to multiples of $\frac{1}{13}$ less than unity, *i.e.*, $\frac{2}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, &c.

Here the order of largeness of the figures is zero of the first period and of the second 1; 2 and 3 of the first and 3 of the second; 4 5, 6 of the second, and 6 of the first; 7 of the first and 8 of the second and lastly 9 of the first.

Hence—

$\cdot\dot{0}7692\dot{3} = \frac{1}{13}$	$\cdot\dot{1}5384\dot{6} = \frac{2}{13}$	$\cdot\dot{2}3076\dot{9} = \frac{3}{13}$	$\cdot\dot{3}0769\dot{2} = \frac{4}{13}$
$\cdot\dot{3}8461\dot{5} = \frac{5}{13}$	$\cdot\dot{4}6153\dot{8} = \frac{6}{13}$	$\cdot\dot{5}3846\dot{1} = \frac{7}{13}$	$\cdot\dot{6}1538\dot{4} = \frac{8}{13}$
$\cdot\dot{6}9230\dot{7} = \frac{9}{13}$	$\cdot\dot{7}6923\dot{0} = \frac{10}{13}$	$\cdot\dot{8}4615\dot{3} = \frac{11}{13}$	$\cdot\dot{9}2307\dot{6} = \frac{12}{13}$

Similarly, when we reduce $\frac{1}{7}$, $\frac{1}{17}$, $\frac{1}{8}$, &c., to recurring decimals, we can at once read off from the answers the decimals equal to all multiples of these fractions lower than unity.

Difficult Questions in Arithmetic.

(Madras University Questions.)

1. A person, after paying an income tax of 1 anna in the rupee devotes $\frac{1}{18}$ of the remainder to purposes of charity, and finds that he has Rs. 5,175 left: what is his income? (1865, A)

2. A book containing between 900 and 1000 pages is divided into four parts, each part being divided into chapters. The whole number of pages in each of the four parts is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the whole number of chapters in the book. (1867)

3. The distance by Railway from Madras to Salem is $206\frac{3}{4}$ miles. A passenger train travelling 20 miles an hour, leaves Madras at 7 a.m., and a special train at 10 a.m. the same day. At what rate must the latter travel, so as just to overtake the former at Jollarpett Junction (132 miles from Madras), and at what hour must a goods train leave Salem for Madras, travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other trains? (1870)

4. A lump composed of gold and silver measures 6 cubic inches and weighs 100 oz.; if a cubic inch of gold weighs 20 oz. and an equal bulk of silver 12 oz., find the weight of gold in the mixture.

5. A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value is increased by one-half, so that after reserving 20 acres for himself, he clears £200 on his purchase money by the sale of the remainder. How many acres were there?

6 A person going from Pondichery to Ootacamund, travels 90 miles by steamer, 330 miles by rail, and 30 miles by horse transit. The journey occupies 30 hours-50 minutes, and the rate of the train is three times that of the horse transit and $1\frac{1}{2}$ times that of the steamer. Find the rate of the train. (1873)

7. A person bought 10 Bank of Madras shares at Rs. 1,540 each, and for 5 years got interest on his investment at the rate of $5\frac{1}{2}$ per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction, and what rate per cent. per annum had he for his money? (1873).

8. Two trains, running at the rate of 25 and 20 miles an hour respectively, on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rates as before, a person sitting in the faster train observes that he passes the other in $31\frac{1}{2}$ seconds; find the lengths of the trains. (1873)

9. *A* barters sugar with *B*, for rice which is worth $1\frac{3}{4}$ annas a measure; but in weighing his sugar uses a false maund weight. *B* discovers this, and to make the exchange fair raises the price of his rice to $2\frac{1}{2}$ annas a measure. Find the real weight of the false maund which *A* uses. (1874)

10. A person pays an income tax of 4*d*. in the pound during the first half of the year, and 3*d*. in the pound during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income. (1874).

11. A peon walks from *A* to *B* at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to *A* by the tramway at the rate of 5 miles an hour. He then finds he has been absent 2 hours 20 minutes. Find the distance from *A* to *B*. (1874)

12. What length of wire will go round the edges of a cube, the surface of which contains 187 yards 54 inches? (1875)

13. A man bequeathed $\frac{5}{11}$ of his estate to one son, $\frac{7}{12}$ of the remainder to another son and the balance to his widow. The children's shares differ by Rs. 1,320, find the widow's share?

14. A train, 132 yards in length, travelling at a uniform speed, overtook a man walking along the line at the rate of 6 miles an hour, and passed him in 12 seconds; 20 minutes later the train overtook a second man, and passed him in 11 seconds. How many hours after the train overtook the second man would the first man also overtake him? (1879)

15. A bankrupt has book-debts equal in amount to his liabilities, but on Rs. 8,640 of such debts he can recover only $8\frac{1}{2}$ annas in the rupee, and on Rs. 6,300 only $5\frac{1}{4}$ annas in the rupee. After allowing Rs. 1,054 11as. for the expenses of bankruptcy, he finds he can pay his creditors 12 annas in the rupee. Find the total amount of his debts. (1881)

16. Two trains measuring 330 feet and 264 feet respectively, run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before, the faster train passes the other in $27\frac{1}{4}$ seconds. Find the speeds of the two trains in miles per hour? (1881)

17. A tea merchant has a rectangular space for storing tea. It is $15\frac{3}{4}$ feet long, $10\frac{1}{2}$ feet broad, and $9\frac{1}{8}$ feet high. He wishes to fill this space with packets of a cubical shape all of the same size. What is the largest size of such cubical packets that can be made to fill it exactly, and what would be the number of such packets?

18. A person holds forty Rs. 500 shares in a concern which pays dividend at the rate of 6 per cent. per annum. When the shares are at Rs. 675 he sells out and invests half the proceeds in 4 per cent. stock at 90. With the other half he buys a house, for which he received an annual rental of Rs. 1,440 subject to a deduction of Re. 0-3-9 per rupee for repairs and taxes. Find the alteration in his annual income? (1888.)

(Calcutta University Questions.)

19. If the price of bricks depends upon their magnitude, and if 100 bricks, of which the length, breadth, and thickness are 16, 8, and 10 inches respectively, cost Rs. 2,9a., what will be the price of 921,600 bricks, which are one-fourth less in every dimension? (1860)

20. *A* and *B* run a race. *A* has a start of 40 yards, and sets off 5 minutes before *B*, at the rate of 10 miles an hour. How soon will *B* overtake him if his rate of running is 12 miles per hour? (1860)

21. A leaky cistern is filled in 5 hours with 30 pails of 3 gallons each, but in 3 hours with 20 pails of 4 gallons each, the pails being poured in at intervals. Find how much the cistern holds, and in what time the water would waste away. (1860)

22. *A* race-course is half a mile long. *A* and *B* run a race, and *A* wins by 10 yards; *C* and *D* run over the same course and *C* wins by 30 yards; *B* and *D* run over it and *B* wins by 20 yards; if *A* and *C* run over it, which should win, and by how much? (1880)

23. A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s* 8*d*. per square foot and the turf 3*s*. per square yard, find the cost of laying out the court? (1894)

24. A schoolmaster divided his scholars, consisting of 221 boys and 143 girls, into the largest possible equal classes, so that each class of boys should number the same as each class of girls. Find the number of classes (Pupil Teachers' Examination.)

25. The population of a parish exceeds 3,000 and is less than 4,000 whether the people are arranged in groups of 8, of 9, of 15, of 18, or of 25, 7 always remain over. Find the exact population? (P.T.E.)

26. Find in grains the least weight which can be expressed by an exact number of ounces in both troy and avoirdupois weight? (R. M. A., Woolwich).

27. The areas of three squares are in the ratio of 1 : 9 : 16 ; the area of the second contains 944,784 square inches ; find the length of a side of each of the others in yards. (Queen's Scholarship Examination.)

28. Express $\sqrt{\dfrac{0\cdot\dot{4}2857\dot{1}\times 0\cdot771428\dot{5}}{0\cdot\dot{2}8571\dot{4}\times 0\cdot0\dot{5}7142\dot{8}}}$ as a vulgar fraction, reducing it to its simplest form. (London Matriculation.)

29. The longer sides of two rectangles are 189 and 244 yards, their shorter sides 45 and 36 yards; find the area of a square that is intermediate in area to the two rectangles, and whose side consists of an exact number of yards. (Q. S. E.)

30. A school-room accommodates 72 children at the rate of 12 square feet of area for each child ; if it had been 12 feet longer, it would have accommodated 90 children ; find the length and breadth of the room. (Q. S. E.)

31. For what sum should a ship worth £7,250 be insured at £3 6*s*. 8*d*. per cent, so that the owner might recover, in case of loss, the value of the ship and insurance? (P. T. E.)

32. A reduction of 20 per cent. in the price of apples would enable a purchaser to obtain 120 more for a sovereign : what may the price be before reduction? (L. M.)

33 On the first of January 1870 a contractor borrows a sum of money at 5 per cent simple interest. At the end of a year the rate of interest is reduced to $4\frac{1}{2}$ per cent The total amount of interest paid up to the end of 1875 is £1,760 ; what was the sum borrowed? (Q. S. E.)

34. What is the least number of years for which simple interest must be reckoned at [illegible] per cent. on £14[illegible] 7*s*. 6*d* so that the interest may be an exact number of pounds sterling? (P. T. E.)

35. £800 is put out at interest at 5 per cent. per annum, and at the end of each year £120 is deducted for the expenses of the next. What is left of the original capital at the end of the sixth year. (Q. S. E.)

36. A job can be finished in 25 days by 30 men ; at the end of each week (consisting of 6 days) 5 men are withdrawn ; how many weeks must the last five men work by themselves to finish the job? (Q. S. E.)

37. A railway train has a journey of 65 miles to perform, and ought to perform it in 3 hours: if its starting be delayed a quarter of an hour how many miles per hour must it increase its speed so as to arrive at the proper time? (Cambridge Previous Exam.)

38. A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour, and passed him completely in 10 seconds It afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking? (Oxford Responsions.)

39. Two boats start to row a race at 3 o'clock. The race is over at $6\frac{2}{3}$ minutes past 3, the losing boat being 40 yards behind at the finish. At 4 minutes past 3 this boat was 760 yards from the winning post. Find the speed of each boat in miles per hour. (Cambridge Previous.)

40 A person rows a distance of $1\frac{1}{2}$ miles down a stream in 20 minutes; but without the aid of the stream it would have taken him half an hour. What is the rate of the stream per hour, and how long would it take him to return against it? (C. P.)

41. Find when first after 2 o'clock the hour and minute hands of a clock will make an angle of 60° with each other. (Educational Department, Ontario.)

42. If the hands of a clock coincide every 66 minutes, how much does the clock gain or lose in a day? (C. P.)

43. Find the value of the series $\frac{1}{2}, \frac{1}{2\cdot 3}, \frac{1}{2\cdot 3\cdot 4}, \ldots$ indefinitely continued, accurate to seven places of decimals. (L. M.)

44. Calculate the value of $\sqrt{3+2\sqrt{2}}$ correctly to two places of decimals. (C. P.)

(Civil Service Papers in Higher Arithmetic.)

45. A cistern can be filled by one of two pipes in 30 minutes and by the other in 36 minutes. They are both opened together for a certain time, but being partially clogged, only $\frac{5}{6}$ of the full quantity of water flows through the former, and only $\frac{9}{10}$ through the latter. The obstructions, however, being suddenly removed, the cistern is filled in $15\frac{1}{2}$ minutes from that moment. How long was it before the full flow of water began?

46. *A* and *B* undertake to do each half of a piece of work. *A* begins at 9 a.m. *B* at 10-30, and both stop at 12, having then done $\frac{1}{3}$ of the work between them; they resume work at 1 p.m., and *A* finishes his share at 4 p.m., when will *B* have finished?

47. A question being proposed in an examination. to find the simple interest on a certain sum of money for $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent. a candidate by mistake reckoned it for $2\frac{1}{4}$ years at $3\frac{1}{2}$ per cent. and so obtaind a result too little by £26 4s. 8*d*. What ought the answer to have been?

48. The profits on a capital of £10,000 used in trade for 4 years, are equivalent to compound interest at 25 per cent. per annum for that time; how much do the profits amount to? A sum of money is

placed out at compound interest at a rate which doubles it in 4 years; how many fold will it be increased in 16 years?

49. A cistern, the cubic contents of which are 360 cubic feet, has two pipes which can empty it in 3 and 4 hours respectively. It has also a third pipe with an orifice of 1 sq. ft. through which water flows into the cistern at the rate of 1 yard per minute. If all the three pipes be opened together when the cistern is full, in what time will it be emptied?

50. A ship is so constructed that if one of its compartments be filled with water, it would just sink. There are two leaks in this compartment, one of which would just fill it in 4 hours, the other in 3 hours. If the compartment were full, the ship's pumps could just empty it in 2 hours How long could the ship be kept afloat after springing the leak?

SOLUTIONS.

1. Let the income be Re. 1 ∴ 16 as. − 1 an. = 15 as. = Re. $\frac{15}{16}$.

$$\tfrac{1}{16}\times\tfrac{15}{16}=\tfrac{15}{256}\,;\quad 1-\left(\tfrac{1}{16}+\tfrac{15}{256}\right)=\tfrac{225}{256}.$$

$\frac{225}{256}$: 1 :: Rs. 5,175 = Rs. 5,888. *Ans.*

2. L. C. M of 20, 40, 60, 80 = 240 ∴ the number of pages in each part must be a multiple of 240, and as the book is divided into 4 parts, the number of pages in the whole book must be a multiple of $4\times240=960$; and as the book contains between 900 and 1,000 pages there are 960 pages of the book. Each part of the book has 240 pages ∴ the number of chapters

$$=\tfrac{240}{20}+\tfrac{240}{40}+\tfrac{240}{60}+\tfrac{240}{80}=12+6+4+2=25.$$

3. The passenger train will reach Jollarpett in $\frac{132}{20}$ or $\frac{33}{5}$ hrs.

∴ the special train must travel 132 miles in $\frac{33}{5}-3$ *or* $\frac{18}{5}$ hrs.

∴ in 1 hr. it must travel $132\times\frac{5}{18}=36\frac{2}{3}$ miles. *Ans.*

The total distance from Salem to Jollarpett $=206\frac{1}{4}-132$ or $74\frac{1}{4}$ miles ∴ the goods train will take $\dfrac{74\frac{1}{4}}{15}$ hrs. or 4 hrs. 59 min. to travel this distance. The other trains reach Jollarpett in $\frac{33}{5}$ or 6 hrs. 36 min. from 7 A.M.

∴ the goods train must leave Salem in (6 hrs. 36 min − 4 hrs. 59 min.) or 1 hr. 37 min. after 7 A.M., *i.e.*, it must leave Salem at 37 min. past 8 A.M. *Ans.*

4. Supposing a cubic inch of gold to weigh 12 oz., we have $12\times6=72$ oz. for the weight of the lump ∴ 100 oz. − 72 oz. or 28 oz. is the excess of the weight of gold, but 20 oz. − 12 oz. or 8 oz. is the excess of the weight of 1 cubic inch of gold, ∴ $\frac{28}{8}$ is the number of cubic inches of gold in the lump ∴ weight of gold $=\frac{28}{8}\times20=70$ oz. *Ans.*

5. £ 500 = C. P. of 20 acres ∴ he gains £500 + £200 or £700 by the sale of the remainder. The gain on 1 acre = £$\frac{25}{2}$. ∴ £700 is the gain on 56 acres, *i.e.*, he sells 56 acres ∴ he bought 56 + 20 or 76 acres. *Ans.*

6. Time required for the whole journey = time taken in travelling $(90\times1\frac{1}{2}+330+30\times3)$ or 555 miles by rail, *i. e.*, in 30 hrs. 50 min. the train travels 555 miles ∴ in 1 hr. it travels 18 miles. *Ans.*

7. The total gain on Rs. 100 = Rs. $5\frac{1}{2} \times 5 - 22\frac{1}{2}$ = Rs. 5

$\therefore$ gain on Rs. 10×1540 = Rs 770

In 5 years the gain is Rs. 5 on Rs. 100 $\therefore$ in one year the gain is Re. 1 $\therefore$ 1 per cent is the rate of interest.

8. The sum of the lengths of the trains = dist. passed over in 8 sec. at the rate of $(20+25)$ or 45 miles an hr. $= \dfrac{8 \times 45 \times 1{,}760}{60 \times 60}$ or 176 yds.

Length of the slower train = distance passed over in $31\frac{1}{2}$ sec. at the rate of $(25-20)$ or 5 miles per hr. $= \dfrac{63}{2} \times \dfrac{5 \times 1{,}760}{60 \times 60} = 77$ yds. *Ans.*

$\therefore$ length of the faster = 176 yds. − 77 yds. = 99 yds. *Ans.*

9. $2\frac{1}{2}$ as. : $1\frac{7}{8}$ as :: 1 md. = 30 seers. *Ans.*

10. On £1 the tax is 4*d*. $\therefore$ on £$\frac{1}{4}$ the tax is 1*d*. for the first half-year, and during the second half-year the tax is 1*d* on £$\frac{1}{3}$ $\therefore$ the gross incomes = £$\frac{1}{4}$: £$\frac{1}{3}$, *i.e.*, 3 : 4. But the gross income for the year = £700 $\therefore$ the gross incomes of the half-years are £300 and £400 $\therefore$ the net income $= 300 \times \frac{236}{240} + 400 \times \frac{237}{240} =$ £295 + £395 = £690. *Ans.*

11. The peon takes $\frac{1}{3} + \frac{1}{5}$ or $\frac{8}{15}$ hr. in walking one mile and coming back by the tramway; the total time taken in walking from *A* to *B* and coming back by tramway $= 2\frac{1}{3}$ hrs. − 1 hr. $= 1\frac{1}{3}$ hr. $\therefore$ distance from *A* to *B* $= 1\frac{1}{3}$ mile $\div \frac{8}{15}$ or $2\frac{1}{2}$ miles. *Ans.*

12. 187 sq. yds. 54 sq. inches $= \dfrac{13{,}467}{8}$ sq. ft. = area of 6 sides

$\therefore$ the area of 1 side $= \dfrac{13{,}467}{8 \times 6} = \dfrac{4{,}489}{16}$ sq. ft.

$\therefore$ the edge $= \sqrt{\dfrac{4489}{16}} = \frac{67}{4}$ ft. : and there are 12 edges

$\therefore$ length of wire $= 12 \times \dfrac{67}{4 \times 3}$ yds. = 67 yds. *Ans.*

13. $1 - \frac{5}{12} = \frac{7}{12}$; $\frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$ share of the second son.
$1 - (\frac{5}{12} + \frac{49}{144}) = \frac{35}{144}$ widow's share. $\frac{5}{12} - \frac{49}{144} = \frac{11}{144}$.
$\frac{11}{144} : \frac{35}{144}$ :: Rs. 1,320 = Rs. 4,200. *Ans.*

14. The first man walks 6 miles in one hr.

$\therefore$ in 12 sec. he walks $35\frac{1}{5}$ yds.

$\therefore$ in 12 sec. the train travels $132 + 35\frac{1}{5} = 167\frac{1}{5}$ yds.

$\therefore$ in 11 sec. the train travels $\frac{11}{12} \times 167\frac{1}{5}$ yds. or $153\frac{4}{15}$ yds.

$\therefore$ in 11 sec. the second man travels $153\frac{4}{15} - 132 = 21\frac{4}{15}$ yds.

$\therefore$ in 12 sec. the second man travels $23\frac{1}{5}$ yds. Now, the first man gains $35\frac{1}{5} - 23\frac{1}{5}$ or 12 yds. over the second man in 12 secs.. $\therefore$ in 1 sec. he gains 1 yd.

In 12 sec. the train travels $167\frac{1}{5}$ yds. $\therefore$ in 20 min. it will travel $9\frac{1}{2}$ miles and in 12 sec. the first man walks $35\frac{1}{5}$ yds. $\therefore$ in 20 min. he will walk 2 miles, *i.e.*, when the train overtakes the second man, the first man is $9\frac{1}{2} - 2$ or $7\frac{1}{2}$ miles behind. Now, the first

man gains $35\frac{1}{5}-23\frac{1}{5}$ or 12yds in 12 sec. over the second man, *i.e.*, 1 yd. in 1 sec $\therefore$ time required=

$$1 \text{ yd.} : 7\tfrac{1}{2}\times 1760 :: 1 \text{ sec.} = \frac{15}{2}\times\frac{1,760}{60\times 60}=3\tfrac{2}{3} \text{ hrs.} \quad \textit{Ans.}$$

15. He gets $8\frac{1}{2}$ as. in the rupee. on Rs. 8,640 $\therefore$ he loses $1-\frac{17}{32}$, or $\frac{15}{32}$ of Rs 8,640 = Rs. 4,050. On Rs 6,300 he gets $5\frac{1}{4}$ as. $\therefore$ he loses Rs. $6,300\times\frac{43}{64}$ or Rs. $4,232\frac{13}{16}$ $\therefore$ he loses (Rs. 4,050 + Rs. $4,232\frac{13}{16}$ + Rs. $1,054\frac{11}{16}$) or Rs $9,387\frac{1}{2}$ altogether. He pays 12 as. in the rupee to the creditors $\therefore$ they lose 4 as. in the rupee.

4 as. : Rs. $9,337\frac{1}{2}$ :: 16 as. = Rs. 37,350. *Ans.*

16. The distance travelled by both in 9sec. = 330 ft. + 264 ft. = 594 ft. $\therefore$ in 1sec. = 66 ft. The distance travelled by the faster − the distance travelled by the slower in $27\frac{1}{2}$ sec. = 594 ft. $\therefore$ in 1 sec. = $21\frac{3}{5}$ft.

$\therefore$ twice the distance travelled by faster in 1 sec. = 66 ft. + $21\frac{3}{5}$ ft. = $87\frac{3}{5}$ ft. $\therefore$ the distance travelled by the faster in 1 sec. = $87\frac{3}{5}\div 2$ = $43\frac{4}{5}$ ft. $\therefore$ in 1 hr. = $29\frac{19}{22}$ miles.

$\therefore$ the distance travelled by the slower in 1 sec. = $43\frac{4}{5}$ ft. − $21\frac{3}{5}$ ft. or $22\frac{1}{5}$ ft. $\therefore$ in 1 hr. = $15\frac{3}{22}$ miles. *Ans.*

17. The length of the edge of each packet = H. C. F. of $15\frac{3}{4}$ ft. $10\frac{1}{2}$ ft., $9\frac{1}{3}$ ft., = $\frac{7}{12}$ ft. $\therefore$ number of packets

$$=\frac{(15\frac{3}{4}\times 10\frac{1}{2}\times 9\frac{1}{3}) \text{ cub. ft.}}{(\frac{7}{12}\times\frac{7}{12}\times\frac{7}{12}) \text{ cub ft.}} = 7,776. \quad \textit{Ans.}$$

18. Rs. 100 : Rs. 20,000 :: Rs. 6 = Rs 1,200 income.

Rs. 675 × 20 = Rs 13,500, half the proceeds.

Rs. 90 : Rs. 13,500 :: Rs. 4 = Rs 600 income on half the proceeds.
Rs. 1,440 × 12as. 3p. = Rs. $1,102\frac{1}{2}$ the annual rental after paying 3as. 9p.

$\therefore$ the alteration in income = Rs. $1,102\frac{1}{2}$ + Rs. 600 − Rs. 1,200 = Rs. 502 8 as. *Ans.*

19. 16 × 10 × 8 = 1280 cubic inches, contents of one brick
$12\times 6\times 7\frac{1}{2}$ = 540 cubic inches contents of the second kind
1.280 × 100 : 540 × 921,600 :: Rs 2 9as. = Rs. 9,963. *Ans.*

20. 60 min. : 5 min. :: 10 miles = $\frac{5}{6}$ mile.

40 yds. = $\frac{40}{1760}$ or $\frac{1}{44}$ mile. $\frac{5}{6}+\frac{1}{44}=\frac{113}{132}$ mile.

B has to gain $\frac{113}{132}$ mile. But he gains 2 miles in 60 min.

$\therefore$ to gain $\frac{113}{132}$ he will take $25\frac{15}{22}$ min. *Ans.*

21. The cistern is filled by 90 gall. in 5 hours and by 80 gall. in 3 hrs. $\therefore$ in 2 hrs. 10 gall. waste away, *i.e.*, there is a waste of 25 galls in 5 hrs, *i.e*, the cistern holds 90 − 25, or 65 gall. $\therefore$ 65 gall. will waste away in 13 hrs. at the rate of 5 gall. per hour.

22. $\frac{1}{2}$ mile = 880 yds. *B* runs 880 yds $\therefore$ *D* runs 860 yds.

$$\therefore \text{ when } D \text{ runs } 850, B \text{ runs } \frac{850\times 880}{860} \text{ yds.}$$

But by the question when *C* runs 880 yds, *D* runs 850 yds.

$$\therefore \text{ when } C \text{ runs } 880 \text{ yds., } B \text{ runs } \frac{850\times 880}{860} \text{ yds.}$$

Again, when *A* runs 880 yds., *B* runs 870 yds.

$$\therefore \text{ when } B \text{ runs } \frac{850\times 880}{860} \text{ yds., } A \text{ runs } \frac{850\times 880\times 880}{860\times 870} \text{ yds, } i.e.,$$

when C runs 880 yds., A runs $\dfrac{850\times880\times880}{860\times870}$ yds. or $879\frac{2861}{3741}$ yds.

$\therefore$ C wins by $(880-879\frac{2861}{3741}$ yds.$)$ or $\frac{880}{3741}$ yds. *Ans.*

23. There are 4 rectangular plots covered with grass and the length of each $=(50$ yds. $-\frac{2}{3}\times3$ yds.$)\div2=22$ yds., and the breadth of each $=(30$ yds $-\frac{2}{3}\times3)\div2=12$ yds. $\therefore$ the area of the grass plots $=4\times12\times22$ or 1,056 sq. yds. $\therefore$ the cost $=1056\times3s.=$ £158 8s.

Now, the area of space to be paved $=50\times30$ sq. yds. -1056 sq. yds. $=444$ sq. yds. $\therefore$ cost $=444\times9\times1s.\ 8d.=$ £333

$\therefore$ total cost $=$ £158 8s. $+$ £333 $=$ £491 8s. *Ans.*

24. The *G. C. M.* of 221 and $143=13$ $\therefore$ number in each class $=13$ $\therefore$ number of classes of boys $=\frac{221}{13}=17$ and number of classes of girls $=\frac{143}{13}=11=$ total number of classes $=17+11=28$. *Ans.*

25. The *L. C. M.* of 8, 9, 15, 18, $25=1800$

$\therefore$ the exact population $=1800\times2+7=3607$. *Ans.*

26. 1 lb. troy $=5760$ grains $\therefore$ 1oz. troy $=\frac{5760}{12}=480$ grs.
1 lb. avoir. $=7000$ grains $=$ 1oz. avoir $=\frac{7000}{16}=437\frac{1}{2}$ grs.

L.C.M of 480 grs. and $437\frac{1}{2}$. grs $=$ 84,000 grs. *Ans.*

27. Area of the first : 944784 sq. in. :: 1 : 9

$\therefore$ area of first $=\dfrac{944,784}{9}$ sq. in. $=104976$ sq. in.

$\therefore$ length of the side of the 1st $=\sqrt{104,976}$ sq. in. $=324$ in. $=9$ yds. *Ans.*

Again, area of first : area of third :: 1 : 16.

$\therefore$ area of third is 16 times area of first $\therefore$ length of the side of the third $=4$ times the length of the side of the first $=4\times9$ yds. $=36$ yds. *Ans.*

28. The fraction $\sqrt{\dfrac{\frac{3}{7}\times\frac{1}{10}\times7\frac{5}{7}}{\frac{2}{7}\times\frac{1}{10}\times\frac{4}{7}}}=\sqrt{\frac{81}{4}}=\frac{9}{2}=4\frac{1}{2}$. *Ans.*

29. Area of the first rect. $=189\times45=8,505$ sq. yds.
" " second " $=244\times36=8,784$ sq. yds.

$\left.\begin{array}{l}\sqrt{8505}=92\cdot2\ldots\ldots\\ \sqrt{8784}=93\cdot7\ldots\ldots\ldots\end{array}\right\}$ $\therefore$ the exact number required $=93$

$\therefore$ area of the square $=93^2=8649$ sq. yds. *Ans.*

30. Area required to accommodate $(90-72)$ or 18 children $=18\times12$ sq. ft. $=216$ sq. ft.

$\therefore$ 216 sq. ft. $=12$ ft. $\times$ breadth $\therefore$ breadth $=\frac{216}{12}$ ft. $=18$ ft. *Ans.*

31. £3 6s. 8d. $=$ £$3\frac{1}{3}$

$\therefore$ amount recovered on £100 $=$ £100 $-$ £$3\frac{1}{3}=$ £$96\frac{2}{3}$

$\therefore$ £$96\frac{2}{3}$: £7,250 :: £100 $=$ £7,500. *Ans.*

32. The reduction is 20 per cent, *i. e.*, $\frac{1}{5}$; $\therefore$ one could get as many apples for £$\frac{4}{5}$ as for £1 at the old price.

$\therefore$ £1 $-$ £$\frac{4}{5}$ or 4s. $=$ price of 120 apples.

$\therefore$ for £$\frac{4}{5}$ or 16s.; 480 apples can be bought.

But the price of 480 apples before reduction was £1 $\therefore$ the price before reduction of 1 apple $=$ £$\frac{1}{480}$ or 2 farthings. *Ans.*

33. The int. for 1 year is £5 on 100. The int. for next 5 years $= 4\frac{1}{4} \times 5 = £21\frac{1}{4}$
$\therefore$ total int. on $100 = £21\frac{1}{4} + £5 = £26\frac{1}{4}$ $£26\frac{1}{4} : £1,760 :: £100 = £6,704$ 15s. $2\frac{6}{7}$d. *Ans.*

34. £145 7s. 6d. $= £145\frac{3}{8} = £\frac{1163}{8}$. $£100 : £\frac{1163}{8} :: £4 = £\frac{1163}{200}$.

In order that the int. may be an exact number of pounds, $\frac{1163}{200}$ must be multiplied by 200
$\therefore$ 200 least number of years. *Ans.*

35.

£800 capital for the 1st year.
5
40·00 int. for 1st year.
800
840
120
720 cap. for 2nd year.
5
36·00 int. for 2nd year.
720
756
120
636 cap. for 3rd year.
5
31·80 int. for 3rd year.
636
667·8
120
547·8 cap. for 4th year.
5
27·390 int. for 4th year.

27·390 int. for 4th year.
547·8
575·19
120
455·19 cap. for 5th year.
5
22·7595 int. for 5th year.
455·19
477·9495
120
357·9495 cap. for 6th year.
5
17·897475 int. for 6th year.
357·9495
375·846975
120
255·846975. £255·846975 left at the end of the 6th year. *Ans.*

36. In 6 days 30 men can do $\frac{6}{25}$ of the work.

	„	6	„	5	„	„	$\frac{1}{25}$	„
∴	„	6	„	25	„	„	$\frac{5}{25}$	„
∴	„	6	„	20	„	„	$\frac{4}{25}$	„
∴	„	6	„	15	„	„	$\frac{3}{25}$	„
∴	„	6	„	10	„	„	$\frac{2}{25}$	„
∴	„	6	„	5	„	„	$\frac{1}{25}$	„

$\frac{1}{25} : \frac{5}{25} :: 1$ week $= 5$ weeks. *Ans.*

37. Speed of the train $= \frac{65}{3}$ miles per hr.

Speed of the train when it is delayed $\frac{1}{4}$ hr. $= \frac{65}{2\frac{3}{4}} = \frac{260}{11}$ miles

∴ increase per hr. $= \frac{260}{11} - \frac{65}{3} = \frac{65}{33}$ miles $= 1\frac{32}{33}$ miles per hr. *Ans.*

38. 10 sec : 60×60 sec. :: 88yds. or $\frac{1}{20}$ mile $= 18$ miles the distance gained by the train in 10 sec.

∴ rate of the train $= 18 + 4 = 22$ miles per hr.

Again, the distance gained by the train in 9sec. $= 88$yds. or $\frac{1}{20}$ mile

∴ in 1 hr. 20 miles is the distance gained.

∴ rate of the second man $= 22 - 20 = 2$ miles per hr. *Ans.*

39. The losing boat goes $700 - 40$ or 660 yds. in $6\frac{2}{5} - 4$ or $2\frac{2}{5}$min.

∴ in 1 hr. it goes $\frac{75}{8}$ miles., *i.e.*, $9\frac{3}{8}$ miles. *Ans.*

$2\frac{2}{5}$min. : $6\frac{2}{5}$min. :: 660 yds $= 1760$yds.

∴ the winning boat goes $1760 + 40$ or 1800yds. in $6\frac{2}{5}$min.

∴ in 1 hr. it will go $9\frac{207}{352}$ miles. *Ans.*

40. Without the aid of the stream the man rows $1\frac{1}{2}$ miles in $\frac{1}{2}$ hr.' *i. e.*, 3 miles in one hr. With the aid of the stream the man rows $1\frac{1}{2}$ miles in 20 min., *i. e.*, $\frac{1}{3}$hr. ∴ in 1 hr. he rows $4\frac{1}{2}$ miles.

∴ the rate of stream $= 4\frac{1}{2} - 3 = 1\frac{1}{2}$ miles per hr.

Again, the man rows $3 - 1\frac{1}{2}$ or $1\frac{1}{2}$ miles per hr. against the stream

∴ in one hour he rows $1\frac{1}{2}$ miles. *Ans.*

41. A circle has 360° and it is divided here into 60 minute spaces

∴ $60° = 10$ minute spaces.

Now, at 2 o'clock the hands are 10 minute spaces apart, ∴ the minute hand must gain $10 + 10$ or 20 minute divisions on the hour hand to make an angle of 60° or to be 10 minute spaces apart.

Hence 55 : 20 :: 60min. $= 21\frac{9}{11}$ past 2 o'clock. *Ans.*

42. The minute hand gains on the hour hand 55 minute spaces in 1 hr. ∴ it gains 60 minute spaces in $\frac{60}{55}$ hrs or $65\frac{5}{11}$. min. Hence the minute hand of the inaccurate clock passes over $65\frac{5}{11}$ minute spaces in 66min. true time. Hence :—

66 min. : 24×60 min. :: $65\frac{5}{11}$ min. $= 1428\frac{12}{121}$ min. spaces.

Now. the minute hand of a correct clock passes over 24×60 minutes or 1440 minutes.

∴ the inaccurate clock loses $1440 - 1428\frac{12}{121} = 11\frac{109}{121}$ min. *Ans.*

43. $\frac{1}{2} = \cdot 5$

$\frac{1}{2.3} = \frac{\cdot 5}{3} = \cdot 166666666$

$\frac{1}{2.3.4} = \frac{\cdot 166666666}{4} = \cdot 041666666$

$\frac{1}{2.3.4.5} = \frac{\cdot 041666666}{5} = \cdot 008333333$

$\frac{1}{2.3.4.5.6} = \frac{\cdot 008333333}{6} = \cdot 001388888$

$\frac{1}{2.3.4.5.6.7} = \frac{\cdot 001388888}{7} = \cdot 000198412$

$\frac{1}{2.3.4.5.6.7.8} = \frac{\cdot 000198412}{8} = \cdot 000024801$

$\frac{1}{2.3.4.5.6.7.8.9} = \frac{\cdot 000024801}{9} = \cdot 000002755$

$\frac{1}{2.3.4.5.6.7.8.9.10} = \frac{\cdot 000002755}{10} = \cdot 000000275$

$\frac{1}{2.3.4.5 \ldots\ldots\ldots 11} = \frac{\cdot 000000275}{11} = \cdot 000000025$

$\frac{1}{2.3 \ldots\ldots\ldots\ldots 12} = \frac{\cdot 000000025}{12} = \cdot 000000002$

The sum of the results $= \cdot 718281823$

$\therefore$ value to seven places $= \cdot 7182818$. *Ans.*

44. $\sqrt{2}=1\cdot 4142 \therefore \sqrt{3+2\sqrt{2}}=\sqrt{3+2\times 1\cdot 4142}=\sqrt{3+2\cdot 8284}=$ $\sqrt{5\cdot 8284}=2\cdot 41$. *Ans.*

45. $\frac{1}{30}+\frac{1}{36}=\frac{11}{180}$ of the cistern filled in 1 *min.* $\therefore$ in $15\frac{1}{3}$ min. $\frac{341}{360}$ of the cistern is filled after the removal of obstruction. $\therefore$ the portion of the cistern filled before the removal of obstruction $=1-\frac{341}{360}=\frac{19}{360}$.

But in 1 min. such portion filled $=\frac{5}{6}\times\frac{1}{30}+\frac{9}{10}+\frac{1}{88}=\frac{19}{360}$ of the cistern. $\therefore$ 1 min. *Ans.*

46. *A* does $\frac{1}{2}$ in 6 hrs., *i.e.*, from 9 A.M. to 12 noon, and again from 1 P.M. to 4 P.M. $\therefore$ he does $\frac{1}{4}$ from 9 A.M. to 12 noon. Now, *A* and *B* do $\frac{1}{3}$ between them, $\because$ *B* does $\frac{1}{3}-\frac{1}{4}$ or $\frac{1}{12}$ in the 3 hrs. $\therefore \frac{1}{12}+\frac{1}{2}=\frac{7}{12}$ is done $\therefore \frac{5}{12}$ is to be done by *B* in the afternoon.

$\frac{1}{12}:\frac{5}{12}::1\frac{1}{2}$ hrs. $=7\frac{1}{2}$ hrs., *i.e.*, *B* finishes at 8-30 P.M. *Ans.*

47. The answer wanted: the answer obtained $::2\frac{1}{2}:2\frac{1}{4}$ *i.e.*, as $\frac{10}{9}$,
$\therefore$ the answer obtained $=\frac{9}{10}$ of the answer wanted.
i.e., the answer obtained is too little by $\frac{1}{10}$ of the answer wanted.
But $\frac{1}{10}$ represents £26 4*s.*8*d.* $\therefore$ the answer wanted $=$ £262 6*s.* 8*d.* *Ans.*

48. $100:1::125=1\cdot 25$, amount of £1 for 1 year
$\therefore (1\cdot 25)^4$ or £2·44140625 is the amount of £1 for 4 years.
$\because$ the amt of £10,000 $=$ £24,414 1*s.* 3*d.* $\therefore$ the profit $=$ £14,414 1*s.* 3*d.*

Let r represent the int. on £1 for year. Then $2=(1+r)^4$.
$\therefore (2)^4=(1+r)^{16}$, *i.e.*, the amt. will be increased 16 fold in 16 yrs.

49. 3 cubic ft. of water enters through the 3rd pipe in 1 min.
$\therefore$ in 1 hr. 180 cubic ft. enters, which is $\frac{1}{2}$ the volume of the cistern.
When the 3 pipes are open the portion emptied in 1 hr. $=\frac{1}{3}+\frac{1}{4}-\frac{1}{2}=\frac{1}{12}$
$\therefore$ 12 hrs. in the required time. *Ans.*

50. With the pumps working and the leaks open, $\frac{1}{4}+\frac{1}{3}-\frac{1}{2}=\frac{1}{12}$ of the compartment is filled in 1 hr. $\therefore$ the ship can be kept afloat for 12 hrs.

Difficult Questions in Algebra.

(Madras University Questions.)

1. Solve the equations—

$x(y+z)=22,\ y(x+z)=40,\ z(x+y)=42.$ (1857)

2. In the following equation show that the value of x is independent of a :—

$$\frac{x+a}{a+b}+\frac{x+b}{a-b}=\frac{(a+b)^2}{a^2-b^2} \quad (1860)$$

3. Find the square root of

(1) $x^6+14x^3-4x^4-28x+4x^2+49$
(2) $a+b+c+2\sqrt{ab}-2\sqrt{bc}-2\sqrt{ac}$ (1863)

4. Find the square root of—

$$a^4+\frac{1}{a^4}+a^2+\frac{1}{a^2}+2\left\{a^3-\frac{1}{a^3}-\left(a-\frac{1}{a}\right)\right\} \quad (1866)$$

5. If $x+\frac{1}{x}=2(a+m),\ x-\frac{1}{x}=2b,\ y+\frac{1}{y}=2(c+n), y-\frac{1}{y}=2d$; find the value of $xy+\frac{1}{xy}$. (1867)

6. State in what cases x^n+a^n will be divisible by $x+a$ and x^2+a^2 respectively; state also the number of terms in the quotient in each case. Shew that the last digit in $3^{2n+1}+2^{2n+1}$ is 5, whatever n may be. (1868)

7. Shew that $\frac{2x^3+5x^2+19}{x^3+5x+12}$ is in its lowest terms.

8. If the two expressions $ax^3-c(3a+b)x^2+(a^3+bc^2)x+d,\ bx^3+c(a-b)x^2+a(c^2-a^2)x-d$, have a common quadratic factor (that is a factor containing x^2 as the highest powers of x) prove that this factor is an exact square. (1871)

9. In the process for finding the *G.C.M.* of two quantities A and B, Q is any remainder and P the preceding divisor, shew that the *G.C.M.* of Q and $P-nQ$ will be the *G.C.M.* of A and B, where n is any factor abitrarily chosen, and not necessarily the quotient arising from the division of P and Q. (1872)

10. Shew that $\frac{(a^2+b^2)(1+a^2b^2)}{a^2b^2}$ is the sum of two squares. (1872)

11. What must be the form of m in order that $a^m - x^m$ may have both $a^n + x^n$ and $a^n - x^n$ for divisors, n being any positive integer? Shew that $2^{4n} - 1$ is divisible by 15. (1875)

12. (i) If an algebraical expression is a common measure of two other algebraical expressions, prove that it will measure the sum or the difference of any multiples of those expressions.

(ii) If $x^2 - (2q-1)x + 2q$ and $x^2 - 2qx + 2(q+1)$ have a common factor, determine q; and hence find the $L.C.M.$ of the two expressions. (1876)

13. Find the cube root of—

$$(x+y)^6 - (x-y)^6 - 12xy(x^2-y^2)^2 \quad (1876)$$

14. Is $(a+b+c)\{a^2-(b+c)a+(b+c)^2\} - 3bc(b+c) = (a+b+c)\{b^2-(a+c)b+(a+c)^2\} - 3ac(a+c)$ an equation or an identity. (1879)

15. Find the square root of $\dfrac{4x^2}{9y^2} - \dfrac{x}{z} - \dfrac{16x^2}{15yz} + \dfrac{9y^2}{16z^2} + \dfrac{6xy}{5z^2} + \dfrac{16x^2}{25z^2}$

16. Solve the equation $\dfrac{(x-a)(x+b)}{x-a+b} = \dfrac{x(x-c)-b(x+c)}{x-b-c}$. (1883)

17. Resolve into 3 factors $(x+1)(x+3)(x+5)(x+7)+15$. (1888)

18. Prove the identity $16s(s-a)(s-b)(s-c) = 2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4$ where $2s = a+b+c$. (Calcutta Univ. 1867.)

19. Solve the equations—

$a^x \cdot a^{y+1} = a^7$, and $a^{2y} \cdot a^{3x+5} = a^{20}$ (Calcutta Univ.) (1879)

20. Express $(x+3a)(x+5a)(x+7a)(x+9a)$ as the difference of two square quantities. (Calcutta Univ.) (1887)

21. What is the only solution of $(x+2a)^2 + y^2 = 0$. (Punjab Univ.)

22. If h is the $H.C.F.$ and l the $L.C.M.$ of two quantities x and y, and if $h+l = x+y$, prove that $h^3 + l^3 = x^3 + y^3$ (Allahabad Univ.)

23. Solve $xy = 12$, $yz = 20$, $zx = 15$. (Oxford Local Exam., 1884.)

24. Solve $z^x = y^{2x}$, $2^z = 2 \times 4^x$, $x+y+z = 16$. (R. M. College, Sandhurst, 1888.)

25. If $a+b-c=0$, prove that $a^2+bc = b^2+ac$. (Army Preliminary, 1887.)

SOLUTIONS.

1. Adding the three equations we get $xy+yz+zx=52$. Now, subtract each from the result.

$\therefore\ xy=10,\ yz=30,\ xz=12\ \therefore\ \dfrac{xy}{yz}=\dfrac{10}{30}\ \therefore\ \dfrac{x}{z}=\frac{1}{3}\ \therefore\ \dfrac{x}{z}\times xz = \frac{1}{3}\times 12$

$\therefore x^2 = 4\ \therefore x = \pm 2\ \therefore\ y = \pm 5$, and $z = \pm 6$. *Ans.*

2. The expression $= \dfrac{x+a}{a+b}+\dfrac{x+b}{a-b}=\dfrac{a+b}{a-b}$

$$\frac{x}{a+b}+\frac{a}{a+b}+\frac{x}{a-b}+\frac{b}{a-b}=\frac{a}{a-b}+\frac{b}{a-b}\ \therefore\ \frac{x}{a+b}+\frac{x}{a-b}=\frac{a}{a-b}-\frac{a}{a+b}$$

$$\therefore\ x\left(\frac{1}{a+b}+\frac{1}{a-b}\right)=a\left(\frac{1}{a-b}-\frac{1}{a+b}\right)\ \therefore\ 2ax=2ab\ \therefore\ x=b.,$$

i. e., in the equation the value of x is independent of a.

3. (i) $(x^6+14x^3+49)-(4x^4+28x)+4x^2$
$= (x^3+7)^2-4x(x^3+7)+4x^2=(x^3+7-2x)^2$
$\therefore$ sq. rt. $=x^3-2x+7$. *Ans.*

(ii) $a+2\sqrt{ab}+b-2\sqrt{c}(\sqrt{a}+\sqrt{b})+c$
$= (\sqrt{a}+\sqrt{b})^2-2\sqrt{c}(\sqrt{a}+\sqrt{b})+(\sqrt{c})^2$
$= (\sqrt{a}+\sqrt{b}-\sqrt{c})^2$ $\therefore$ *sq. rt.* $=\sqrt{a}+\sqrt{b}-\sqrt{c}$.

4. $a^4+2+\dfrac{1}{a^4}+a^2-2+\dfrac{1}{a^2}+2\left(a^3-\dfrac{1}{a^3}\right)-2\left(a-\dfrac{1}{a}\right)$

$$=\left(a^2+\frac{1}{a^2}\right)^2+\left(a-\frac{1}{a}\right)^2+2\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}-1\right)$$

$$=\left(a^2+\frac{1}{a^2}\right)^2+\left(a-\frac{1}{a}\right)^2+2\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}\right)$$

$$=\left(a^2+\frac{1}{a^2}+a-\frac{1}{a}\right)^2\ \therefore\ \textit{sq. rt.}=a^2+\frac{1}{a^2}+a-\frac{1}{a}.\ \textit{Ans.}$$

5. $\left(x+\dfrac{1}{x}\right)\left(y+\dfrac{1}{y}\right)+\left(x-\dfrac{1}{x}\right)\left(y-\dfrac{1}{y}\right)$

$$=xy+\frac{y}{x}+\frac{x}{y}+\frac{1}{xy}+xy-\frac{y}{x}-\frac{x}{y}+\frac{1}{xy}=2\left(xy+\frac{1}{xy}\right)$$

$$\therefore\ xy+\frac{1}{xy}=\tfrac{1}{2}\left\{\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)\right\}$$

$= \frac{1}{2}\{2(a+m)\times 2(c+m)+2b\times 2d\} = \frac{1}{2}\{4(a+m)(c+m)+4bd\}$
$= 2\{(a+m)(c+m)+bd\}$ *Ans.*

6. x^n+a^n is divisible by $x+a$ if n be any odd whole number, the number of terms in the quotient being n.

x^n+a^n is divisible by x^2+a^2 if $\frac{n}{2}$ be any odd number, the number of terms in the quotient being $\frac{n}{2}$

$2n+1$ is odd $\therefore 3^{2n+1}+2^{2n+1}$ is divisible by $3+2$ i. e., by 5. i.e., the last digit must end in 5.

7. *H.C.F.* of the two expressions = a factor of their difference, *i.e.*, $2x^3+5x^2+19-2x^3-10x-24 = 5x^2-10x-5$ *i.e.* $5(x^2-2x^2-1)$ these two factors do not divide the numerator as well as the denominator exactly $\therefore$ the fraction is in its lowest terms.

8. The common quadratic factor is a factor of the sum of the expressions; *viz.*, of

$$x^3(a+b)+x^2\{c(a-b)-c(3a+b)\}+x\{(a^3+bc^2)+a(c^2-a^2)\}\because \text{ or}$$

$$x^3(a+b)-2x^2c(a+b)+xc^2(a+b),$$

i.e., $x(a+b)(x^2-2cx+c^2)$. Of the 3 factors the common quadratic factor is $x^2-2cx+c^2$, which is an exact square.

9. Every remainder in the course of our work contains the *G.C.M.* . *G.C.M.* of A and B, is the *G.C.M.* of P and Q, and we know that if a quantity divides P and Q it also divides $P-nQ$. $\because$ *G.C.M.* of P and Q is the *G.C.M.* of Q and $P-nQ$. Hence *G.C.M.* of Q and $P-nQ$ is the *G.C.M.* of A and B,

10. The expression $=\dfrac{a^2+b^2}{a^2b^2}\left(1+a^2b^2\right)=\left(\dfrac{1}{b^2}+\dfrac{1}{a^2}\right)\left(1+a^2b^2\right)$

$$=\frac{1}{a^2}+\frac{1}{b^2}+a^2+b^2 \;=a^2+2+\frac{1}{a^2}+b^2-2+\frac{1}{b^2}$$

$$=\left(a+\frac{1}{a}\right)^2+\left(b-\frac{1}{b}\right)^2 \text{ or } =a^2-2+\frac{1}{a^2}+b^2+2+\frac{1}{b^2}$$

$$=\left(a-\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \quad \textit{Ans.}$$

11. a^m-x^m is divisible by $a^{2n}-x^{2n}$ if $\dfrac{m}{2n}$ be an integer, *i.e.*, m must be an even multiple of n. $2^{4n}-1=(2^4)^n-1=16^n-1$.
Now whether n is even or odd, 16^n-1 is divisible by 15.

12. If P measures A and B, it will also measure $mA\pm nB$.

Let A be equal to xP and $B=yP$. Now P divides A and B

$\therefore$ $mA\pm nB=mxP\pm nyP$. *i.e.*, P divides $mA\pm nB$.

The common factor is a factor of the difference of the two expressions, *viz.*, of—

$x^2-(2q-1)x+2q-x^2+2qx-2q-2$
i.e., of $x-2$ *i.e.* $x-2$ is the common factor.

Now dividing $x^2-(2q-1)x+2q$ by $x-2$ we get a remainder $2q-6$ and this remainder must be zero $\because 2q=6 \because q=3$ *Ans.*

Now putting the value of q in both expressions, we get,
$x^2-5x+6=(x-2)(x-3)$ and $x^2-6x+8=(x-2)(x-4)$
$\therefore$ *L.C.M.* $=(x-2)(x-3)(x-4)$ *Ans.*

13. The expression may be put in the form $a^3-b^3-3ab(a-b)$ thus :—

$(x+y)^6-(x-y)^6-3(x+y)^2(x-y)^2(4xy)$ but $4xy=(x+y)^2-(x-y)^2$
$\therefore =\{(x+y)^2\}^3-\{(x-y)^2\}^3-3(x+y)^2(x-y)^2\{(x+y)^2-(x-y)^2\}$
$=\{(x+y)^2-(x-y)^2\}^3=(4xy)^3$ $\therefore$ cube root $=4xy$. *Ans.*

14. $\{a+(b+c)\}\{a^2-(b+c)a+(b+c)^2\}-3bc\,(b+c)$
$=a^3+(b+c)^3-3bc(b+c)=a^3+b^3+c^3$
$\{b+(a+c)\}\{b^2-(a+c)b+(a+c)^2\}-3ac(a+c)$
$=b^3+(a+c)^3-3ac(a+c)=b^3+a^3+c^3$
Hence we have an identity.

15. $\left(\frac{2x}{3y}\right)^2-\frac{x}{z}-\frac{16x^2}{15yz}+\frac{6xy}{5z^2}+\left(\frac{3y}{4z}\right)^2+\left(\frac{4x}{5z}\right)^2$
$=\left\{\left(\frac{2x}{3y}\right)^2-2.\left(\frac{2x}{3y}\right)\left(\frac{3y}{4z}\right)+\left(\frac{3y}{4z}\right)^2\right\}+\left(\frac{4x}{5z}\right)^2-2\left(\frac{4x}{5z}\right)$
$\times\left(\frac{2x}{3y}-\frac{3y}{4z}\right)$
$=\left(\frac{2x}{3y}-\frac{3y}{4z}\right)^2-2\left(\frac{4x}{5z}\right)\left(\frac{2x}{3y}-\frac{3y}{4z}\right)+\left(-\frac{4x}{5z}\right)^2$
$=\left(\frac{2x}{3y}-\frac{3y}{4z}-\frac{4x}{5z}\right)$ $\therefore$ sq. rt. $=\frac{2x}{3y}-\frac{3y}{4z}-\frac{4x}{5z}$. *Ans.*

16 $\frac{x^2+x(b-a)-ab}{x+(b-a)}=\frac{x^2-x(b+c)-bc}{x-(b+c)}$ $\therefore$ dividing out, we get
$x-\frac{ab}{x+b-a}=x-\frac{bc}{x-(b+c)}$ $\therefore$ $\frac{a}{x+b-a}=\frac{c}{x-b-c}$
$\therefore$ $a(x-b-c)=c(x+b-a)$ hence $x=\frac{b(a+c)}{a-c}$. *Ans.*

17. The product of any 4 consecutive odd or even numbers together with 16 is a square.
Hence the expression may be put thus:—
$(x+1)(x+3)(x+5)(x+7)+16-1$
$=\{(x+1)(x+7)(x+3)(x+5)+16\}-1$
$=(x^2+8x+11)^2-1=(x^2+8x+12)(x^2+8x+10)$
$=(x+6)(x+2)(x^2+8x+10)$. *Ans.*

18. $16s(s-a)(s-b)(s-c)=2s(2s-2a)(2s-2b)(2s-2c)$
But $2s=a+b+c$
$\therefore=(a+b+c\;(b+c-a)(a+c-b)(a+b-c)$
$=\{(b+c)^2-a^2\}\{a^2-(b-c)^2\}$
$=a^2\{(b+c)^2+(b-c)^2\}-(b^2-c^2)^2-a^4=a^2(2b^2+2c^2)-b^4+2b^2c^2$
$-c^4-a^4$
$=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$. *Q. E. D.*

19. $a^{x+y+1}=a^7$ and $a^{2y+3x+5}=a^{20}$

$\therefore$ $x+y+1=7$ and $2y+3x+5=20$
Hence solving the equations we get $x=3$, $y=3$. *Ans.*

20. $(x+3a)(x+9a)(x+5a)(x+7a)$
$=(x^2+12ax+27a^2)(x^2+12ax+35a^2)$
$=\{(x^2+12ax+31a^2)-4a^2\}\{(x^2+12ax+31a^2)+4a^2\}$
$=(x^2+12ax+31ax)^2-16a^4$. Q. E D.

21. If the sum of two or more squares is zero, then each square is zero. $\because$ $(x+2a)^2=0$ and $y^2=0$ $\therefore$ $x+2a=0$ and $y=0$ $\therefore$ $x=-2a$, and $y=0$

22. $L.\ C.\ M.=\dfrac{\text{Product of two quantities}}{G.\ C.\ M.}$ $\therefore$ $l=\dfrac{xy}{h}$

$\therefore$ $hl=xy$ and $h+l=x+y$. $h^3+l^3 = (h+l)(h^2-hl+l^2)$ $=(h+l)\{(h+l)^2-3hl\}=(x+y)\{(x+y)^2-3xy\}=(x+y)^3-3xy(x+y)$ $=x^3+y^3$. Q. E. D.

23. $\dfrac{xy}{yz}=\dfrac{12}{20}$ $\therefore$ $\dfrac{x}{z}=\dfrac{3}{5}$ $\therefore$ $\dfrac{x}{z}\times zx=\dfrac{3}{5}\times15$ $\therefore$ $x^2=9$.
$\therefore$ $x=\pm3$; hence $y=\pm4$, and $z=\pm5$.

24. $z^x=y^{2x}$ $\therefore$ $z^x=(y^2)^x$ $\therefore$ $z=y^2$.
$2^z=2\times4^x=2^z=2\times2^{2x}$ $\therefore$ $2^z=2^{2x+1}$.

$\therefore$ $2x+1=z=y^2$ $\therefore$ $2x=y^2-1$ $\therefore$ $x=\dfrac{y^2-1}{2}$. Putting the values of x and z in the 3rd equation we get

$\dfrac{y^2-1}{2}+y+y^2=16$ $\therefore$ $3y^2+2y-33=0$.

$\therefore$ $(y-3)(3y+11)=0$ $\therefore$ $y=3$ or $-\frac{11}{3}$
$\therefore$ $x=4$, or $\frac{56}{9}$, and $z=9$ or $\frac{121}{9}$. *Ans.*

25. $a+b-c=0$ $\therefore$ $a+b=c$; $\therefore$ a^2+bc
$=a^2+b(a+b)=a^2+ab+b^2=a(a+b)+b^2=ac+b^2$. Q E.D.

Finish

1898.

Arithmetic and Algebra.

1. A carriage has four wheels, the circumference of each of the two larger being 10 ft. 6 in. and that of each of the two smaller 6 ft. 5 in. Find the length of a journey during which the smaller wheels revolve 1,120 times oftener than the larger. **8**

2. State the rule for reducing a mixed recurring decimal to a vulgar fraction, and shew its truth by means of an example, **9**

Add $\cdot 0\dot{6}$ to $1\cdot 1\dot{5}\dot{6}$, and divide the sum by $\cdot 507\dot{7}441$.

3. If I put by Rs. 120 at the beginning of each year, how much shall I have at the end of three years, allowing compound interest at 5 per cent. per annum? **8**

4. A wine merchant mixes three qualities of spirits worth, respectively, 15*s.*, 16*s.*, 17*s.* per gallon, in the proportion 1 : 3 : 2, respectively, first adding $\frac{1}{8}$ of a gallon of water to each gallon of spirit. Find at what rate he must sell in order to make 15 per cent. profit. **9**

5. A person having bought a certain amount of $2\frac{3}{4}$ per cent. stock at 95, afterwards sold it, and with the proceeds bought $3\frac{1}{3}$ per cent. stock; he obtained £900 less stock than before, but his income was unchanged. How much money did he originally invest? **10**

6. Define an algebraical *fraction*, and explain the term *reciprocal*. **8**

If p be the difference between any proper fraction and unity, q the difference between its reciprocal and unity, prove that $pq = q - p$.

7. Resolve $ab\,(a + b) + bc\,(b + c) + ca\,(c + a) + 2abc$ into factors. **8**

Find the value of $xy + yz + zx + 2xyz$ when

$$x=\frac{a}{b+c}, y=\frac{b}{c+a}, z=\frac{c}{a+b}.$$

8. Find the highest common factor of 10

$x^4-5x^3+5x^2-x-12$ and $x^4-x^3-4x^2+13x-15$;
and the lowest common multiple of

$b^3+c^3-a^3+3abc$ and $b^2+c^2-a^2+2bc$.

9. Simplify 13

$(1+m+n)^3+(1-m-n)^3-(1-m+n)^3-(1+m-n)^3$;
and extract the fourth root of—

$$\left(x^4+\frac{1}{x^4}\right)-8\left(x^3+\frac{1}{x^3}\right)+28\left(x^2+\frac{1}{x^2}\right)-56\left(x+\frac{1}{x}\right)+70.$$

10. Solve the equations— 10

(i) $(x-a)^2+(x-b)^2=2\{(x-a-b)^2-ab\}$;

(ii) $\frac{1}{2}(5x+3y+7)=\frac{1}{3}(4x-5y+6)=\frac{1}{5}(3x+7y+9)$.

11. At the review of an army, the troops were drawn up in a solid mass 40 deep, and there were just one-fourth as many men in front as there were spectators. Had the depth, however, been increased by 5, and the spectators drawn up with the army, the number of men in front would have been 100 fewer than before. Find the number of troops. 7

1898.

Euclid.

1. Construct a triangle of which the sides shall be equal to three given straight lines, provided that any two of these lines are together greater than the third. 13

Construct a parallelogram, one side and the two diagonals of which are given in length.

2. If a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another, and the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side together equal to two right angles. 11

If the straight line bisecting an exterior angle of a triangle be parallel to a side of the triangle, the triangle shall be isosceles.

3. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part. 16

On a given straight line as hypotenuse construct a right-angled triangle, so that the square on one side may be equal to the rectangle contained by the hypotenuse and the other side.

4. If two circles touch one another externally, the straight line which joins their centres shall pass through the point of contact. 14

Three circles ADE, BDF, CEF, touch externally two and two at the points D, E, F; P, Q, R are their respective centres. DE and DF produced meet the circle CEF in G and H, prove that the straight line GH passes through R and is parallel to PQ.

5. In equal circles, equal arcs are subtended by equal straight lines. 10

In the circle $ACBD$, the arc ACB is equal to the arc CBD: prove that the triangles ACB, CBD are equal.

6. Describe a circle about a given triangle. 14

From an angular point of a triangle three straight lines are drawn, one to the centre of the circle inscribed in it, another to the centre of the circle described about it, and the third perpendicular to the opposite side: prove that the first bisects the angle between the second and third.

7. $ABCDE$ is a regular pentagon; join AC and BD intersecting at O: shew that AO is equal to DO, and that the rectangle AC, CO is equal to the square on BC. 12

8. Inscribe an equilateral and equiangular hexagon in a given circle. 10

SOLUTIONS.

1. The smaller makes $\frac{10 \text{ ft. } 6 \text{ in.}}{6 \text{ ft. } 5 \text{ in.}}$ revolution more than the larger, *i. e.*, $1\frac{7}{11}$ more.

$1\frac{7}{11} : 1{,}120 :: \frac{126}{12 \times 3 \times 1{,}760}$ mile : length of the journey. $\therefore$ length of the journey $= 3\frac{1}{2}$ miles.

2. (*a*) To convert a mixed circulating decimal into a vulgar fraction, subtract the non-recurring part from the whole decimal, *i. e.*, the digits from the decimal point down to the end of the first period and write down the remainder as the numerator; for the denominator write as many *nines* as there are digits recurring, followed by as many *ciphers* as there are non-recurring digits.

Ex.—Convert $\cdot 1\dot{2}3\dot{4}$ to a vulgar fraction.

$10{,}000$ times $\cdot 1\dot{2}3\dot{4} = 1234{\cdot}234234\ldots$

10 times $\cdot 1\dot{2}3\dot{4} = \quad 1{\cdot}234234\ldots$

$\therefore$ by subtraction, $9{,}990$ times $\cdot 1\dot{2}3\dot{4} = 1233$.

$\therefore \cdot 1\dot{2}3\dot{4} = \frac{1233}{9990}$. *Q. E. D.*

(*b*) $$\frac{\cdot 0\dot{6} + 1{\cdot}4\dot{5}\dot{6}}{\cdot 5077441} = \frac{\frac{6}{90} + 1\frac{452}{990}}{\frac{5077436}{9990000}}$$

$$= \frac{\frac{1}{15} + 1\frac{226}{495}}{\frac{5077436}{9990000}} = \frac{754}{495} \times \frac{9990000}{5077436} = 3 \textit{ Ans.}$$

3. Rs. 100 : Rs. 120 :: Rs. 5 Int.

$\therefore$ Interest for the 1st year = Rs. 6.

$\therefore$ Capital for the 2nd year = Rs 120 + Rs. 120 + Rs. 6.
= Rs. 246.

Rs. 100 : Rs. 246 :: Rs. 5 Int.

$\therefore$ Interest for the 2nd year = Rs. $\frac{123}{10}$

$\therefore$ Capital for the 3rd year = Rs. 120 + Rs 246 + Rs. $\frac{123}{10}$
= Rs. 378·3.

Rs. 100 : Rs. 378·3 :: Rs. 5 Int.

$\therefore$ Interest for the 3rd year = Rs. 18·915.

$\therefore$ At the end of 3 years I shall have Rs. 378·3 + Rs. 18·915 = Rs 397·215.

Rs. 397-3-$5\frac{7}{25}$ *Ans.*

4. $1\frac{1}{8}$ gallon of the first, in which there is 1 gallon of pure spirit and $\frac{1}{8}$ water.

$\therefore$ $3\frac{3}{8}$ gallons of the second in which there are 3 gallons pure spirit and $\frac{3}{8}$ water.

$\therefore$ $2\frac{2}{8}$ gallons of the third, in which there are 2 gallons pure spirit and $\frac{2}{8}$ water. $\therefore$ total mixture $= 1\frac{1}{8} + 3\frac{3}{8} + 2\frac{2}{8} = 6\frac{3}{4}$.

Total cost $= 1 \times 15s. + 3 \times 16s. + 2 \times 17s. = 97s.$

$100 : 97 :: 115 : x$.

$\therefore$ S. P. of $6\frac{3}{4}$ gallons $= \frac{2231}{20}\ s.$

$\therefore$ S. P. of 1 gallon $= \frac{2231}{20} \times \frac{4}{27} = \frac{2231}{135} = 16\frac{71}{135}s.$

$= 16s.\ 6\frac{14}{35}d.$

S. P. $16s.\ 6\frac{14}{35}d.$ *Ans.*

5. Let £100 be the amount of stock bought.

First income £ $2\frac{3}{4}$.

Second income is the same.

$\therefore 3\frac{1}{2} : 2\frac{3}{4} ::$ 100 stock : stock held in the second case.

$\therefore$ stock $= £\frac{550}{7}$

$£100 - £\frac{550}{7} = £\frac{150}{7}$ stock less.

$\therefore$ $£\frac{150}{7} : £900 :: £95$ investment : x.

$\therefore$ Original investment $= \frac{900 \times 95 \times 7}{150} = £3,990$

£3,990 *Ans.*

6. (*a*) The *algebraical fraction* $\frac{a}{b}$, where a and b may have any numerical values, is defined to be a quantity which when multiplied by b, becomes equal to a.

If the product of two quantities be equal to unity, each of the quantities is said to be the *reciprocal* of the other: thus 3 is the reciprocal of $\frac{1}{3}$: a is the reciprocal of $\frac{1}{a}$, &c.

(*b*) Let $\frac{a}{b}$ be any proper fraction.

Then by hyp. $1 - \frac{a}{b} = p$ and $\frac{b}{a} - 1 = q$.

$$\therefore pq = \left(1 - \frac{a}{b}\right)\left(\frac{b}{a} - 1\right) = \frac{b}{a} - 1 - 1 + \frac{a}{b}$$

$$= \left(\frac{b}{a} - 1\right) - \left(1 - \frac{a}{b}\right) = q - p$$

Q. E. D.

7. (*i*) The expression:

$$= a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2\,abc$$
$$= a^2b + a^2c + ab^2 + 2\,abc + c^2a + b^2c + bc^2$$
$$= a^2(b + c) + a(b^2 + 2\,bc + c^2) + bc(b + c)$$
$$= (b + c)\{a^2 + a(b + c) + bc\}$$
$$= (b + c)(a + b)(a + c)$$

$(a + b)(b + c)(c + a)$ *Ans.*

(ii) Substituting the values of x, y, z in the given expression we get

$$\frac{ab}{(b+c)(c+a)}+\frac{bc}{(c+a)(a+b)}+\frac{ac}{(a+b)(b+c)}+\frac{2\,abc}{(a+b)(b+c)(c+a)}$$

$$=\frac{ab(a+b)+bc(b+c)+ca(c+a)+2abc}{(a+b)\ (b+c)\ (c+a)}$$

$$=\frac{(a+b)(b+c)(c+a)}{(a+b)(b+c)(c+a)}(vide\ \text{VII, I above})=1\ Ans.$$

8. (a) $x+1$ is a factor of $x^4-5x^3+5x^2-x-12$.

$\therefore x^4-5x^3+5x^2-x-12=(x+1)\ (x^3-6x^2+11x-12)$

Again on trial the second factor vanishes, when $x=4$

$\therefore$ $x-4$ is a factor of $x^3-6x^2+11x-12$

$\therefore$ $x^3-6x^2+11x-12=(x-4)\ (x^2-2x+3)$

Now the expression $x^4-x^3-4x^2+13-15$ is exactly divided by x^2-2x+3, and not by each of $(x-4)$ and $(x-1)$

$\therefore x^2-2x+3$ is the H.C.F. of the two expressions.

x^2-2x+3 *Ans.*

(b) $b^3+c^3-a^3+3abc$

$=(b+c-a)\ (b^2+c^2+a^2-bc+ab+ac)$ ....(I)

$b^2+c^2-a^2+2bc=b^2+c^2+2bc-a^2$

$=(b+c)^2-a^2=(b+c+a)(b+c-a)$.........(II)

$\therefore$ the L. C. M. of (I) & (II) $=(b+c-a)(b+c+a)$ $(b^2+c^2+a^2-bc+ab+ac)$ *Ans.*

9. (a) $(1+m+n)^3=1+3(m+n)+3(m+n)^2+(m+n)^3$

$+\ (1-m-n)^3=1-3(m+n)+3(m+n)^2-(m+n)^3$

$-\ (1-m+n)^3=-1+3(m+n)-3(m-n)^2+(m-n)^3$

$-\ (1+m-n)^3=-1-3(m-n)-3(m-n)^2-(m-n)^3$

$\therefore$ the given expression

$=6(m+n)^2-6(m-n)^2$

$=6\ \{(m+n)^2-(m-n)^2\}=6\cdot4\ mn=24mn.$ *Ans.*

$$
\begin{array}{l|l|l}
 & x^4-8x^3+28x^2-56x+70-\frac{56}{x}+\frac{28}{x^2}-\frac{5}{x^3}+\frac{1}{x^4} & x^2-4x+6-\frac{4}{x}+\frac{1}{x^2} \\
 & x^4 & \\
\hline
2x^2-4x & -8x^3+28x^2-56x & \\
 & -8x^3+16x^2 & \\
\hline
2x^2-8x+6 & 12x^2-56x+70 & \\
 & 12x^2-48x+36 & \\
\hline
2x^2-8x+12-\frac{4}{x} & -8x+34-\frac{56}{x}+\frac{28}{x^2} & \\
 & -8x+32-\frac{48}{x}+\frac{16}{x^2} & \\
\hline
2x^2-8x+12-\frac{8}{x}+\frac{1}{x^2} & 2-\frac{8}{x}+\frac{12}{x^2}-\frac{8}{x^3}+\frac{1}{x^4} & \\
 & 2-\frac{8}{x}+\frac{12}{x}-\frac{8}{x^3}+\frac{1}{x^4} & \\
\hline
\end{array}
$$

$$x^2-4x+6-\frac{4}{x}+\frac{1}{x^2}=\left(x^2+\frac{1}{x^2}+2\right)-4\left(x+\frac{1}{x}\right)+4$$

$$=\left(x+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)+4=\left\{\left(x+\frac{1}{x}\right)-2\right\}^2$$

$\therefore$ Fourth root $= x+\frac{1}{x}-2.$ *Ans.*

X. (i) $(x-a)^2+(x-b)^2=2\{(x-a-b)^2-ab\}$

$\therefore\ (x-a)^2+(x-b)^2=2\{(x-a)^2-2b\ (x-a)+b^2-ab\}$

$\therefore\ (x-a)^2+(x-b)^2=2(x-a)^2-4b(x-a)+2b^2-2ab.$

$\therefore\ (x-b)^2-(x-a)^2+4b(x-a)=2b^2-2ab.$

$\therefore\ x^2-2bx+b^2-x^2+2ax-a^2+4bx-4ab=2b^2-2ab.$

$\therefore\ 2bx+2ax=a^2+2ab+b^2\ \therefore\ 2x(b+a)=(a+b)^2$

$\therefore\ x=\dfrac{a+b}{2}$ *Ans.*

(ii) $\frac{1}{2}(5x+3y+7)=\frac{1}{3}(4x+5y+6)$............ (i)

$\frac{1}{2}(5x+3y+7)=\frac{1}{5}(3x+7y+9)$..... ......(ii)

$3(5x+3y+7)=2(4x+5y+6)$............ (i)

and $5(5x+3y+7)=2(3x+7y+9)$............(ii)

$15x+9y+21=8x+10y+12$, *i.e.*, $7x-y=-9$

$25x+15y+35=6x+14y+18$, *i.e.*, $19x+y=-17$

Adding the two, we get

$26x=-26\ \therefore\ x=-1$

Substituting the value of x in $7x-y=-9$, we get

$-7-y=-9\ \therefore\ y=2$

$x=-1;\ y=2.$ *Ans.*

XI. Let x be the number of men in the front.

$\therefore\ 40x=$ no. of troops; and $4x=$ no. of spectators.

$\therefore\ \dfrac{40x+4x}{45}$ is the no. in the front row of the second arrangement.

$\therefore\ \dfrac{44x}{45}=x-100$, whence $x=4{,}500$.

$\therefore$ No. of troops $=40x=40\times 4{,}500=180{,}000$. *Ans.*

1899.

Arithmetic and Algebra.

1, Explain the terms *composite number* and *common multiple.* 9

(*a*) Find the least number which must be added to seven thousand and one million nine hundred and seven thousand and sixty one, in order that the sum may be a multiple of seven hundred and nine thousand four hundred and eighty.

(*b*) Find the Least Common Multiple of 1,160, 2,948, 3,886.

2. If in France the railway fare for a distance of 384 kilometres is 25·28 francs, how does this rate of charge compare with the English Parliamentary rate of 1*d*. per mile? Given one metre = 1 yard $3\frac{1}{2}$ inches, £1 = 25·2 francs. 7

3. *A* walks to a place at the rate of $4\frac{1}{2}$ miles per hour; at 8 miles from his destination he meets *B*, and turns back with him (walking at *B*'s rate) for a mile: if *A* is half an hour late at his destination, what is *B*'s rate? And at what rate should *A* have walked after parting with *B*, so as to arrive at the proper time? 9

4. A trader's debts amount to £5,174 15*s*.; he has assets sufficient to pay his creditors 16*s*. 6*d*. in the pound. Some creditors, however, have the right to be paid in full, and in consequence the others receive only 15*s*. in the pound. Find how much is paid in full. 11

5. A man has an income of £415 derived from capital invested in 4 per cent. stock; he sells out his stock at 102, and re-invests the proceeds in 5 per cent. stock. What price must he pay for the latter, if his new income is £425? 9

6. (*a*) Divide— 11

$$x^4 - 3(a+1)x^3 + 2(3a+1)x^2 + 3(a+1)(a^2-1)x + (a^2-1)^2 \text{ by } x^2 - (a+3)x - a^2 + 1.$$

(*b*) Resolve into factors—

$$12x^2 + x - 35; \quad (2b-a)^3 + (2a-b)^3 - (a+b)^3.$$

7. Prove that the product of the Highest Common Factor and the Lowest Common Multiple of two expressions is equal to the product of the expressions themselves. 10

Find the H. C. F. of

$$x^4 + 5x^3 - 36x^2 + 50x + 48 \text{ and } x^4 + x^3 - 12x^2 - 2x + 80.$$

8. Simplify— **12**

(i) $$\frac{a^2}{\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{c}\right)}+\frac{b^2}{\left(\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}-\frac{1}{a}\right)}+\frac{c^2}{\left(\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}-\frac{1}{b}\right)};$$

(ii) $$\frac{\sqrt{64x^6-48x^4+12x^2-1}\ \sqrt{16x^4-64x^3+24x^2+80x+25}}{4x^2-12x-7}$$

9. Solve the equations— **11**

(i) $$\frac{21}{4}\left(\frac{2x}{3}-\frac{5}{18}\right)+\frac{7x-3\frac{3}{4}}{12}=2\frac{19}{144}-\frac{14-15x}{3};$$

(ii) $$(1+p)\ (x-py)=2p^2\left(\frac{x}{1+p}+\frac{y}{1-p}\right)=\frac{2p^2}{1-p}.$$

10. Two passengers have together 7 maunds of luggage, and for the excess above the weight allowed free one of them is charged Rs. 3 and the other Rs. 5. If all the luggage had belonged to one passenger he would have been charged Rs. 11. What amount of luggage is each passenger allowed free of charge? **11**

1899

Euclid.

1. If, at a point in a straight line, two straight lines on opposite sides of it make the adjacent angles together equal to two right angles, they are in the same straight line. **12**

ABCD is a quadrilateral whose opposite sides are equal, and *O* is the middle point of the diagonal *AC*. Prove that *BOD* is a straight line.

2. Triangles on the same base, and between the same parallels, are equal. **12**

From a point *P* in the side *AB* of a triangle *ABC*, a straight line *PQR* is drawn parallel and equal to *BC*, meeting the side *AC* in *Q*. Prove that the triangle *AQR* is equal to the triangle *BPQ*.

3. If a straight line be divided into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line. 13

D is any point in the side *BC* of an equilateral triangle *ABC*. Prove that the square on *BC* is equal to the rectangle contained by *BD*, *DC*, together with the square on *AD*.

4. In any triangle, the square on the side opposite an acute angle is less than the squares on the sides containing it by twice the rectangle contained by either of these sides and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle. 15

Prove that the sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the diagonals.

5. From a given point without the circumference of a given circle, draw a tangent to the circle. 15

If any two adjacent sides of a quadrilateral described about a circle are equal, then the diagonals of the quadrilateral are at right angles to each other.

6. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the tangent are equal to the angles in the alternate segments of the circle. 13

If two circles intersect, prove that the angles subtended at the points of intersection by a common tangent are supplementary.

7. Inscribe a circle in a given triangle. 14

A circle whose centre is *O* is inscribed in a triangle *ABC*, and *AO* produced meets the opposite side in *D*. Prove that *AO* is greater than *OD*.

8. Describe a square about a given circle. 6

SOLUTIONS.

1. A *composite number* is a number which has factors each greater than one: thus, 4, 6, 8, etc., are composite numbers.

   A *common multiple* of two or more numbers is a number which is exactly divisible by each of them: thus, 12 is a common multiple of 2, 3, 4, and 6.

(*a*) $\frac{7001907061}{709480} = 9869\frac{48941}{709480}$

$\therefore$ The number to be added to 7001907061 to make it a multiple of $709480 = 709480 - 48941 = 660539$.

(*b*) Resolve the numbers into prime fractors, and then find the L.C.M., thus :—

$$\begin{array}{r|r} 2 & 1160 \\ 2 & 580 \\ 29 & 290 \\ 2 & 10 \\ & 5 \end{array} \qquad \begin{array}{r|r} 2 & 2948 \\ 2 & 1474 \\ 11 & 737 \\ & 67 \end{array} \qquad \begin{array}{r|r} 2 & 3836 \\ 29 & 1943 \\ & 67 \end{array}$$

$1160 = 2^3 \times 5 \times 29$, $2948 = 2^2 \times 11 \times 67$

$3836 = 2 \times 29 \times 67$

$\therefore$ L.C.M. $= 2^3 \times 5 \times 11 \times 29 \times 67 = 854920$. *Ans.*

2. 1 metre $= 1$ yd. $3\frac{1}{2}$ in. $= 39\frac{1}{2}$ in.

384 kilometres $= 384000$ metres $= 384000 \times \frac{79}{2}$ in.

$= \frac{384000 \times 79}{2 \times 12 \times 3 \times 1760}$ miles $= \frac{7900}{33}$ miles.

£1 $= 25{\cdot}2$ francs $\therefore$ $25{\cdot}28$ francs $= £\frac{25{\cdot}28}{25{\cdot}2} = £\frac{316}{315}$

In France, the Railway fare for a distance of $\frac{7900}{33}$ miles.

$= £\frac{316}{315}$ $\therefore$ the fare for 1 mile $= £\frac{316}{315} \times \frac{33}{7900}$

$= £\frac{11}{105 \times 25} = \frac{176}{175}d.$

In England, the rate for one mile is 1 *d*,

$\therefore$ French rate : English rate $:: \frac{176}{175} : 1$

i. e., $:: 176 : 175$ $\qquad$ $176 : 175$. *Ans.*

3. A turns back with B, walking at B's rate for a mile, and walks one mile more at his own rate, hence he is half an hour late.

6. (*a*)

$$
x^2-(a+3)x-(a^2-1) \;\big)\; x^4-3(a+1)x^3+2(3a+1)x^2+3(a+1)(a^2-1)x+(a^2-1)^2 \;\big(\; x^2-2ax-(a^2-1)
$$

$$
\begin{array}{l}
x^4-(a+3)x^3-(a^2-1)x^2 \\
\hline
-2ax^3+(a^2+6a+1)x^2+3(a+1)(a^2-1)x+(a^2-1)^2 \\
-2ax^3+2a(a+3)\,x^2+2a(a^2-1)x \\
\hline
-(a^2-1)x^2+(a^2-1)\,(a+3)x+(a^2-1)^2 \\
-(a^2-1)x^2+(a^2-1)\,(a+3)x+(a^2-1)^2 \\
\hline
\end{array}
$$

$x^2-2ax-a^2+1$ *Ans.*

(*b*) $12x^2+x-35$

$=12x^2+21x-20x-35=3x(4x+7)-5(4x+7).$

$=(3x-5)(4x+7)$ *Ans.*

$(2b-a)^3+(2a-b)^3-(a+b)^3$

This expression is of the form $x^3+y^3+z^3$

Now, if $x+y+z=0$, $x^3+y^3+z^3=3xyz$

Let $2b-a=x$, $2a-b=y$ and $-(a+b)=z$

$\therefore\; x+y+z=2b-a+2a-b-a-b=0$

$\therefore$ The expression $=3(2b-a)\,(2a-b)-(a+b)$

$=3(a+b)\,(2a-b)\,(a-2b)$ *Ans.*

7. Let A and B be the two expressions, and let x be their H. C. F.

Let a and b be the respective quotients when A and B are divided by x $\therefore A=ax$, $B=bx$. Then since a and b have no common factor the L. C. M. of A and B, say y, is abx.

$\therefore AB=ax.\ bx=x.\ abx=xy$ Q. E. D.

$x^4+x^3-12x^2-2x+80$) $x^4+5x^3-36x^2+50x+48$ (1

$x^4+x^3-12x^2-2x+80$

$4x^3-24x^2+52x-32$

$4(x^3-6x^2+13x-8$) $x^4+x^3-12x^2-2x+80$ ($x+7$

$x^4-6x^3+13x^2-8x$

$7x^3-25x^2+6x+80$

$7x^3-42x^2+91x-56$

$17x^2-85x+136$

$17(x^2-5x+8$) $x^3-6x^2+13x-8$ ($x-1.$

x^3-5x^2+8x

$-x^2+5x-8$

$-x^2+5x+8$

x^2-5x+8 *Ans.*

8. (*i*) The expression :

$$=\frac{a^2}{\left(\frac{b-a}{ab}\right)\left(\frac{c-a}{ac}\right)}+\frac{b^2}{\left(\frac{c-b}{bc}\right)\left(\frac{a-b}{ab}\right)}+\frac{c^2}{\left(\frac{a-c}{ac}\right)\left(\frac{b-c}{bc}\right)}$$

$$=\frac{a^4bc}{(b-a)(c-a)}+\frac{b^4ca}{(c-b)(a-b)}+\frac{c^4ab}{(a-c)(b-c)}$$

$$=-abc\left\{\frac{a^3}{(a-b)(c-a)}+\frac{b^3}{(a-b)(b-c)}+\frac{c^3}{(b-c)(c-a)}\right\}$$

$$=-abc\left\{\frac{a^3(b-c)+b^3(c-a)+c^3(a-b)}{(a-b)(b-c)(c-a)}\right\}$$

Now, $a^3(b-c)+b^3(c-a)+c^3(a-b)$

$$=a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2)$$
$$=(b-c)\{a^3-a(b^2+bc+c^2)+bc(b+c)\}$$
$$=(b-c)\{-b^2(a-c)-bc(a-c)+a(a^2-c^2)\}$$
$$=(b-c)(a-c)\{-b^2-bc+a(a+c)\}$$
$$=(b-c)(a-c)\{c(a-b)+(a^2-b^2)\}$$
$$=(b-c)(a-c)(a-b)(c+a+b)$$

$\therefore$ The expression equals

$$-abc\left\{-\frac{(a-b)(b-c)(c-a)(a+b+c)}{(a-b)(b-c)(c-a)}\right.$$
$$=abc(a+b+c) \text{ Ans.}$$

II. $64x^6-48x^4+12x^2-1$

$$=(4x^2)^3-3(4x^2)^2\times 1+3.-4x^2(1)^2-(1)^3$$
$$=(4x^2-1)^3 \qquad \therefore \sqrt[3]{(4x^2-1)^3}=4x^2-1 \textit{ Ans.}$$
$$\sqrt{16x^4-64x^3+24x^2+80x+25}=4x^2-8x-5$$

$\therefore$ The fraction equals

$$\frac{4x^2-1-(4x^2-8x-5)}{4x^2-12x-7}=\frac{8x+4}{(2x+1)(2x-7)}$$
$$=\frac{4(2x+1)}{(2x+1)(2x-7)}=\frac{4}{2x-7} \textit{ Ans.}$$

IX. (i) $\frac{7x}{2}-\frac{35}{24}+\frac{7x}{12}-\frac{5}{16}=\frac{307}{144}-\frac{14}{3}+5x$

$\therefore$ $504x-210+84x-45=307-672+720x$

$\therefore$ $504x+84x-720x=307-672+210+45$

$\therefore$ $-132x=-110 \quad \therefore x=\frac{110}{132}=\frac{5}{6}$ *Ans.*

(ii.) $(1+p)(x-py)=\frac{2p^2}{1-p}$ (i); $2p^2\left(\frac{x}{1+p}+\frac{y}{1-p}\right)$
$=\frac{2p^2}{1-p}\cdots\cdots$(ii)

$x-py=\frac{2p^2}{1-p^2}$ (i) ; $\frac{x}{1+p}+\frac{y}{1-p}=\frac{1}{1-p}$ (ii)

Multiply (i) by $\frac{1}{1+p}$ and subtract it from (ii)

$$\therefore \quad \frac{x}{1+p}+\frac{y}{1-p}=\frac{1}{1-p}$$

$$\frac{x}{1+p}-\frac{py}{1+p}=\frac{2p^2}{(1-p^2)(1+p)}$$

$$\frac{y}{1-p}+\frac{py}{1+p}=\frac{1}{1-p}-\frac{2p^2}{(1-p^2)(1+p)}$$

$$\therefore \quad \frac{y+py+py-p^2y}{1-p^2}=\frac{(1+p)^2-2p^2}{(1-p^2)(1+p)}$$

$$\therefore \quad \frac{y(1+2p-p^2)}{1-p^2}=\frac{1+2p-p^2}{(1-p^2)(1+p)}$$

$$\therefore \quad y=\frac{1}{1+p}$$

Substituting the value of y in (ii,) we get

$$\frac{x}{1+p}+\frac{1}{1-p^2}=\frac{1}{1-p}$$

$$\therefore \quad \frac{x}{1+p}=\frac{1}{1-p}-\frac{1}{1-p^2}=\frac{1+p-1}{1-p^2}=\frac{p}{1-p^2}$$

$$\therefore \quad x=\frac{p}{1-p} \qquad x=\frac{p}{1-p};\ y=\frac{1}{1+p}\ \textit{Ans.}$$

X. Let x maunds be the luggage allowed free to each passenger.

Then $(3+5)$ Rs. = charge for $(7-2x)$ maunds.

$\therefore$ Rs. $\frac{8}{7-2x}$ = charge for 1 maund.

Again, Rs. 11 is the charge for $(7-x)$ maunds.

$\therefore$ Rs. $\frac{11}{7-x}$ is the charge for 1 maund.

$\therefore \quad \frac{8}{7-2x}=\frac{11}{7-x}$ whence $x=1\frac{1}{2}$

$1\frac{1}{2}$ maunds free of charge. *Ans.*

1900

Arithmetic and Algebra.

I. (*a*) Prove that the Least Common Multiple of two numbers is equal to their product divided by their Greatest Common Measure. Find whether the rule is true for three numbers. 8

(*b*) A company of soldiers is formed into 6 equal rows, after a time it is re-arranged into 7 equal rows, and finally into 8 equal rows. Find the least number of soldiers above 900 which the company may contain.

II. Reduce the weight of— 8

$$\frac{3{\cdot}4\dot{4}\dot{5} \times 3{\cdot}4\dot{4}\dot{5} - 1{\cdot}5\dot{5}\dot{4} \times 1{\cdot}\dot{5}\dot{5}\dot{4}}{4{\cdot}\dot{1} \times {\cdot}\dot{4}0\dot{5}}$$

cubic feet of water to the decimal of a ton: it being known that one cubic foot of water weighs 62·37 lbs. avoir.

III. If 38 men working 6 hours a day do a piece of work in 12 days, find in what time 57 men working 8 hours a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour as 3 men of the second set can do in $1\frac{1}{3}$ hours. 9

IV. A person invested Rs. 15,147 in 4 per cent. stock and Rs. 12,954 in 6 per cent. stock; when the stocks were at Rs. 86, anna 1, and Rs. 102, respectively; what income did he derive from these investments? He afterwards transferred at the above rates a certain sum of money from the 6 per cent. stock to 4 per cent. stock, and then found that the income from each stock was the same. How much stock had he finally in the 6 per cent? 11

V. (a) Find the square root of ·0001083681 9

(b) A stone dropped down a shaft falls through a number of feet equal to 16·1 times the square of the number of seconds during which it is falling: find to two places of decimals the number of seconds that the stone will take to reach the bottom of a mine 1,104 yards deep.

VI (a) Divide $2x^4-6ax^3+(4a^2+ab-2b^2)x^2+3ab^2x$ $-a^2b^2$ by $x^2-(2a-b)x-ab$. 11

(b) Resolve into factors:—

$x^2+6x-187$ and $x^4-5x^3+9x^2-7x+2$.

VII. Simplify:— 11

(i) $\dfrac{x^3+2x^2-29x+30}{x^3-3x^2-34x+120}$

(ii) $\dfrac{3a+2b+2c}{(a-b)(a-c)}+\dfrac{3b+2c+2a}{(b-c)(b-a)}+\dfrac{3c+2a+2b}{(c-a)(c-b)}$

VIII. Find the square root of— 10

$$x^6-\frac{3}{2}x^5+\frac{2}{3}x^3+\frac{9}{16}x^2-\frac{1}{2}x+\frac{1}{9}$$

and the cube root of—

$$8+36x+42x^2-9x^3-21x^4+9x^5-x^6$$

IX. Upon what axioms does the process of solving a simple equation depend? 12

(i) Solve $\frac{x}{5}-\left\{3x+6-\frac{4}{5}(x+10)\right\}=$

$$2\tfrac{1}{4}-11\left(9-\frac{5}{12}x\right)$$

(ii) $\dfrac{2x+3y}{5a+b}=\dfrac{ab}{a^2-b^2}=\dfrac{ax+by}{a^2+b^2}$

X. Two cyclists ride from A to B, a distance of 55 miles, and the first arrives 30 minutes before the second. They then ride from B to A, the first giving the second a start of 4 miles, and yet arriving 6 minutes before him. Find the rate of each cyclist in miles per hour. 11

1900

EUCLID.

I. If one angle of a triangle be greater than a second angle, the side opposite to the first angle shall be greater than the side opposite to the second. 13

The base of a triangle whose sides are unequal is divided into two parts by the straight line bisecting the vertical angle: prove that the greater part is adjacent to the greater side.

II. In any right-angled triangle the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle. 15

In a given straight line AB find a point D such that the difference of the squares on AD, BD may be equal to the square on a given straight line Y which is not greater than AB.

III. A, B, C, D, E are points on a straight line such that AB, BC, CD, DE are all equal, and O is any point outside the straight line, prove that the difference of the squares on OA and OE is twice the difference of the squares on OB and OD. 11

IV. Describe a square that shall be equal to a given rectilineal figure. 14

Construct a right-angled isosceles triangle equal to a given rectilineal figure.

V. If two circles touch one another externally, the straight line which joins their centres shall pass through the point of contact. 11

Three equal circles with centres A, B, C touch each other externally at the points D, E, F: prove that the area of the triangle ABC is four times the area of the triangle DEF.

VI. If a straight line touch a circle the radius drawn from the centre to the point of contact shall be perpendicular to the line touching the circle. 13

A quadrilateral is formed by the diameter of a semi-circle, the tangents at its extremities, and any third tangent. Prove that its area is half that of the rectangle contained by the diameter and the side opposite to the diameter.

VII. If from any point without a circle there be drawn two straight lines, one of which cuts the circle and the other meets it, and if the rectangle contained by the whole line which cuts 7

1 c. ft. of water weighs $\frac{6237}{100}$ lbs.

$\therefore$ $\frac{312}{55}$ c. ft. weigh $\frac{312}{55} \times \frac{6237}{100}$ lbs. $= \frac{3159}{20000}$ of a ton

$=$ ·15795 of a ton. *Ans.*

III. Three men can do in $1\frac{1}{2}$ hours as much work as $3 \times 1\frac{1}{2}$ or $4\frac{1}{2}$ men can do in 1 hour $\therefore$ 2 men of the first set can do as much work in 1 hour as $4\frac{1}{2}$ men of the second set in the same time. $\therefore$ 38 men of the first set can do as much work as$(4\frac{1}{2} \times 19)$ men of the second can do in the same time:—

$$\left.\begin{array}{lll} \text{Inverse} & 4\frac{1}{2} \times 19 \text{ men} : 57 \text{ men} \\ \text{Direct} & 1 \text{ work} : 2 \text{ work.} \\ \text{Inverse} & 6 \text{ hours} : 8 \text{ hours.} \end{array}\right\} :: 12 \text{ days.}$$

$\therefore$ number of days $=27$ 27 *Ans.*

IV. Rs. $86\frac{1}{16}$: Rs. 15,147 :: Rs. 4 income : x

$\therefore$ $x=$Rs. 704 income in the first case.

Rs. 102 : Rs. 12,954 :: Rs. 6 income : x.

$\therefore$ $x =$ Rs. 762 income in the second case.

$\therefore$ Total Income$=$Rs. 704$+$Rs. 762$=$Rs. 1,466. *Ans.*

The man invests Rs. $86\frac{1}{16}$ to obtain Rs. 4 income;

Income Rs. 6 : Income Rs. 4 :: Investment Rs. 102 : x.

$\therefore$ $x=$ Rs. 68 Investment.

i. e.—In order to get the same income from the second investment, *viz.* Rs. 4, he ought to invest Rs. 68. Rs. 15,147 + Rs. 12,954 = Rs. 28,101 total investment.

Rs. $86\frac{1}{16}$ + Rs. 68 = Rs. $154\frac{1}{16}$ total investment which produces Rs. 4 income.

$\therefore$ Rs. $154\frac{1}{16}$: Rs. 28,101 :: Rs. 68 : x.

$x=$Rs. $\frac{62,016}{5}$ investment in the second case.

Rs. 102 : Rs. $\frac{62,016}{5}$ :: Rs. 100 Stock : x.

$\therefore$ $x =$ Rs, 12,160 stock in the 6 per cents. *Ans.*

V. (a) $\sqrt{\cdot 0001083681} = \cdot 01041$. *Ans.*

(b) 16·1 times square of the number of seconds $= 1104 \times 3$.

$\therefore$ square of the number of seconds $= \dfrac{3312}{16\cdot 1}$

$= \dfrac{33120}{161} = \dfrac{1440}{7} = 205\cdot\dot{7}1428\dot{5}$.

$\therefore$ number of seconds $= \sqrt{205\cdot\dot{7}1428\dot{5}} = 14\cdot 34 \;\ldots$

14·34 Seconds. *Ans.*

VI. (a)

$$x^2-(2a-b)-ab \,\big)\, 2x^4-6ax^3+(4a^2+ab-2b^2)x^2+3ab^2x-a^2b^2 \,\big(\, 2x^2-2(a+b)x+ab$$
$$-2x^4 \mp 2(2a-b)x^3 \mp 2abx^2$$
$$-2(a+b)x^3+(4a^2+3ab-2b^2)x^2+3ab^2x-a^2b^2$$
$$\mp 2(a+b)x^3 \pm 2(2a-b)(a+b)x^2 \pm 2ab(a+b)x$$
$$abx^2-ab(2a-b)x-a^2b^2$$
$$abx^2 \mp ab(2a-b)x \mp a^2b^2$$

$2x^2-2(a+b)x+ab$. *Ans.*

(b) $x^2+6x-187 = x^2+17x-11x-187$
$= x(x+17)-11(x+17)$
$= (x-11)(x+17)$

$(x-11)(x+17)$ *Ans.*

If, in any expression consisting of four or more terms involving several powers of any quantity, say x, the sum of the Co-efficients of all the terms be equal to zero, then $x-1$ is a factor of the expression.

In $x^4-5x^3+9x^2-7x+2$, the sum of all the co-efficients is zero, $\therefore$ $x-1$ is a factor of the expression : hence the exp.

$$=x^4-x^3-4x^3+4x^2+5x^2-5x-2x+2$$
$$=x^3(x-1)-4x^2(x-1)+5x(x-1)-2(x-1)$$
$$=(x-1)(x^3-4x^2+5x-2)$$

Again the sum of all the co-efficients of x^3-4x^2+5x-2 $=0$ $\therefore$ $x-1$ is a factor of the expression : hence the exp.

$$=x^3-x^2-3x^2+3x+2x-2$$
$$=x^2(x-1)-3x(x-1)+2(x-1)$$
$$=(x-1)(x^2-3x+2)=(x-1)(x-1)(x-2)$$

$\therefore$ the factors of the original exp. $=(x-1)^3(x-2)$ *Ans.*

VII. (*i*) If, in any expression consisting of four or more terms involving several powers of any quantity, say x, the sum of the Co-efficients of the odd terms be equal to the sum of the Co-efficients of the even terms, then $x+1$ is a factor of the expression.

$\therefore$ $x+1$ is a factor of the numerator.

$\therefore$ the numerator $=x^3+x^2+x^2+x-30x-30$

$$=x^2(x+1)+x(x+1)-30(x+1)$$
$$=(x+1)(x^2+x-30)$$

By trial, we find that the denominator vanishes when $x=4$ $\therefore$ $x-4$ is a factor of the denominator $\therefore$ The denominator

$$=x^3-4x^2+x^2-4x-30x+120$$
$$=x^2(x-4)+x(x-4)-30(x-4)$$
$$=(x-4)(x^2+x-30)$$

$\therefore$ The fr. $=\dfrac{(x+1)(x^2+x-30)}{(x-4)(x^2+x-30)}=\dfrac{x+1}{x-4}$; $\dfrac{x+1}{x-4}$ *Ans.*

The expression

$$=\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}$$

$$+\frac{2a+2b+2c}{(a-b)(a-c)}+\frac{2b+2c+2a}{(b-c)(b-a)}+\frac{2c+2a+2b}{(c-a)(c-b)}$$

Now $\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}$

$$=-\frac{a}{(a-b)(c-a)}-\frac{b}{(b-c)(a-b)}-\frac{c}{(c-a)(b-c)}$$

$$=\frac{-a(b-c)-b(c-a)-c(a-b)}{(a-b)(b-c)(c-a)}=0$$

Again. $\frac{2a+2b+2c}{(a-b)(a-c)}+\frac{2b+2c+2a}{(b-c)(b-a)}+\frac{2c+2a+2b}{(c-a)(c-b)}$

$$=(2a+2b+2c)\left\{-\frac{1}{(a-b)(c-a)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(c-a)(b-c)}\right\}$$

$$=(2a+2b+2c)\left\{\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}\right\}$$

$=(2a+2b+2c)\times 0=0$ $\therefore$ The whole expression $=0$

0 *Ans.*

VIII. (a)

$$\begin{array}{r|l}
 & x^6-\frac{3}{2}x^4+\frac{2}{3}x^3+\frac{9}{16}x^2-\frac{x}{2}+\frac{1}{9}\ \ \lfloor x^3-\frac{3}{4}x+\frac{1}{3} \\
 & x^6 \\
\hline
2x^3-\frac{3}{4}x & -\frac{3}{2}x^4+\frac{2}{3}x^3+\frac{9}{16}x^2-\frac{x}{2}+\frac{1}{9} \\
 & -\frac{3}{2}x^4\qquad+\frac{9}{16}x^2 \\
\hline
2x^3-\frac{3}{2}x+\frac{1}{3} & \frac{2}{3}x^3-\frac{x}{2}+\frac{1}{9} \\
 & \frac{2}{3}x^3-\frac{x}{2}+\frac{1}{9} \\
\hline
\end{array}$$

$x^3-\frac{3}{4}x+\frac{1}{3}$ *Ans.*

(*b*)

$$\begin{array}{r|l}
& 8+36x+42x^2-9x^3-21x^4+9x^5-x^6 \;(\underline{2+3x-x} \\
& \underline{8} \\
& \quad 36x+42x^2-9x^3-21x^4+9x^5-x^6 \\
3\times 2^2 = \quad 12 & \\
3\times 2\times 3x = \quad +18x & \\
(3x)^2 = \quad +9x^2 & \\
\overline{12+18x+9x^2} & \underline{36x+54x^2+27x^3} \\
& \quad -12x^2-36x^3-21x^4+9x^5-x^6 \\
3(2+3x)^2 = 12+36x+27x^2 & \\
3(2+3x)\times -x^2 = \quad -6x^2-9x^3 & \\
(-x^2)^2 = \quad +x^4 & \underline{-12x^2-36x^3-21x^4+9x^5-x^6} \\
\overline{12+36x+21x^2-9x^3+x^4} & \quad \cdots\cdots \\
& \qquad 2+3x-x^2 \; \textit{Ans.}
\end{array}$$

IX. The process of solving a simple equation depends on the following axioms:—

(1) If equals be added to equals, the sums are equal.

(2) If equals be taken from equals, the remainders are equal.

(3) If equals be multiplied by equals, the products are equal.

(4) If equals be divided by equals, the quotients are equal.

$$\therefore\quad \frac{x}{5}-3x-6+\frac{4x}{5}+8=\frac{9}{4}-99+\frac{55}{12}x$$

$$\therefore\quad \frac{x}{5}-3x+\frac{4x}{5}-\frac{55x}{12}=\frac{9}{4}-99+6-8$$

$$\therefore\quad \frac{12x-180x+48x-275x}{60}=-98\tfrac{3}{4}=-\frac{395}{4}$$

$$\therefore\quad -\frac{395x}{60}=-\frac{395}{4}\qquad\therefore\quad x=15\qquad 15\ \textit{Ans.}$$

(ii) $$\frac{2x+3y}{5a+b}=\frac{ab}{a^2-b^2};\ \frac{ax+by}{a^2+b^2}=\frac{ab}{a^2-b^2}$$

$$2x+3y=\frac{ab(5a+b)}{a^2-b^2}\ (i);\qquad ax+by=\frac{ab(a^2+b^2)}{a^2-b^2}\ (ii)$$

Multiply (*i*) by a, (*ii*) by 2; and subtract (*ii*) from (*i*)

$$\therefore\quad 2ax+3ay=\frac{a^2b(5a+b)}{a^2-b^2}$$

$$2ax+2by=\frac{2ab(a^2+b^2)}{a^2-b^2}$$

$$\therefore\quad y(3a-2b)=\frac{a^2b(5a+b)}{a^2-b^2}-\frac{2ab(a^2+b^2)}{a^2-b^2}$$

$$=\frac{ab(5a^2+ab-2a^2-2b^2)}{a^2-b^2}$$

$$=\frac{ab(3a^2+ab-2b^2)}{a^2-b^2}=\frac{ab(3a-2b)(a+b)}{a^2-b^2}$$

$$=\frac{ab(3a-2b)}{a-b}$$

$$\therefore\quad y=\frac{ab(3a-2b)}{a-b}\times\frac{1}{3a-2b}=\frac{ab}{a-b}$$

Substituting the value of y in (*i*) we get

$$2x+\frac{3ab}{a-b}=\frac{ab(5a+b)}{a^2-b^2}$$

$$\therefore \quad 2x=\frac{ab(5a+b)}{a^2-b^2}-\frac{3ab}{a-b}$$

$$=\frac{ab(5a+b)-3ab(a+b)}{a^2-b^2}=\frac{ab(5a+b-3a-3b)}{a^2-b^2}$$

$$=\frac{ab(2a-2b)}{a^2-b^2}=\frac{2ab(a-b)}{a^2-b^2}=\frac{ab}{a+b}$$

$$x=\frac{ab}{a+b}; \quad y=\frac{ab}{a-b} \quad \textit{Ans.}$$

X. Let x miles per hour be the rate of the 1st cyclist.

Let y miles per hour be the rate of the 2nd cyclist.

The 1st takes $\frac{55}{x}$ miles an hour to ride the whole distance.

The 2nd takes $\frac{55}{y}$ miles an hour to ride the whole distance

$$\therefore \frac{55}{y}-\frac{55}{x}=\frac{1}{2} \text{ ....}(i).$$

Again, the second cyclist is given a start of 4 miles;

$\therefore$ He has to ride $55-4=51$ miles.

$$\therefore \frac{51}{y}-\frac{55}{x}=\frac{1}{10}\text{.........}(ii)$$

Subtracting (ii) from (i) we get,

$$\frac{55}{y}-\frac{51}{y}=\frac{1}{2}-\frac{1}{10} \quad \therefore \frac{4}{y}=\frac{4}{10} \quad \therefore y=10$$

Substituting the value of y in (i) we get,

$$\frac{55}{10}-\frac{55}{x}=\frac{1}{2}, \text{ hence } x=11$$

Rate of the first cyclist $=$ 11 miles an hr.

Rate of the 2nd cyclist $=$ 10 miles an hr. *Ans.*

1901.

Arithmetic and Algebra.

I. Simplify— **9**

$$\frac{\cdot\dot{9}4\dot{5} \times \cdot 6\dot{1}4285\dot{7}}{1\cdot\dot{1}6\dot{2}} + \frac{\cdot\dot{8}5714\dot{2} \times \cdot\dot{5}6\dot{7}}{\cdot 1\dot{9}4\dot{5}}$$

II. A tea-merchant has a rectangular space for storing tea. It is $30\frac{1}{4}$ ft. long, $16\frac{1}{2}$ ft. broad, $27\frac{1}{3}$ ft. high. He wishes to fill this space with packets of cubical shape, all of the same size. What is the largest size of such cubical packets that can be made to fill the space exactly, and what will be the number of such packets? **10**

III. *A*, *B*, *C* enter into partnership. *A* contributes a certain sum for 8 months, and claims $\frac{8}{47}$ths of the profit. *B's* capital is in the trade for 11 months, and *C* advances Rs. 425 for 7 months and claims $\frac{17}{47}$ths of the profit. How much did *A* and *B* contribute? **8**

IV. 640 horses are conveyed in transports to the seat of war at a cost of Rs. 5,432 for food. A storm occurs just after $\frac{3}{8}$ths of the voyage is completed, in which 32 horses are killed. If the cost of the food of each horse be 7 annas per day, what is the length of the voyage? **8**

V. A person borrows two equal sums at the same time at 6 and 5 per cent., respectively, and finds that, if he repays the former sum with interest on a certain date 6 months before he repays the latter, he will have to pay in each case the same amount, *viz.*, Rs. 1,380. Find the amount borrowed and the time for which interest is paid. **10**

VI. (*a*) Subtract $\dfrac{x+5}{x^2+5x-6}$ from $\dfrac{x+6}{x^2+3x-10}$ and **11**
divide the difference by $1+\dfrac{2(x^2+4x-8)}{x^2+11x+30}$

(*b*) Resolve into factors :—

(1) $4x^2-9y^2-6x-9y$,

(2) $x^4-5x^3+x^2+21x-18$.

VII. State and prove the rule for finding the Lowest **10**
Common Multiple of two algebraical expressions.

Find the G. C. M. of—

$4x^3-20x^2+17x-4$, $2x^3-15x^2+31x-12$, and $4x^3-16x^2+13x-3$.

VIII. Find the square root of— **10**

$a^2x^6+6abx^4-2acx^3+9b^2x^2-6bcx+c^2$ and the cube root of $x^6+\dfrac{27}{x^6}-6\left(x^4+\dfrac{9}{x^4}\right)+21\left(x^2+\dfrac{3}{x^2}\right)-44$

IX. Solve the following equations :— **12**

(i) $\dfrac{1\cdot05x+10}{50}+\dfrac{1\cdot35x-2}{20}-\dfrac{1\cdot5x-18}{10}+\dfrac{1\cdot5x-3}{15}=1\cdot854$

(ii) $\dfrac{(a-b)x+(a+b)y}{a^2-b^2}=\dfrac{ab}{a-b}=\dfrac{ab(x-y)-(a^2y-b^2x)}{2ab^2}$

X. A man travels part of a journey on a bicycle, and **12**
then for the last 72 miles takes a train which travels four times as fast as he did on his bicycle, and arrives at his destination in $3\frac{1}{2}$ hours from the start. If he had travelled the whole way in the train, he would have saved $1\frac{1}{2}$ hours. Find the length of journey in miles.

Euclid.

I. At a given point in a given straight line make a rectilineal angle equal to a given rectilineal angle. 13

Construct a triangle having given the base, one of the angles at the base, and the sum of the remaining sides.

II. Prove that the opposites angles of a parallelogram are equal, and that a diagonal bisects it. 13

If two parallelograms have a common diagonal, shew that their other angular points are at the corners of another parallelogram.

III. If a straight line be divided into any two parts, the square on the line made up of the whole and one part is equal to four times the rectangle contained by the whole and that part, with the square on the other part. 14

By means of this proposition prove that the square on a straight line is nine times the square on one-third of the line.

IV. $ABCD$ is a parallelogram, of which AC and BD are the diagonals; P is a point such that the sum of the squares on PA and PC is equal to the sum of those on PB and PD. Prove that $ABCD$ is a rectangle. 13

V. If one circle touch another internally, the straight line joining their centres, if produced, pass through the point of contact. 13

A circle is described on the radius of another circle as diameter. Prove that any chord of the greater circle drawn from the point of contact is bisected by the lesser circle.

VI. The angle at the centre of a circle is double the angle at the circumference, when the angle stand on the same arc. 13

Two circles, whose centres are A and B, intersect in C and D; a common tangent EF is drawn to them on the same side of AB on which C is; prove that the angle ACB is double the angle EDF.

VII. Inscribe a regular pentagon in a given circle. 13

$ABCDE$ is a regular pentagon and AC, BE intersect at H, shew that $AB=CH=EH$.

VIII. Inscribe a regular hexagon in a given circle. 8

SOLUTIONS.

1. $$\frac{\frac{945}{999}\times\frac{6\frac{1}{7}}{10}}{1\frac{162}{999}}+\frac{\frac{6}{7}\times\frac{567}{999}}{\frac{1,944}{9,990}}$$

$$=\frac{35}{37}\times\frac{43}{70}\times\frac{999}{1161}+\frac{6}{7}\times\frac{567}{999}\times\frac{9,990}{1,944}=\frac{1}{2}+\frac{5}{2}=3 \quad \textit{Ans.}$$

2. H. C. F. of $30\frac{1}{4}$, $16\frac{1}{2}$, $7\frac{1}{3}$ or $\frac{121}{4}$, $\frac{33}{2}$, $\frac{22}{3}=\frac{11}{12}$, *i. e.*, 11 inches $\therefore$ the length of the edge of each packet = 11 inches,

$\therefore$ number of packets $=(30\frac{1}{4}\times16\frac{1}{2}\times7\frac{1}{3})\div\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}$

$=4,752$. 11 inches each way: 4,752. *Ans.*

3. $1-\left(\frac{8}{47}+\frac{17}{47}\right)=\frac{22}{47}$ $\therefore$ the profits are in the ratio of $\frac{8}{47}, \frac{22}{47}, \frac{17}{47}$, *i. e.*, 8 : 22 : 17.

C advances Rs. 425 for 7 months, *i. e.*, advances Rs. 425 × 7 for 1 month.

$\therefore$ 17 : 8 :: 425 × 7 : A's capital for 1 month.

$\therefore$ A's capital for 1 month = Rs. 1,400.

$\therefore$ his capital for 8 months = Rs. $\frac{1,400}{8}$ = Rs. 175.

17 : 22 :: 425 × 7 : B's capital for 1 month.

$\therefore$ B's capital for 1 month = Rs. 3,850.

$\therefore$ his capital for 11 months = Rs. $\frac{3850}{11}$ = Rs. 350.

A's capital, Rs. 175 ; B's capital, Rs. 350. *Ans.*

4. The length of the voyage in days

$= 640\times7\times\frac{2}{5}$ of the voyage $+ 608\times7\times\frac{3}{5}$ of the voyage

$= 5432 \times 16$

$\therefore \frac{21728}{5}$ of the journey in days $= 5,432\times 16.$

$\therefore$ the length of the journey in days$=\dfrac{5432\times 16\times 5}{21728}=20.$

20 days. *Ans.*

5. Proceed as in Q. 4, 1891.

Rs. 1,200 sum borrowed. *Ans.*

Rs. $2\frac{1}{2}$ years, and 3 years. *Ans.*

VI. (*a*) $\dfrac{x+6}{x^2+3x-10}-\dfrac{x+5}{x^2+5x-6}$

$$=\frac{(x+6)(x^2+5x-6)-(x+5)(x^2+3x-10)}{(x^2+3x-10)(x^2+5x-6)}$$

$$=\frac{3x^2+19x+14}{(x^2+3x-10)(x^2+5x-6)}$$

$$1+\frac{2(x^2+4x-8)}{x^2+11x+30}=\frac{x^2+11x+30+2(x^2+4x-8)}{x^2+11x+30}$$

$$=\frac{3x^2+19x+14}{x^2+11x+30}$$

$$\frac{3x^2+19x+14}{(x^2+3x-10)(x^2+5x-6)}\div\frac{3x^2+19x+14}{x^2+11x+30}$$

$$=\frac{3x^2+19x+14}{(x+5)(x-2)(x+6)(x-1)}\times\frac{(x+6)(x+5)}{3x^2+19x+14}$$

$$=\frac{1}{(x-1)(x-2)}.$$

$\dfrac{1}{(x-1)(x-2)}$. *Ans.*

(*b*) (i) $4x^2-9y^2-6x-9y$

$=(2x-3y)(2x+3y)-3(2x+3y)$

$=(2x+3y)(2x-3y-3).$

$(2x+3y)(2x-3y-3)$. *Ans.*

(ii) $x^4-5x^3+x^2+21x-18$

As the sum of the coefficients all the terms$=0$ $x-1$ is a factor.

$\therefore = x^4-x^3-4x^3+4x^2-3x^2+3x+18x-18$

$=x^3(x-1)-4x^2(x-1)-3x(x-1)+18(x-1)$

$=(x-1)(x^3-4x^2-3x+18)$

Again, $x^3-4x^2-3x+18$

$=x^3+2x^2-6x^2-12x+9x+18$

$=x^2(x+2)-6x(x+2)+9(x+2)$

$=(x+2)(x^2-6x+9)$ $\therefore$ the factors of the expression

$=(x+2)(x-3)^2$

$=(x-1)(x+2)(x-3)^2.$ *Ans.*

VII(a) Rule :—L. C. M. of two Expressions $=\dfrac{\text{Product of the Exp.}}{\text{H. C. F.}}$

Proof :—Let A and B be the two Expressions and let x be the H. C. F.

Divide A and B by x, and let the respective quotients be a and b.

$\therefore$ $A=ax$ and $B=bx$. Now, since a and b have no common factors, every common multiple of A and B must contain a. x. b. as a factor. $\therefore$ L. C. M. $=abx$; but $abx=a(xb)=\dfrac{A}{x}.B.$

$\therefore$ the required L. C. M. $=\dfrac{A\times B}{H.}$ Q. E. D.

(b) On trial, we find that the first expression vanishes when $x=4$, hence $x-4$ is a factor of the expression.

$\therefore 4x^3-20x^2+17x-4$

$=4x^3-16x^2-4x^2+16x+x-4$

$=4x^2(x-4)-4x(x-4)+1(x-4)$

$=(x-4)(4x^2-4x+1)$

$=(x-4)(2x-1)(2x-1)$ (a)

Again, by trial, we find that $x-4$ is a factor of the second expression.

$\therefore$ $2x^3-15x^2+31x-12$

$=2x^3-8x^2-7x^2+28x+3x-12$

$=2x^2(x-4)-7x(x-4)+3(x-4)$

$=(x-4)(2x^2-7x+3)$

$=(x-4)(2x-1)(x-3)$ (b)

By trial, we find that $x-3$ is a factor of the third expression.

$$\therefore\ 4x^3-16x^2+13x-3$$

$$=4x^3-12x^2-4x^2+12x+x-3$$

$$=4x^2(x-3)-4x(x-3)+1(x-3)$$

$$=(x-3)(4x^2-4x+1)$$

$$=(x-3)(2x-1)(2x-1) \qquad (c)$$

$\therefore$ L. C. M. of a, b, $c=(x-3)(x-4)(2x-1)^2$. *Ans.*

VIII (a)

$$\begin{array}{r|l}
 & a^2x^6+6abx^4-2acx^3+9b^2x^2-6bcx+c^2\ \underline{(\ ax^3+3bx-c} \\
 & a^2x^6 \\
\hline
 & \qquad 6abx^4-2acx^3+9b^2x^2-6bcx+c^2 \\
2ax^3+3bx & \qquad 6abx^4 \qquad\quad +9b^2x^2 \\
\hline
 & \qquad\qquad -2acx^3 \qquad\quad -6bcx+c^2 \\
2ax^3+6bx-c & \qquad\qquad -2acx^3 \qquad\quad -6bcx+c^2 \\
\hline
\end{array}$$

$ax^3+3bx-c$. *Ans.*

(*b*)

$$x^6-6x^4+21x^2-44+\frac{63}{x^2}-\frac{54}{x^4}+\frac{27}{x^6}\ \Big|\ x^2-2+\frac{3}{x^2}$$

$$x^6$$

$$-6x^4+21x^2-44+\frac{63}{x^2}-\frac{54}{x^4}+\frac{27}{x^6}$$

$$\begin{aligned}
3(x^2)^2 &= 3x^4\\
3\times x^2\times -2 &= -6x^2\\
(-2)^2 &= +4\\
&\overline{3x^4-6x^2+4}
\end{aligned}$$

$$-6x^4+12x^2-8$$

$$9x^2-36+\frac{63}{x^2}-\frac{54}{x^4}+\frac{27}{x^6}$$

$$\begin{aligned}
3(x^2-2)^2 &= 3x^4-12x^2+12\\
3(x^2-2)\times\frac{3}{x^2}+ &\quad +9-\frac{18}{x^2}\\
\left(\frac{3}{x^2}\right)^2 &= +\frac{9}{x^4}\\
&\overline{3x^4-12x^2+21-\frac{18}{x^2}+\frac{9}{x^4}}
\end{aligned}$$

$$9x^2-36+\frac{63}{x^2}-\frac{54}{x^4}+\frac{27}{x^6}$$

$$x^2-2+\frac{3}{x^2}.\quad \textit{Ans.}$$

IX. (i) $\frac{1\frac{5}{100}x}{50}+\frac{10}{50}+\frac{1\frac{35}{100}x}{20}-\frac{2}{20}-\frac{1\frac{5}{10}x}{10}+\frac{18}{10}+\frac{1\frac{5}{10}x}{15}-\frac{3}{15}$

$-\frac{3}{15}=1\frac{854}{1000}$

$$\therefore \frac{21x}{1000}+\frac{1}{5}+\frac{27x}{400}-\frac{1}{10}-\frac{3x}{20}+\frac{9}{5}+\frac{x}{10}-\frac{1}{5}=\frac{927}{500}$$

$$\therefore \frac{21x}{1,000}+\frac{27x}{400}-\frac{3x}{20}+\frac{x}{10}=\frac{927}{500}+\frac{1}{10}-\frac{1}{5}$$

$$\therefore \frac{42x+135x-300x+200x}{2,000}=\frac{77}{500}$$

$\therefore \frac{7,7x}{2.000}=\frac{77}{500}$ $\quad\therefore x=4.$ $\qquad$ 4. *Ans.*

(ii) $(a-b)x+(a+b)y=ab(a+b)$..............(i)

$b(a+b)x-a(a+b)y=\frac{2a^2b^3}{a-b}$..................(ii)

Multiply (i) by a, and add (i) and (ii)

$a(a-b)x+a(a+b)y=a^2b(a+b)$

$b(a+b)x-a(a+b)y=\frac{2a^2b^3}{a-b}$

$$x(a^2-ab+ab+b^2) = a^2b(a+b)+\frac{2a^2b^3}{a-b}$$

$$=\frac{a^4b-a^2b^3+2a^2b^3}{a-b}$$

$$=\frac{a^2b(a^2-b^2+2b^2)}{a-b}=\frac{a^2b(a^2+b^2)}{a-b}$$

$\therefore x(a^2+b^2)=\frac{a^2b(a^2+b^2)}{a-b}$ $\qquad \therefore x=\frac{a^2b}{a-b}$

Substituting the value of x in (i) we get

$(a-b)\times\frac{a^2b}{a-b}+(a+b)y=ab(a+b)$

$\therefore a^2b+(a+b)y=ab(a+b)$

$\therefore (a+b)y = ab(a+b)-a^2b = ab^2$

$\therefore y=\frac{ab^2}{a+b}$: $x=\frac{a^2b}{a-b}$; $y=\frac{ab^2}{a+b}$. *Ans.*

X. Let x be the length of the journey in miles, and let y miles per hour be the rate at which the man goes on his bicycle.

$\therefore$ $4y$ miles per hour is the rate of the train.

$$\frac{x-72}{y}+\frac{72}{4y}=\frac{7}{2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\frac{x}{4y}=3\tfrac{1}{2}-1\tfrac{1}{2}=2 \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

from (*ii*) $x=8y$.

Substituting the value of x in (*i*), we get

$$\frac{8y-72}{y}+\frac{72}{4y}=\frac{7}{2}, \text{ hence } y=12 \quad \therefore\ x=8\times 12=96.$$

96 miles. *Ans.*

1902.

Arithmetic and Algebra.

1. (*a*) Find the value of— 9

$$\frac{2{\cdot}8\times 2{\cdot}\dot{2}\dot{7}}{1{\cdot}1\dot{3}\dot{6}}+\frac{4{\cdot}\dot{4}-2{\cdot}8\dot{3}}{1{\cdot}\dot{6}+2{\cdot}\dot{6}2\dot{9}}\times\frac{6{\cdot}8\times 3}{2{\cdot}25}$$

(*b*) Find the cube root of 2,924·207.

2. A contractor undertakes to execute a piece of work in 30 days, and engages what he considers a sufficient number of men for the purpose. It turns out, however, that four of his men do, respectively, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{12}$ less than an average day's work, while three of them do, respectively, $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{8}$ more than an average day's work. For how many days must he engage 3 additional men, so that the work may be completed in the specified time? 7

3. The cost of freight on a certain quantity of goods was 15 per cent., and that of duty, 10 per cent. on the original outlay. The goods were sold at a loss of 5 per cent., but if they had been sold for £3 more, there would have been a gain of 1 per cent. What was the cost of the goods? 9

4. A man invests £864 in the following manner. One-half he invests in the $5\frac{1}{2}$ per cent. at 8 premium and the other half in Bank shares at 116 premium. At the end of one year he sells out his $5\frac{1}{2}$ per cents. at 12 premium, and his Bank shares at 130 premium, and invests the whole of the proceeds in $4\frac{1}{2}$ per cents. at $90\frac{1}{8}$. His annual income is now £1 less than it was before. What rate of interest did the Bank shares pay? 10

5. Three pipes A, B, and C connected with a tank are opened at 1, 2, and 3 o'clock, respectively. A alone would fill it in 3 hours, and B alone in 4 hours, while C can empty it in 1 hour. At what o'clock will the tank be empty? 9

6. (a) Find the expression which divides $x^8+2x^4y^4+y^8$, so that x^2+xy+y^2 is the quotient, and $x^6y^2+2x^4y^4+x^2y^6$ the remainder. 14

(b) Resolve into factors—

x^6+27y^6 and $(a-b)^3-2(b-c)^3-(a-3b+2c)^3$

7. If $x+y+z=-xyz$, find the value of $\frac{x}{1+x^2}+$ 11

$$\frac{y}{1+y^2}+\frac{z}{1+z^2}+\frac{4xyz}{(1+x^2)(1+y^2)(1+z^2)}$$

8. (a) Obtain an expression which will divide both $3x^3+10x^2+10x+7$ and $6x^3+17x^2+x-14$ without a remainder. 11

(b) Find the $G.\ C.\ M.$ of $2x^4+x^3+x^2-7x+3$, $3x^4+7x^3+9x^2-x-6$ and $6x^4-7x^3-4x^2+7x-2$.

9. Find the number values of a and b if $x^4+4x^3+10x^2+ax+b$ be a perfect square. 10

10. Find a number consisting of two digits such t its square root is equal to the sum of the digits, and ss than the number obtained by inverting the digits by 9. 14

1902.

EUCLID.

1. Draw a straight line perpendicular to a given straight line from a given point without it. Given two points one on each side of a given straight line: find a point in the straight line such that the angle contained by two straight lines drawn from it to the given points may be bisected by the given straight line. 13

2. Equal triangles on the same base and on the same side of it are between the same parallels. 16

The perimeter of an isosceles triangle is less than that of any other triangle of equal area standing on the same base.

3. If a straight line be divided into two parts, the squares on the whole line and on one part are together equal to twice the rectangle contained by the whole and that part, with the square on the other part. 11

Shew how to produce a given straight line, so that the sum of the squares on the given straight line and on the whole straight line thus produced, may be equal to twice the rectangle contained by the whole straight line thus produced and the part produced.

4. In an obtuse-angled triangle, if a perpendicular be drawn from one of the acute angles to the opposite side produced, the square on the side opposite the obtuse angle is greater than the squares on the sides containing it by twice the rectangle contained by the side on which when produced the perpendicular falls, and the straight line intercepted outside the triangle, between the perpendicular and the obtuse angle. Describe an isosceles obtuse-angled triangle such that the square on the largest side may be equal to three times the square on either of the equal sides. 14

5. The angles in the same segment of a circle are equal. 13

Two opposite sides of a quadrilateral inscribed in a circle are produced to meet, and a perpendicular is drawn from a point of intersection of the diagonals to the bisector of the angle between the produced sides: shew that this perpendicular will bisect the angle between the diagonals.

6. In equal circles equal angles, either at the centres or at the circumferences, stand on equal arcs. 15

It two chords of a circle intersect at right angles, the sum of the squares on the segments is equal to the square on the diameter.

7. If a given circle touch another given circle internally, and if two parallel diameters be drawn, prove that the point of contact of the two circles and an extremity of each diameter lie in the same straight line. 11

8. Inscribe a circle in a given regular pentagon. 7

SOLUTIONS.

(1) *a*—

$$\frac{2\frac{8}{10}\times 2\frac{27}{99}}{1\frac{135}{990}}+\frac{4\frac{4}{9}-2\frac{75}{90}}{1\frac{6}{9}+2\frac{629}{999}}\text{ of }\frac{6\frac{8}{10}\times 3}{2\frac{25}{100}}$$

$$=\frac{\frac{14}{5}\times\frac{25}{11}}{\frac{25}{22}}+\frac{\frac{40}{9}-\frac{17}{6}}{\frac{5}{3}+\frac{71}{27}}\text{ of }\frac{\frac{34}{5}\times 3}{\frac{9}{4}}$$

$$=\frac{14}{5}\times\frac{25}{11}\times\frac{22}{25}+\frac{29}{18}\times\frac{27}{116}\times\frac{102}{5}\times\frac{4}{9}$$

$$=\frac{28}{5}+\frac{17}{5}=\frac{45}{5}=9 \qquad\qquad 9\ \textit{Ans.}$$

(*b*) $\sqrt[3]{2924\cdot207} = 14\cdot3$ *Ans.*

(2) Four of his men do $\frac{1}{3}+\frac{1}{4}+\frac{2}{5}+\frac{1}{12}=\frac{64}{60}$ or $\frac{16}{15}$ less than an average day's work.

3 of his men do $\frac{1}{6}+\frac{1}{2}+\frac{1}{5}=\frac{26}{30}$ or $\frac{13}{15}$ more.

$\therefore \frac{16}{15}-\frac{13}{15}=\frac{1}{5}$ of an average day's work remains to be done.

In 30 days, $30\times\frac{1}{5}=6$ days' average work remains to be done.

$\therefore$ 3 additional men must be employed for $\frac{6}{3}$ or 2 days to complete the work in the specified time.

2 days. *Ans.*

(3) Let £100 be the C. P. of goods.

$\therefore$ total c. p.=£100+£15+£10=£125.

The s. p. at a loss of 5 per cent. :—

c. p. c. p. s. p.

£100 : £125 :: £95 : x $\therefore x$=£118$\frac{3}{4}$ s. p. in the first case.

c.p. c p. s.p.

£100 : £25 ::£101 = £126$\frac{1}{4}$ s. p. in the second case.

£126$\frac{1}{4}$−£118$\frac{3}{4}$=£7$\frac{1}{2}$ difference between the two s. p.

diff. diff. c.p.

£7$\frac{1}{2}$: £3 :: £100 : x $\therefore$ c. p. = £40.

£40. *Ans.*

(4) Investment—

$£108 : \frac{£864}{2} :: \frac{11}{2}$: Income.

$\therefore$ Income in the first case $= £22$.

$£108 : \frac{£864}{2} : £112$: sum realised by selling the $5\frac{1}{2}$ per cents. $\therefore$ Sum realised $= £448$.

$£216 : \frac{£864}{2} :: £230$: sum realised by selling the Bank shares. $\therefore$ Sum realised $= £460$.

Total sum realised $= £448 + £460 = £908$.

cash cash income

$£90\frac{4}{5} : £908 :: \frac{9}{2}$: Income.

$\therefore$ Income in the second case $= £45$.

This new income is £1 less than it was before

$\therefore$ the former income $= £46$.

$£46 - £22 = £24$ income derived from the Bank shares.

cash cash income

$\frac{£864}{2} : £216 :: £24$: Interest on each Bank share.

$\therefore$ Interest on Bank Share $= 12$.

12% *Ans.*

(5) Before C is opened, A runs for 2 hours and B for 1 hour.

A fills $\frac{1}{3}$ of the tank in 1 hour and B $\frac{1}{4}$ of the tank in 1 hour.

$\therefore$ $\frac{2}{3}+\frac{1}{4}$ or $\frac{11}{12}$ of the tank is filled before 3 o'clock.

$1-\frac{1}{3}-\frac{1}{4}$ or $\frac{5}{12}$ of the tank is emptied in 1 hour when all are open.

$\frac{5}{12} : \frac{11}{12} :: 1$ hr. $: x$ $\therefore x = \frac{11}{5} = 2\frac{1}{5}$ hrs. $= 2$ hrs. 12 min.

The tank will be empty at 12′ past 5 o'clock.

6. (*a*) Divisor × Quotient = Dividend − Remainder.

$$\therefore \text{ Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}.$$

$$\text{Divisor} = \frac{x^8 2x^4y^4 + y^8 - (x^6y^2 + 2x^4y^4 + x^2y^6)}{x^2 + xy + y^2}$$

$$\frac{x^8 - x^6y^2 - x^2y^6 + y^8}{x^2 + xy + y^2} =$$

$$= \frac{(x^6 - y^6)(x^2 - y^2)}{x^2 + xy + y^2} = \frac{(x^3 + y^3)(x^3 - y^3)(x^2 - y^2)}{x^2 + xy + y^2}$$

$$= \frac{(x^3 + y^3)(x - y)(x^2 + xy + y^2)(x - y)(x + y)}{x^2 + xy + y^2}$$

$$= (x^3 + y^3)(x - y)^2(x + y) \qquad \textit{Ans.}$$

(*b*) $x^6 + 27y^6 = (x^2)^3 + (3y^2)^3 = (x^2 + 3y^2)(x^4 - 3x^2y^2 + 9y^4)$

$$= (x^2 + 3y^2)\{(x^2 + 3y^2)^2 - 9x^2y^2\}$$

$$= (x^2 + 3y^2)(x^2 + 3xy + 3y^2)(x^2 - 3xy + 3y^2)$$

Ans.

$$(a - b)^3 - 2(b - c)^3 - (a - 3b + 2c)^3$$

$$= (a - b)^3 + \{-8(b - c)^3\} + \{-(a - 3b + 2c)^3\} + 6(b - c)^3$$

Now, if $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$,

but $(a - b) - 2(b - c) - (a - 3b + 2c) = 0$

$\therefore$ the first three terms of the exp. $= 6(a - b)(b - c)(a - 3b + 2c)$

$$\therefore 6(a - b)(b - c)(a - 3b + 2c) + 6(b - c)^3$$

$$= 6(b - c)\{(a - b)(a - 3b + 2c) + (b - c)^2\}$$

$$= 6(b - c)\Big[(a - b)\{(a - b) - 2(b - c)\} + (b - c)^2\Big]$$

$$= 6(b - c)\Big[(a - b)^2 - 2(a - b)(b - c) + (b - c)^2\Big]$$

$$= 6(b - c)\{(a - b) - (b - c)\}^2$$

$$= 6(b - c)(a - 2b + c)^2 \qquad 6(b - c)(a - 2b + c)^2 \qquad \textit{Ans.}$$

The expression—

$$= \frac{x(1+y^2)(1+z^2)+y(1+x^2)(1+z^2)+z(1+x^2)(1+y^2)+4xyz}{(1+x^2)(1+y^2)(1+z^2)}$$

$= x(1+y^2+z^2+y^2z^2)+y(1+x^2+z^2+x^2z^2)+z(1+x^2$
$\quad +y^2+x^2y^2)+4xyz.$

By arranging the terms and splitting $4xyz$, we get

$(x+y+z+xyz)+(xy^2+x^2y-xyz)+(y^2z+z^2y+xyz)$
$+(xz^2+x^2z+xyz)+xy^2z^2+yx^2z^2+zx^2y^2$

$= (x+y+z+xyz)+xy\ (y+x+z)+yz(y+z+x)+zx$
$\quad (z+x+y)+xyz(xy+yz+zx)$

$= (-xyz+xyz)+(x+y+z)\ (xy+yz+zx)+xyz(xy+$
$\quad yz+zx)$

$= 0+(xy+yz+zx)(x+y+z+xyz)$

$= 0+(xy+yz+zx)(-xyz+xyz)=0+0=0$

$\therefore \dfrac{0}{(1+x^2)(1+y^2)(1+z^2)}=0$ 0 *Ans.*

8. (*a*) $3x^3+10x^2+10x+7$

$=3x^3+7x^2+3x^2+7x+3x+7$

$=x^2(3x+7)+x(3x+7)+1(3x+7)$

$=(3x+7)(x^2+x+1)$...................... (a)

$6x^3+17x^2+x-14$

$=6x^3+14x^2+3x^2+7x-6x-14$

$=2x^2(3x+7)+x(3x+7)-2(3x+7)$

$=(3x+7)(2x^2+x-2)$...................... (b)

$\therefore$ the H. C. F. of a and $b=3x+7$.

(*b*) By inspection, $x-1$ is a factor of the first expression

$\therefore$ The exp. $=2x^4-2x^3+3x^3-3x^2+4x^2-4x-3x+3$

$=2x^3(x-1)+3x^2(x-1)+4x(x-1)-3(x-1)$

$=(x-1)(2x^3+3x^2+4x-3)$

And $2x^3+3x^2+4x-3$

$=2x^3-x^2+4x^2-2x+6x-3.$

$= x^2(2x-1)+2x(2x-1)+3(2x-1)$

$= (2x-1)(x^2+2x+3)$

$\therefore$ the first exp. $=(x-1)(2x-1)(x^2+2x+3)$......(i)

Again, by inspection $x+1$ is a factor of the 2nd exp.

$\therefore$ the expression

$= 3x^3(x+1)+4x^2(x+1)+5x(x+1)-6(x+1)$

$= (x+1)(3x^3+4x^2+5x-6)$

Also, $3x^3+4x^2+5x-6$

$= 3x^3-2x^2+6x^2-4x+9x-6$

$= x^2(3x-2)+2x(3x-2)+3(3x-2)$

$= (3x-2)(x^2+2x+3)$

$\therefore$ the 2nd exp. $=(x+1)(3x-2)(x^2+2x+3)$......(ii)

Again, by inspection x^2-1 is a factor of the 3rd exp.

$\therefore$ the expression $= 6x^2(x^2-1)-7x(x^2-1)+2(x^2-1)$

$= (x^2-1)(6x^2-7x+2)$

$= (x+1)(x-1)(3x-2)(2x-1)$.....................(iii)

$\therefore$ the L. C. M. of (i), (ii), (iii) is

$(x+1)(x-1)(3x-2)(2x-1)(x^2+2x+3)$. *Ans.*

IX.

$$
\begin{array}{r|l}
 & x^4+4x^3+10x^2+ax+b\,(\,x^2+2x+3 \\
 & x^4 \\
\hline
2x^2+2x & \quad 4x^3+10x^2+ax+b \\
 & \quad 4x^3+4x^2 \\
\cline{2-2}
2x^2+4x+3 & \qquad 6x^2+ax+b \\
 & \qquad 6x^2\pm 12x\pm 9 \\
\cline{2-2}
 & \qquad x(a-12)+b-9
\end{array}
$$

In order that the given expression may be a perfect square the remainder $x(a-12)+b-9$ should be equal to 0.

$\therefore x(a-12)+b-9=0$, *i.e.*, $a-12=0$, $b-9=0$,

i.e., $a=12$, $b=9$ *Ans.*

X. Let x be the digit in the tens' place.

Let y be the digit in the units' place.

$\therefore$ the number formed $= 10x+y$

and the number formed by inverting the digits $= 10y+x$

$\sqrt{10x+y}=x+y$..................(i)

$\sqrt{10x+y}<10y+x$ by 9, *i e.*, $\sqrt{10x+y}=10y+x-9$.........(ii)

From (i) and (ii), $10y+x-9=x+y \therefore y=1$

Substituting the value of y in (i), we get

$\sqrt{10x+1}=x+1$. Squaring both sides, we get

$10x+1=x^2+2x+1 \quad \therefore x^2-8x=0 \quad \therefore x(x-8)=0$

$\therefore x=0$, or 8

x cannot be 0 as it is here a digit in the tens' place and stands first $\therefore x=8$ $\therefore$ the number $= 81$ *Ans.*

1903.

Arithmetic and Algebra.

1. Explain what is meant by the division of one fraction by another; prove by means of an example how the result may be obtained. 12

(*a*) Find the value of—

$$\left\{\tfrac{1}{8}\times\left(\tfrac{1}{10}-\tfrac{1}{11}\right)\div\frac{\tfrac{1}{7}-\tfrac{1}{9}\div\left(\tfrac{4}{9}+\tfrac{4}{11}\right)}{\left(\tfrac{1}{7}+\tfrac{1}{9}\right)\div\left(\tfrac{4}{9}-\tfrac{4}{11}\right)}\right\}\times\frac{\tfrac{1}{8}+\tfrac{1}{7}\div\left(\tfrac{1}{7}-\tfrac{1}{9}\right)}{\left(\tfrac{1}{8}+\tfrac{1}{7}\right)\div\tfrac{1}{7}-\tfrac{1}{9}}$$

(*b*) Simplify—

$$\frac{\cdot7\times\cdot7+\cdot07\times\cdot07+\cdot007\times\cdot007}{\cdot14\times\cdot14+1\cdot4\times1\cdot4+14\times14}$$

2. A railway train 770 feet long overtook a man walking along the line at the rate of $3\frac{1}{2}$ miles an hour, and passed him in 30 seconds. The train reached the next station 20 minutes after it had passed the man. In what time did the man reach the station? 9

3. An alloy contains 12 parts by weight of lead, 4 of antimony, and 1 of tin. How much of this alloy must be 8

taken, and how much lead and tin must be added to it, to make up 9 cwt. of type metal, which consists of 14 parts of lead, 3 of antimony, and 1 of tin ?

4. A person invested equal sums of money in 3 per cents. at $97\frac{1}{2}$, and in $3\frac{1}{2}$ per cents. at $102\frac{1}{2}$. His resulting income being £259 10*s*. How much did he invest ? 7

5. A bankrupt, whose liabilities amount to Rs. 5,016 10 8, has in his favour two bills for equal sums, due one year and two years hence, and is thus in a position to pay 4 annas in the rupee. Find the sum for which bills were drawn, simple interest being reckoned at 5 per cent. per annum. 9

6. If $xy\ (x+1)=1$, prove that $\dfrac{1}{x^3\,y^3}-x^3-y^3=3$. 6

7. Define *quotient*, *power*, giving an example of each. 11
Simplify and factorize—

$$a(b+c)\ (b^2+c^2-a^2)+b\ (c+a)\ (c^2+a^2-b^2) +c\ (a+b)\ (a^2+b^2-c^2)\text{; and divide}$$

$$8x^9-39x^7+66x^5-43x^3+8 \text{ by } (x+1)^3$$

8. Find the H. C. F. and L. C. M. of— 8

$$4y^4-5y^2+1 \text{ and } 4y^4+4y^3+y^2-1.$$

9. Simplify— 13

$$\left(y+\frac{m-xy}{x-y}\right)\left(x-\frac{m-xy}{x-y}\right)+\left(\frac{m-xy}{x-y}\right)^2\text{; and prove that}$$

$$\left(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}\right)^2=\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}-2.$$

10. Solve the equations— 11

$$(a)\ 1{\cdot}2x-\frac{{\cdot}18x-{\cdot}025}{{\cdot}5}={\cdot}4x+4{\cdot}45\ ;$$

$$(b)\ \frac{1}{x}+\frac{1}{y}=\frac{1}{2},\quad \frac{21a}{x}-\frac{12b}{y}=\frac{56a-10b}{7}.$$

11. A bag contains £8 12*s*. in half crowns and shillings ; if six half crowns are added, the number of half crowns is thrice the number of shillings. How many e there of each ? 6

Euclid.

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal, then the angle which is contained by the two sides of the one shall be equal to the angle which is contained by the two sides of the other. 13

The vertex of an isosceles triangle, the point of intersection of the bisectors of the angles at the base, and the point of intersection of the bisectors of the angles between the base and the sides produced, are in one and the same straight line.

2. Triangles on equal bases and between the same parallels are equal in area. 12

In the triangle ABC, AB is produced beyond B to P, BC beyond C to Q and CA beyond A to R, so that BP, CQ, AR are double of AB, BC, CA, respectively: prove that the triangle PQR is nineteen times the triangle ABC in area.

3. If a straight line is divided into any two parts, the square on the whole line is equal to the sum of the squares on the two parts, together with twice the rectangle contained by the two parts. 14

In the triangle ABC, AD is drawn perpendicular to BC; if the rectangle BD, DC be equal to the square on AD, prove that the angle BAC is a right angle.

4. In any quadrilateral, if two opposite sides be bisected, the sum of the squares on the other two sides, together with squares on the diagonals, shall be equal to the sum of the squares on the bisected sides, together with four times the squares on the line joining the points of bisection. 9

5. The diameter is the greatest chord in a circle and of others, that which is nearer to the centre, is greater than one more remote; conversely, the greater chord is nearer to the centre, than the less. 15

If a chord of a circle be bisected by another chord, and this chord again by another and so on, prove that the chord continually increases in length.

6. The angle in a semicircle is a right angle, the angle in a segment greater than a semi-circle, is less than a right angle, and the angle in a segment less than a semi-circle is greater than a right angle. 15

In the triangle ABC, BE and CF are drawn perpendicular to the opposite sides, and D is the middle point of BC; shew that each of the angles FED, EFD is equal to the angle BAC.

7. Circumscribe a circle about a given triangle. 14

The angle BAC of a triangle ABC is half a right angle and O is the centre of the circumscribed circle of the triangle; if BO, CO (or these produced), meet the opposite sides in E, F, respectively, prove that BC is a tangent to the circumscribed circle of the triangle ABE and ACF.

8. Shew how to inscribe in a given circle regular polygons of fifteen and of ten sides. 8

Solutions.

1. The division of one fraction by another is the finding of the number, whole or fractional, by which the divisor must be multiplied in order that the result may be the dividend.

Thus, to divide $\frac{5}{8}$ by $\frac{9}{7}$ is to find a number such that when multiplied by $\frac{9}{7}$ the result may be $\frac{5}{8}$.

$\therefore$ $\frac{9}{7}$ $\times$ the required no. $=\frac{5}{8}$ Multiply both sides by $\frac{7}{9}$

$\therefore$ $\frac{7}{9}\times\frac{9}{7}$ of the required no. $=\frac{5}{8}\times\frac{7}{9}$ $\therefore$ the required no. $=\frac{35}{72}$

(*a*) The expression—

$$=\left\{\frac{1}{8}\text{ of }\left(\frac{11-10}{110}\right)\div\frac{\frac{2}{7}-\frac{1}{9}\times\frac{99}{80}}{\frac{16}{63}\times\frac{99}{8}}\right\}\times\frac{\frac{1}{3}+\frac{1}{7}\times\frac{63}{2}}{\frac{10}{21}\times\frac{7}{1}-\frac{1}{9}}$$

$$=\left\{\frac{1}{8}\text{ of }\frac{1}{110}\div\frac{\frac{3}{560}}{\frac{22}{7}}\right\}\times\frac{\frac{1}{3}+\frac{9}{2}}{\frac{10}{3}-\frac{1}{9}}$$

$$=\left\{\frac{1}{8}\times\frac{1}{110}\times\frac{560}{3}\times\frac{22}{7}\right\}\times\frac{20}{6}\times\frac{9}{20}$$

$$=\frac{2}{3}\times\frac{20}{6}\times\frac{9}{20}=1$$

1 *Ans.*

(*b*) The expression—

$$=\frac{\cdot49+\cdot0049+\cdot000049}{\cdot0196+1\cdot96+196}$$

$$=\frac{49}{100}+\frac{49}{10000}+\frac{49}{1000000}=\frac{494949}{1000000}\div\frac{1979796}{10000}$$

$$=\frac{494949}{1000000}\times\frac{10000}{1979796}=\frac{1}{400}=\cdot0025$$

$\cdot0025$ *Ans.*

(2) The rate at which the train passes the man is 770 feet in 30 seconds, *i. e.*, $\frac{7}{48}$ mile in 30 seconds, *i.e.*, $\frac{35}{2}$ miles per hour.

$\therefore$ the rate of the train $=\frac{35}{2}$ miles $+\frac{7}{2}$ miles $=21$ miles per hour.

Now the train takes 20 minutes in reaching the next station.

$\therefore$ the distance of the next station from the point where the train passes the man $=21\times\frac{20}{60}$ miles $=7$ miles.

$\therefore$ the man reaches the station in $7\div\frac{7}{2}$, or 2 hours.

2 hours. *Ans.*

(3) In 17 parts of alloy there are 12 parts by weight of lead, 4 of antimony, and 1 of tin.

In 18 parts of type-metal there are 14 parts of lead, 3 of antimony, and 1 of tin.

The quantity of antimony in the alloy taken is the same as that in 9 cwts. of type-metal.

$\therefore \frac{4}{17}$ of the alloy $= \frac{3}{18}$ of 9 cwts. $\therefore$ the alloy $= \frac{3}{18} \times 9 \times \frac{17}{4}$ cwts. $= \dfrac{51}{8}$ cwts. $= 6\frac{3}{8}$ cwts.

Again, the quantity of lead in 9 cwts. of type-metal $= \frac{14}{18}$ of 9 cwts. $= 7$ cwts.

And the quantity of lead in $6\frac{3}{8}$ cwts. of the alloy $= \frac{12}{17}$ of $6\frac{3}{8}$ cwts. $= 4\frac{1}{2}$ cwts.

$\therefore$ the quantity of lead added $= (7 - 4\frac{1}{2})$ cwts. $= 2\frac{1}{2}$ cwts.

Again, the quantity of tin in 9 cwts. of type-metal

$= \frac{1}{18}$ of 9 cwts. $= \frac{1}{2}$ cwt.

And the quantity of tin in $6\frac{3}{8}$ cwts. of the alloy

$= \frac{1}{17}$ of $6\frac{3}{8}$ cwts. $= \frac{3}{8}$ cwt.

$\therefore$ the quantity of tin added $= (\frac{1}{2} - \frac{3}{8})$ cwt. $= \frac{1}{8}$ cwt.

$$\left.\begin{array}{ll} \text{The alloy to be taken} & = 6\frac{3}{8} \text{ cwts.} \\ \text{Lead to be added} & = 2\frac{1}{2} \text{ cwts.} \\ \text{Tin to be added} & = \frac{1}{8} \text{ cwt.} \end{array}\right\} \textit{Ans.}$$

(4) Let £$97\frac{1}{2}$ be invested in each.

$\therefore$ income from the first investment $=$ £3.

£$102\frac{1}{2}$ cash : £$97\frac{1}{2}$ cash :: £$3\frac{1}{2}$ income : income from the 2nd investment.

Income in the 2nd case $=$ £ $\frac{273}{82}$

$\therefore$ total income from both the sources $=$ £3 $+$ £$\frac{273}{82}$ $=$ £$\frac{519}{82}$.

£$\frac{519}{82}$ total income : £$259\frac{1}{2}$ :: £$97\frac{1}{2} + 97\frac{1}{2}$ total investment : total investment. Total investment $=$ £7,995. £7,995 *Ans.*

(5) Liabilities $=$ Rs. $5{,}016\frac{2}{3}$.

The bankrupt is in a position to pay 4 ans. in the rupee.

$\therefore$ his assets are Rs. $5{,}016\frac{2}{3} \div 4 =$ Rs. 1,254-2-8 $=$ Rs. $\frac{7525}{6}$ $\therefore$ the P. W. of the two bills $=$ Rs. $\frac{7525}{6}$

Let Rs. 105 be the amount of each of the bills.

$\therefore$ the P. W. of the first bill, *i. e.*, the bill due 1 year hence, is Rs. 100.

Rs. 110 amount : Rs. 105 amount :: Rs. 100 P. W.: P. W. of the second bill due 2 years hence.

$\therefore$ P. W. of the 2nd bill $=$ Rs. $\frac{1050}{11}$

$\therefore$ the P. W. of the two bills $=$ Rs. 100 $+$ Rs. $\frac{1050}{11}$

$=$ Rs. $\frac{2150}{11}$

Rs. $\frac{2150}{11}$ total P. W. : Rs. $\frac{7525}{6}$ total P. W. :: Rs. 105 amount : amount of each bill.

$\therefore$ amount $=$ Rs. $\frac{2695}{4} =$ Rs. 673-12.

Each bill was drawn for Rs. 673-12. *Ans.*

VI. $xy(x+y)=1 \therefore \frac{1}{xy}=x+y \therefore \frac{1}{x^3y^3}=(x+y)^3$

$$=x^3+y^3+3xy\,(x+y)$$
$$=x^3+y^3+3 \quad (\because xy\,(x+y)=1)$$

$\therefore \frac{1}{x^3y^3}-x^3-y^3=3.$ *Q. E. D.*

VII. The quantity which tells how many times the divisor is contained in the dividend is called the *quotient*:

Or, the *quotient* is the quantity by which the divisor must be multiplied to produce the dividend:

Thus, if x^2 be the divisor and x^5 the dividend, then x^3 is the quantity by which x^2 the divisor must be multiplied to produce the dividend x^5.

If a quantity be multiplied by itself any number of times, the product is called a *power* of that quantity.

Thus, $a \times a$ (i. e., a^2) is called the second *power* of a ; $a \times a \times a$. (i. e., a^3) is called the third *power* of a, &c.

The expression :

$$= a(b+c)(b^2+c^2) - a^3(b+c)$$
$$+ b(c+a)(c^2+a^2) - b^3(c+a)$$
$$+ c(a+b)(a^2+b^2) - c^3(a+b)$$
$$= ab^3 + abc^2 + acb^2 + ac^3 - a^3b - a^3c + bc^3 + bca^2 + bac^2$$
$$+ ba^3 - b^3c - b^3a + a^3c + acb^2 + bca^2 + cb^3 - c^3a - c^3b$$
$$= 2abc^2 + 2acb^2 + 2cba^2 = 2abc(c+b+a)$$
$$2abc(a+b+c) \qquad Ans.$$

The sum of the coefficients of all the terms of the expression

$8x^9 - 39x^7 + 66x^5 - 43x^3 + 8$ is 0 $\therefore$ $x-1$ is a factor,

$$\therefore \text{ the dividend} = (x-1)\ (8x^8 + 8x^7 - 31x^6 - 31x^5 + 35x^4$$
$$+ 35x^3 - 8x^2 - 8x - 8)$$
$$= (x-1)(x-1)(8x^7 + 16x^6 - 15x^5 - 46x^4$$
$$- 11x^3 + 24x^2 + 16x + 8)$$
$$= (x-1)(x-1)(x-1)(8x^6 + 24x^5 + 9x^4$$
$$- 37x^3 - 48x^2 - 24x - 8)$$

$$\therefore \frac{\text{dividend}}{\text{divisor}}, \textit{ i. e.}, \text{ quotient}$$

$$= 8x^6 + 24x^5 + 9x^4 - 37x^3 - 48x^2 - 24x - 8 \qquad Ans.$$

VIII. $4y^4 - 4y^2 - y^2 + 1$

$$= 4y^2(y^2-1) - 1(y^2-1)$$
$$= (y^2-1)(4y^2-1)$$
$$= (y+1)(y-1)(2y+1)(2y-1) \ldots\ldots\ldots\ldots\ldots\ldots (a)$$
$$4y^4 + 4y^3 + y^2 - 1$$
$$= 4y^3(y+1) + (y+1)(y-1)$$

$= (y+1)(4y^3+y+1)$

$= (y+1)\{4y^3-2y^2+2y^2-y+2y-1\}$

$= (y+1)\{2y^2(2y-1)+y(2y-1)+1(2y-1)\}$

$= (y+1)(2y-1)(2y^2+y+1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$

$\therefore$ the H. C. F. of a and $b=(y+1)(2y-1)$

and the L. C. M. of a and $b=$

$(y-1)(y+1)(2y-1)(2y+1)(2y^2+y+1)$ *Ans.*

IX. $\left(y+\frac{m-xy}{x-y}\right)\left(x-\frac{m-xy}{x-y}\right)+\left(\frac{m-xy}{x-y}\right)^2$

$$=\left(\frac{xy-y^2+m-xy}{x-y}\right)\left(\frac{x^2-xy-m+xy}{x-y}\right)+\left(\frac{m-xy}{x-y}\right)^2$$

$$=\frac{(m-y^2)(x^2-m)}{(x-y)^2}+\left(\frac{m-xy}{x-y}\right)^2$$

$$=\frac{mx^2-x^2y^2-m^2+my^2+m^2-2mxy+x^2y^2}{(x-y)^2}$$

$$=\frac{mx^2-2mxy+my^2}{(x-y)^2}$$

$$=\frac{m(x^2-2xy+y^2)}{(x-y)^2}=\frac{m(x-y)^2}{(x-y)^2}=m. \qquad m \quad \textit{Ans.}$$

$$\left(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}\right)^2$$

$$=\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}+\frac{2ab}{(b-c)(c-a)}$$
$$+\frac{2bc}{(c-a)(a-b)}+\frac{2ca}{(a-b)(b-c)}$$

$$=\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}+2\left\{\frac{ab}{(b-c)(c-a)}\right.$$
$$\left.+\frac{bc}{(c-a)(a-b)}+\frac{ca}{(a-b)(b-c)}\right\}$$

Now $\frac{ab}{(b-c)(c-a)}+\frac{bc}{(c-a)(a-b)}+\frac{ca}{(a-b)(b-c)}$

$$=\frac{ab(a-b)+bc(b-c)+ca(c-a)}{(a-b)(b-c)(c-a)}$$

$$=\frac{a^2(b-c)-a(b^2-c^2)+bc(b-c)}{(a-b)\ (b-c)\ (c-a)}$$

$$=\frac{(b-c)\ \{(a^2-a\ \ (b+c)+bc\}}{(a-b)(b-c)(c-a)}=\frac{(b-c)\ (a-b)\ (a-c)}{(a-b)(b-c)(c-a)}$$

$$=\frac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=-1$$

$$\therefore \text{ the given exp.} = \frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}-2$$

Ans.

X. $\cdot 6x-\cdot 18x+\cdot 025=\cdot 2x+2\cdot 225$

(*a*) $\therefore \cdot 6x-\cdot 18x-\cdot 2x=2\cdot 225-\cdot 025$

$\therefore \cdot 22x=2\cdot 2 \qquad \therefore x=\frac{2\cdot 2}{\cdot 22}=10 \qquad 10$ *Ans.*

(*b*) Multiplying the first by $12b$, and adding the two, we get—

$$\frac{12b}{x}+\frac{12b}{y}=6b\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$\frac{21a}{x}-\frac{12b}{y}=\frac{56a-10b}{7}\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\therefore \frac{21a+12b}{x}=\frac{56a-10b+42b}{7}=\frac{56a-32b}{7}$$

$$\therefore \frac{3(7a+4b)}{x}=\frac{8(7a+4b)}{7} \quad \therefore \frac{3}{x}=\frac{8}{7} \quad \therefore x=2\frac{5}{8}$$

Substituting the value of x in (i) we get—

$$\frac{8}{21}+\frac{1}{y}=\frac{1}{2} \qquad \therefore \frac{1}{y}=\frac{1}{2}-\frac{8}{21}=\frac{5}{42} \qquad \therefore y=8\frac{2}{5}$$

$x=2\frac{5}{8}, y = 8\frac{2}{5}$ *Ans.*

XI. Let x be the number of half-crowns.

$\therefore\ x+6=3\times$ no. of shillings.

$\therefore$ no. of shillings $=\dfrac{x+6}{3}$

$\therefore\ \dfrac{5x}{2}+\dfrac{x+6}{3}=172$

$\therefore\ 15x+2x+12=1032$

$\therefore\ 17x=1032-12=1020\ \therefore\ x=60$

$\therefore$ no. of shillings $=\dfrac{x+6}{3}=\dfrac{60+6}{3}=22$

no. of half-crowns, 60 ; no. of shillings, 22 *Ans.*

1904.

Arithmetic and Algebra.

1. (*a*) State the rule for the addition of fractions **12**
with different denominators, and give the reason for it.

Reduce the following expression to its simplest form:—

$$\frac{\frac{1}{3}+\frac{3}{5}+\frac{7}{240}-\frac{1}{3}\text{ of }\frac{3}{5}\text{ of }\frac{7}{240}}{1-\frac{1}{3}\text{ of }\frac{3}{5}-\frac{3}{5}\text{ of }\frac{71}{240}-\frac{7}{240}\text{ of }\frac{1}{3}}$$

(*b*) Find by any method the product of ·6931471 and ·4342945 true to six places of decimals.

2. Two railway passengers have together $9\frac{4}{5}$ maunds **8**
of luggage, and are charged for the excess above the weight allowed Rs. 2-14 and Rs. 6-4, respectively; if the luggage had all belonged to one person, he would have been charged Rs. 10-11. Find how much luggage is allowed free of charge to a passenger, and how much luggage each of the above passengers had.

3. Two pipes *A* and *B* can fill a cistern in 20 and **8**
15 minutes, respectively, and a pipe *C* can empty it in 10 minutes. If *A* be opened first for one minute, then *B* for one minute, then *C* for one minute, and so on alternately, find in what time the cistern would be full.

4. A sum of money is divided among 60 persons, men, women and children; the collective sums of the men's, the women's, the children's shares are as 5 : 4 : 3, respectively, while their individual shares are as 3 : 2 : 1, respectively. Find the number of men, women and children. 7

5. A man had 4 per cent. Railway Preference Stock which brought him in £664 a year; he sold out at $119\frac{1}{4}$, and invested in Ordinary Stock at 145, paying $\frac{1}{4}$ per cent. brokerage on each transaction. If the Ordinary Stock paid a dividend of 6 per cent., what was his gain or loss in income? 9

6. If $a^2=b^2+c^2$, prove that 6

$$(a+b+c)\,(b+c-a)\,(c+a-b)\,(a+b-c)=4b^2c^2$$

7. Divide $x^3-b\,(4a+b)\,x+a^3+2a^2b+3ab^2+6b^3$ by $x+a+2b$ 5

8. If F be a common factor of the algebraical expressions A and B, prove that F will divide $mA+nB$ exactly. 9

Find the H. C. F. of $x^4-4x^3-11x^2-50x+16$ and

$$x^4-12x^3+29x^2+46x-16$$

9. (a) Simplify— 13

$$\frac{(a+b)^3-c^3}{(a+b)^2-c^2}+\frac{(b+c)^3-a^3}{(b+c)^2-a^2}+\frac{(c+a)^3-b^3}{(c+a)^2-b^2}-2(a+b+c)$$

(b) Prove the following equality—

$$\frac{a^2}{a^2x-x^3}+\frac{b^2}{b^2x-x^3}+\frac{c^2}{c^2x-x^3}=\frac{x}{a^2-x^2}+\frac{x}{b^2-x^2}+\frac{x}{c^2-x^2}+\frac{3}{x}$$

10. Extract the square root of 7

$$(2n-1)\,(2n-3)\,(2n-5)\,(2n-7)+16$$

11. Find the value of x from $\dfrac{(x+12)^3}{x^3}=\dfrac{x+24}{x-12}$ 6

12. A boatman rows 42 miles up a river and back again in 14 hours; he finds that he can row 7 miles with the stream in the same time as 3 miles against it. Find the rate at which the river flows. 10

Euclid.

1. If two triangles have two angles of one respectively equal to two angles of the other and a side of one equal to a side of the other, these sides being opposite to equal angles in each, then shall the triangles be equal in all respects. 14

ABC is an isosceles triangle right-angled at *A*; the bisector of the angle *ABC* meets *AC* in *D*. Prove that *AD* is equal to the difference between *BC* and *AB*.

2. The complements of the parallelograms about the diagonal of any parallelogram are equal to one another. 14

ABCD is a parallelogram, and any straight line *HKL* parallel to *AB* meets *AD*, *AC*, *BC* in *H*, *K*, *L*, respectively. Prove that the triangle *AHL* is equal in area to the triangle *AKD*.

3. If a straight line be divided into any two parts, the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part. 14

The straight line *AB* is bisected at *C*, and *D* is any point in *CB*. Prove that $AD^2 = DB^2 + 4AC, CD$.

4. Two radii of a circle at right angles to one another when produced are cut by a straight line which touches the circle. Prove that the tangents drawn from the points of section are parallel to one another. 9

5. In equal circles the chords, which cut off equal arcs, are equal. 13

A triangle *ABC* is inscribed in a circle, and the angle at *A* is bisected by *AE* meeting the circumference in *E*; also the angle at *C* is bisected by *CI* meeting *AE* in *I*. Prove that *EB*, *EC*, *EI* are all equal.

6. On a given straight line describe a segment of a circle which shall contain an angle equal to a given angle. 6

7. About a given circle describe a square. 15

If a polygon described about a circle is equiangular, prove that it is also equilateral.

8. Describe an isosceles triangle having each of the angles at the base double of the third angle. 15

If *ABC* be a triangle, such that the angles at *B* and *C* are ch double of the angle at *A*, shew that the square on *AB* exceeds the square on *BC* by the rectangle *AB*, *BC*.

Solutions.

(1) When there are several fractions having different denominators we should first convert them into equivalent fractions and then add them up.

(1) $\frac{1}{3}+\frac{3}{5}+\frac{7}{249}-\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{7}{249}$

$$=\frac{1}{3}+\frac{3}{5}+\frac{7}{249}-\frac{7}{5\times 249}$$

$$=\frac{5\times 249+9\times 249+7\times 3\times 5-7\times 3}{3\times 5\times 249}$$

$$=\frac{249\ (5+9)\ +21\ (5-1)}{3\times 5\times 249}$$

$$=\frac{249\times 14+21\times 4}{3\times 5\times 249}=\frac{42\ (83+2)}{5\times 3\times 249}=\frac{\overset{14}{\cancel{42}}\times\overset{17}{\cancel{85}}}{\cancel{5}\times\cancel{3}\times 249}=\frac{14\times 17}{249}$$

$1-\frac{1}{3}$ of $\frac{3}{5}-\frac{3}{5}$ of $\frac{7}{249}$ $-$ $\frac{7}{249}$ of $\frac{1}{3}$

$$=1-\frac{1}{5}-\frac{7}{5\times 83}-\frac{7}{249\times 3}$$

$$=\frac{4}{5}-\frac{7}{83}\left(\frac{1}{5}+\frac{1}{9}\right)=\frac{4}{5}-\frac{7}{83}\times\frac{14}{5\times 9}=\frac{4}{5}-\frac{98}{15\times 249}$$

$$=\frac{12\times 249-98}{15\times 249}=\frac{2988-98}{15\times 249}=\frac{2880}{15\times 249}=\frac{578}{3\times 249}$$

$$\therefore \text{ The given fr.}=\frac{\overset{7}{\cancel{14}}\times\cancel{17}}{\cancel{249}}\times\frac{3\times\cancel{249}}{\underset{17}{\cancel{578}}}=\frac{21}{17}=1\frac{4}{17}\ \textit{Ans.}$$

B.
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   ·6931471
   ·4342945
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     34657355
    27725884
   62383239
  13862942
 27725884
20794413
27725884
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·30102997322095   Ans.
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(2) (Rs. 2-14 + Rs. 6-4) = Rs. 9-2 is the charge for $9\frac{4}{5}$ maunds with two tickets, and Rs. 10-11 is the charge for $9\frac{4}{5}$ with one ticket, *i. e.* (Rs. 10-11—Rs. 9-2) Rs. 1-9 more.

Therefore if they had no ticket they would have been charged (Rs. 10-11 + Rs. 1-9) = Rs. 12-4 for $9\frac{4}{5}$ maunds. Therefore Re. 1-9 is charged for (Rs. $12\frac{1}{4}$: Rs. $1\frac{9}{16}$ :: $9\frac{4}{5}$ maunds) $1\frac{1}{4}$ maund of luggage which is allowed free of charge to a passenger. (1) *Ans.*

The first passenger is charged Rs. 2-14 for (Rs. $12\frac{1}{4}$: Rs. $2\frac{7}{8}$ :: $9\frac{4}{5}$ amunds) $2\frac{3}{10}$ maunds.

Therefore he has $(2\frac{3}{10} + 1\frac{1}{4}) = 3\frac{11}{20}$ maunds.

The second passenger has $(9\frac{4}{5} - 3\frac{11}{20}) = 6\frac{1}{4}$ maunds.

(3) $\frac{1}{20} + \frac{1}{15} = \frac{7}{60}$ of the cistern is filled in the last two minutes.

$1 - \frac{7}{60} = \frac{53}{60}$ of the cistern is filled in the time previous to the last two minutes.

Now $(\frac{1}{20} + \frac{1}{15} - \frac{1}{10})$ or $\frac{1}{60}$ of the cistern is filled in three minutes.

Hence, $\frac{1}{60} : \frac{53}{60} :: 3$ m. : $\left(3 \times \frac{53}{60} \times \frac{60}{1}\right)$ 159 minutes.

$\therefore$ 159 + 2 = 161 minutes = 2 hours 41 minutes. *Ans.*

(4) Dividing £1 in the proportion of 5, 4 and 3 we have £$\frac{5}{12}$ = the sum received by men, £$\frac{1}{3}$ = the sum received by women, and £$\frac{1}{4}$ = the sum received by children.

Since the individual shares are as 3 : 2 : 1, the numbers representing the proportion of the numbers of men, women and children are as $\frac{5}{12 \times 3} : \frac{1}{3 \times 2} : \frac{1}{4 \times 1}$. *i. e.*, 5 : 6 : 9

Hence dividing 60 in the proportion of 5 : 6 : 9 we have 15 : 18 : 27. *Ans.*

(5) Amount of 4 p. c., Stock is (4 : 664 :: 100) 16,600£ and money obtained from the sale of 4 p. c. stock is (100 : 16,600 :: 119) 166 × £119.

The proceeds invested in the 6 per cent. at $145\frac{1}{4}$ bring him an nual income of ($145\frac{1}{4}$: 166 × 119 :: 6) 816£ *i. e.* (816 − 664), 152£ more. *Ans.*

6. $(a+b+c)\ (b+c-a)\ (c+a-b)\ (a+b-c)$

$=\{(b+c)+a\}\ \{(b+c)-a\}\ \{a-(b-c)\}\ \{a+(b-c)\}$

$=\{(b+c)^2-a^2\}\ \{a^2-(b-c)^2\}$

$=(b^2+2bc+c^2-b^2-c^2)\ \ (b^2+c^2-b^2+2bc-c^2)$

$=2bc\times 2bc=4b^2c^2$ *Q. E. D.*

7. By actual division we have $x^2-ax-2bx+a^2+3b^2$, or

$x^3-4abx-b^2x+a^3+2a^2b+3ab^2+6b^3$

$=x^3+ax^2+2bx^2-ax^2-a^2x-2abx-2bx^2-2abx$
$-4b^2x+a^2x+a^3+2a^2b+3b^2x+3ab^2+6b^3$

$=x^2\ (x+a+2b)\ -ax\ (x+a+2b)-2bx\cdot(x+a+2b)$
$+a^2(x+a+2b)+3b^2(x+a+2b)$

$=(x^2-ax-2bx+a^2+3b^2)\ \ (x+a+2b)$

$=\dfrac{(x^2-ax-2bx+a^2+3b^2)(x+a+2b)}{x+a+2b}$

$=x^2-ax-2bx+a^2+3b^2$ *Ans.*

8. Let $A=Fa$ and $B=Fb$ $\therefore$ $Am=Fam$ and $Bn=Fbn$

$\therefore Am+Bn=F\ (am+bn)$ *Q. E. D.*

$x^4-4x^3-11x^2-50x+16$

$=x^4-7x^3+2x^2+3x^3-21x^2+6x+8x^2-56x+16$

$=x^2(x^2-7x+2)+3x(x^2-7x+2)+8(x^2-7x+2)$

$=(x^2+3x+8)\ (x^2-7x+2)$.....................i

$x^4-12x^3+29x^2+46x-16$

$=x^4-7x^3+2x^2-5x^3+35x^2-10x-8x^2+56x-16$

$=x^2(x^2-7x+2)-5x(x^2-7x+2)-8(x^2-7x+2)$

$=(x^2-5x-8)\ (x^2-7x+2)$............... ii

$\therefore$H. C. F. of i and ii.

$=x^2-7x+2$

9 (a). $\dfrac{(a+b)^3-c^3}{(a+b)^2-c^2}+\dfrac{(b+c)^3-a^3}{(b+c)^2-a^2}+\dfrac{(c+a)^3-b^3}{(c+a)^2-b^2}$

$$=\frac{(a+b)^2+c\,(a+b)+c^2}{(a+b)+c}+\frac{(b+c)^2+a(b+c)+a^2}{(b+c)+a}$$

$$+\frac{(c+a)^2+b\,(c+a)+b^2}{(c+a)+b}$$

$$=a+b+\frac{c^2}{a+b+c}+b+c+\frac{a^2}{a+b+c}+c+a+\frac{b^2}{a+b+c}$$

$$=2a+2b+2c+\frac{a^2+b^2+c^2}{a+b+c}$$

Therefore $2(a+b+c)+\dfrac{a^2+b^2+c^2}{a+b+c}-2(a+b+c)$

$$=\frac{a^2+b^2+c^2}{a+b+c} \qquad \textit{Ans.}$$

9 (b). $\dfrac{a^2}{a^2x-x^3}=\dfrac{x^2+a^2-x^2}{x(a^2-x^2)}=\dfrac{x^2}{x(a^2-x^2)}+\dfrac{a^2-x^2}{x(a^2-x^2)}$

$$=\frac{x}{a^2-x^2}+\frac{1}{x}$$

Similarly $\dfrac{b^2}{b^2x-x^3}=\dfrac{x}{b^2-x^2}+\dfrac{1}{x}$ and $\dfrac{c^2}{c^2x-x^3}=\dfrac{x}{c^2-x^2}+\dfrac{1}{x}$

$$\therefore \frac{a^2}{a^2x-x^3}+\frac{b^2}{c^2x-x^3}+\frac{c^2}{c^2x-x^3}=\frac{x}{a^2-x^2}+\frac{x}{b^2-x^2}+\frac{x}{c^2-x^2}+\frac{3}{x}$$

Q. E. D.

10. $\sqrt{(2n-1)\,(2n-3)\,(2n-5)\,(2n-7)+16}$

$$=\sqrt{(2n-1)\,(2n-7)\,(2n-3)\,(2n-5)+16}$$

$$=\sqrt{(4n^2-16n+7)\,(4n^2-16n+15)+16}$$

$$=\sqrt{(4n^2-16n+11-4)\,(4n^2-16n+11+4)+16}$$

$$\sqrt{(4n^2-16n+11)^2-(4)^2+16}$$

$$\sqrt{(4n^2-16n+11)^2}$$

$4n^2-16n+11.$

11. $\dfrac{(x+12)^3}{x^3}=\dfrac{x+24}{x-12}$

$\therefore (x+12)^3 (x-12)=(x+24) (x^3)$
$= (x^2-144) (x^2+24x+144)=x^4+24x^3$
$= x^4+24x (x^2-144)-(144)^2=x^4+24x^3$
$= x^4+24x^3-144\times 24x-(144)^2=x^4+24x^3$
$= x^4+24x^3-144\times 24x-(144)^2-x^4-24x^3=0$
$= -144\times 24x-(144)^2=0$
$\therefore -144\times 24x=(144)^2$
$\therefore -x=\dfrac{144\times 144}{144\times 24}=6 \therefore x= -6$ *Ans.*

12. Let x miles per hour be the rate of rowing.
and y „
$\therefore x+y$ „